STEAM TURBINES

WITH AN APPENDIX ON

GAS TURBINES

AND THE FUTURE OF

HEAT ENGINES

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AUTHORIZED TRANSLATION FROM THE SECOND ENLARGED AND REVISED GERMAN EDITION

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AUTHOR'S PREFACE TO THE FIRST EDITION.

At the suggestion of friends in practical life, the author presents this work. It was first published in the *Zeitschrift des Vereins deutscher Ingenieure*, 1903, but is here greatly amplified. Thanks to the assistance of many turbine builders, the author is in a position to give a further series of important constructive details that may be generally unknown, and hopes to have approached a step nearer to the goal of creating a reference work on steam turbine construction.

In the present state of steam turbine building, we must give most weight to the discussion of the scientific principles involved in this important type of motor. We engineers of course know that machine building, through widely extended practical experimenting, has solved problems, with the utmost ease, which baffled scientific investigation for years. But this "cut and try method," as engineers ironically term it, is often extremely costly; and one of the most important questions of all technical activity, that of efficiency, should lead us not to underestimate the results of scientific technical work, particularly in such new territory as this.

Time and again it is urged that machine building be placed on an entirely practical experimental basis. Such a beginning would not be impossible, but it would not be economical, and therefore not technical. Industry cannot do without scientific coöperation, not from idealism, but because under certain circumstances it is a "cheapest method" of accomplishing our purpose. As opposed to the above very one-sided argument, we may also call attention to the considerable sacrifices which engineers and machine builders have made in vain, due to incomplete understanding of the scientific principles involved in the problem undertaken. The great majority of those concerned will witness with indifference the economic loss of one of their number conducting a fruitless experi-

ment; but to thoughtful men, such an occurrence, which is, unfortunately, only too frequent, is regarded as a common loss, even without considering the fact that no one would like to be placed in a similar position. Steam turbine building, especially, affords numerous examples of the necessity of combining constructive activity with scientific principles. For instance, we might refer to the importance of previously determining the exact dimensions of a rotating wheel whose periphery reaches almost the velocity of a bullet, in order that the existing stresses in the material do not at any place exceed the allowable limits. Or, again, how disadvantageous it would be, in the case of the horizontally rotating disc wheels that have lately come into use, and whose diameters attain considerable dimensions, to determine, experimentally, after first constructing, how much the discs deflect on account of their own weight, and how much they again straighten out due to their centrifugal force; the latter a point of great importance, if we consider the scraping of the blades in the narrow clearance. What dangers the designer risks when he attempts the building of steam turbines without having exact knowledge of the phenomena of the so-called critical velocity! Finally, we could ask, is it "economical" to even apply for a patent for a certain turbine system in which the greater part of the attainable work is destroyed before the steam has even reached the rotating wheel?

Obviously, on the other hand, we should not expect of the busy engineer in practical life that he should closely follow the complicated details of scientific work. It is advisable also for technical students to acquire fundamental ideas as far as possible before they undertake the consideration of difficult problems. Nevertheless, *everyone concerned may properly be invited as a reader to profit by the results of scientific investigation*, and this purpose, it is hoped, this present little treatise may serve. Constant care has been taken to check the results by experiment, to have them as reliable as possible. It may be mentioned that besides the experiments already published, others have been made on the flow from nozzles into the atmosphere, on the resistances of turbine blades, and also a series of experiments on the critical numbers of revolution of multiple loaded shafts.

Among the things that have been added, it may arouse interest to note, the discovery of the hitherto unknown critical velocity of

the "second degree," the action of the "resonance" of the number of revolutions on the vibrations of the foundation, the deflection of horizontal discs and the action of their centrifugal forces, the exact solution of the question as to the pressure distribution in a flow of an elastic fluid, etc.

The presentation had to be very much abbreviated, and often the development was only indicated, still to the closely interested reader the proof also will probably be everywhere evident.

In order to facilitate the study the book is divided into three parts. In the first, the principles peculiar to turbine are discussed. In the second are found investigations requiring more advanced mathematicai preparation. The third part is greatly amplified and gives a short résumé of the mechanics of heat; for there is no doubt but that a thorough understanding of the energy transformation in a steam turbine can be gained only by having thermodynamic foundation. The abstract theory of unresisted flow must be given up when we deal with actual problems; and in order to accomplish this, no means will suffice but a thorough understanding of entropy, which, with the help of our entropy diagram, will permit of the easy solution of all heat problems. To encourage the practical engineer, to freshen up on the somewhat forgotten principles of thermodynamics, the fundamental laws of this science have been briefly derived for heat motors. To the thoughtful student, the discussions in these chapters may be recommended as an introduction. I have made use of the opportunity to present the second fundamental law of thermodynamics from one of the modern points of view, starting from perpetual motion of the second type. The derivation given by *Planck*, which still shows some obscurity on close inspection, has been replaced, I hope, by a more satisfactory presentation. The more this second fundamental law has been attacked, the stronger has it proved itself, and for this reason we caution inventors not to attempt any violation of this fundamental law of our science.

The conclusion is a short review of the latest suggestions for the work-processes of heat engines, and because of the recent advances in the coal-gas producer it seemed opportune also to discuss the gas turbine.

THE AUTHOR.

ZÜRICH, *August, 1903.*

AUTHOR'S PREFACE TO THE SECOND EDITION.

The second edition differs from the first in the following details:

First, in the interest of readers who are in practical life, an elementary introduction to the theory of steam turbines has been undertaken, which avoids following the changes of heat conditions, nevertheless with the aid of empirical formulæ takes into account a large part of the friction occurrences.

The larger part of the time at my disposal I spent in carrying out a series of experiments on the frictional resistances of turbine wheels in air, from which, with fair accuracy, the resistances in steam may be determined. These experiments have at least partially filled the greatest gap in the theory of the steam turbine, and have furnished a reliable foundation for future analyses of experimental results.

With the permission of *Prof. Mollier* of Dresden, there has been added his excellent diagram of the "heat contents" of steam, exceedingly useful in turbine calculations.

A further addition, that may be useful to designers just at present, is the reports of the comprehensive experiments on the many-stage impulse turbines of *Zölly* and *Rateau*. As the author was given full freedom in conducting his experiments, researches of a scientific character could also be made, and we might say that we know more about the performance of this turbine type under various conditions of running than that of any other system.

The immense strides made by the steam turbine in mechanical engineering enabled the author to present other reports besides those on the *Zölly* turbine, namely, on the new turbines of *Riedler-Stumpf*, *Lindmark*, *Gelpke* and *Schulz*; but the most recent constructions could not be taken up.

The text has been revised thoroughly so far as time would

allow, and considerably simplified. An investigation of the influences which unequal heating exercises upon the stresses of disc wheels has been introduced because it was of special practical importance. A discussion of the marine turbine and its gyroscopic action seemed imperative. The critical number of revolutions of the "second degree" are derived in a simpler manner, and the beautiful investigations of *Dunkerley* on the critical number of revolutions of shafts have been taken up. Turbine design has received special emphasis, and the author is pleased to be able to present a number of working drawings. Special thanks are due to the liberal-minded firms that have risen superior to the prevalent policy of maintaining secrecy of methods and of design.

The steam turbine has also brought about practical results in commercial life, as it gave rise to the establishment of the well-known "community of interest" which was hitherto considered unattainable in the circles of European machine-building. The intense competition to be expected between the various systems permits us to say that the designer, through the force of circumstances, is compelled to strive for the highest degree of practical and scientific perfection, in order to insure every possible success. May also the second edition of this book serve his purposes!

THE AUTHOR.

ZÜRICH, *April, 1904.*

TRANSLATOR'S PREFACE.

In presenting this authorized first English edition of the second German edition of *Prof. Stodola's "Steam Turbines,"* the translator desires to express his thanks to *Prof. J. F. Klein* of Lehigh University, not only for suggesting the translation itself, but for much valuable assistance during its progress. He also gratefully acknowledges the courtesy extended him by *Prof. Stodola* in authorizing the translation itself.

All formulæ in the German edition which are expressed in French units are given in both the French and English units, so that they may be applied to problems by English engineers, at the same time permitting the checking of results published in European engineering journals in the French units. The experiments conducted by the author and European engineers and scientists are left in their original units, but all important results from these experiments have been recalculated for the English system.

The translator has also constructed a large diagram, or graphical table, from which can be read directly all steam properties in English units. This table will be of great value to designers, by saving time in calculating problems relating to the various properties of saturated and superheated steam. The two diagrams accompanying the German edition have also been given for the benefit of those who would compare data expressed in French units with those expressed in English units.

There are works on the Steam Turbine in English, but none in which the subject is so exhaustively treated from a purely scientific standpoint as in *Prof. Stodola's* work, now considered the standard authority on the subject in Europe; and there is no branch of machine design in which a complete knowledge of the theory involved is so necessary as in Turbine Design. For this reason, it is hoped that this volume will prove helpful, not only to designers and engineers, but to students of technical schools, and to those interested generally in turbine development.

THE TRANSLATOR.

Bethlehem, Pa., *September, 1904.*

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NOTATIONS OFTEN USED.

q Heat of the liquid per unit weight.
r Heat of vaporization per unit weight.
x Fraction of steam weight in mixture — Quality of steam.
T Absolute temperature.
T_s Absolute temperature at the curve of constant steam weight.
$\lambda = q + xr = q + r + c_p\,(T - T_s)$ "Heat contents," Total heat.
p Absolute pressures; lbs. per sq. ft. or kg. per sq. m.
v Specific volumes; cu. ft. per lb. or cu. m. per kg.
u { Internal energy per unit weight (in heat units).
Peripheral velocity (in velocity diagram).
$A = \frac{1}{778}$ (English units) or $\frac{1}{424}$ (French units). Mechanical equivalent of heat.
c Absolute steam velocity.
w Relative (also absolute) steam velocity.
Q Quantity of heat.
R Work of friction (in heat units).
Z Loss of kinetic energy (in heat units).
S { Entropy (in heat units).
Shearing force (in strength of materials).
f, F Cross-sections.
G Weight of steam flowing through per second.
M Mass of steam flowing through per second.
$\gamma = \frac{1}{v}$ Specific weight.
$\mu = \frac{\gamma}{g}$ Specific mass.
ζ Coefficient of resistance.
σ_r, σ_t Radial and tangential stresses in a rotating disc.
ω Angular velocity.
ω_k Critical velocity.
m Ratio of elastic elongation to cross-section contraction.
$\nu = \frac{1}{m}$.
E Modulus of elasticity.
ξ Radial expansion of a rotating disc.
J Moment of inertia of area referred to the neutral axis.
θ Mass moment of inertia.
t { Time.
Temperature in F.° or C.°
η Efficiency.
h H "Drop" of heat.
L Mechanical work.
N Power in h.p.

I.

ELEMENTARY THEORY OF THE STEAM TURBINE.

1. THE ADIABATIC CHANGE OF CONDITION OF STEAM.

WHEN steam flows in a non-conducting conduit with negligibly small friction and eddy current losses, it undergoes a so-called reversible adiabatic change of condition. Zeuner proved that if the steam is in a saturated or only slightly moist condition the equation

$$p\ v^k = constant \quad . \ (1)$$

is true.

k is a constant, for saturated initial condition of steam $= 1.135$, for superheated steam $= 1.3$.

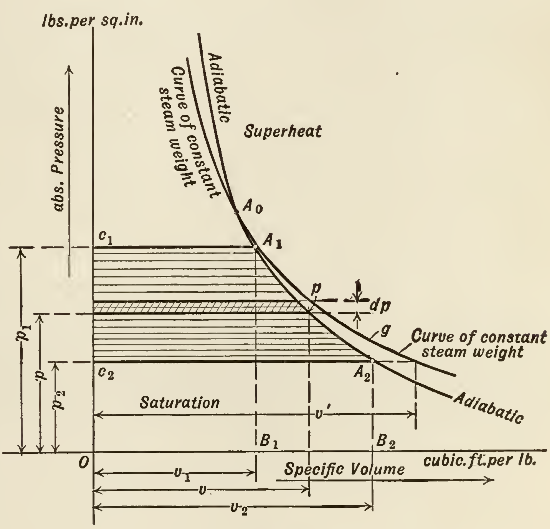

Fig. 1.

In the French units

p = pressure in kilograms per square meter.

v = specific volume, or volume of one kilogram in cubic meters.

In the English units

p = pressure in pounds per square foot.

v = specific volume, or volume of one pound in cubic feet.

A curve drawn with v as abscissa and p as ordinate, will be a hyperbolic curve for adiabatic change of condition, as shown in Fig. 1. This curve will have a slight bend at A_0 in passing from the superheated to the saturated condition. The volume v' of "dry" saturated steam at pressure p is given, according to Zeuner, by the equation

$$p\,(v')^{\mu} = K' \quad . \quad . \quad . \quad . \quad (2)$$

where $\mu = 1.0646.$

In the French units $K' = 1.7617$, when p is expressed in kilograms per square centimeter, and v' in cubic meters per kilogram.

In the English units $K' = 480.2$, when p is given in pounds per square inch, and v' in cubic feet per pound.

The curve g representing equation 2 is

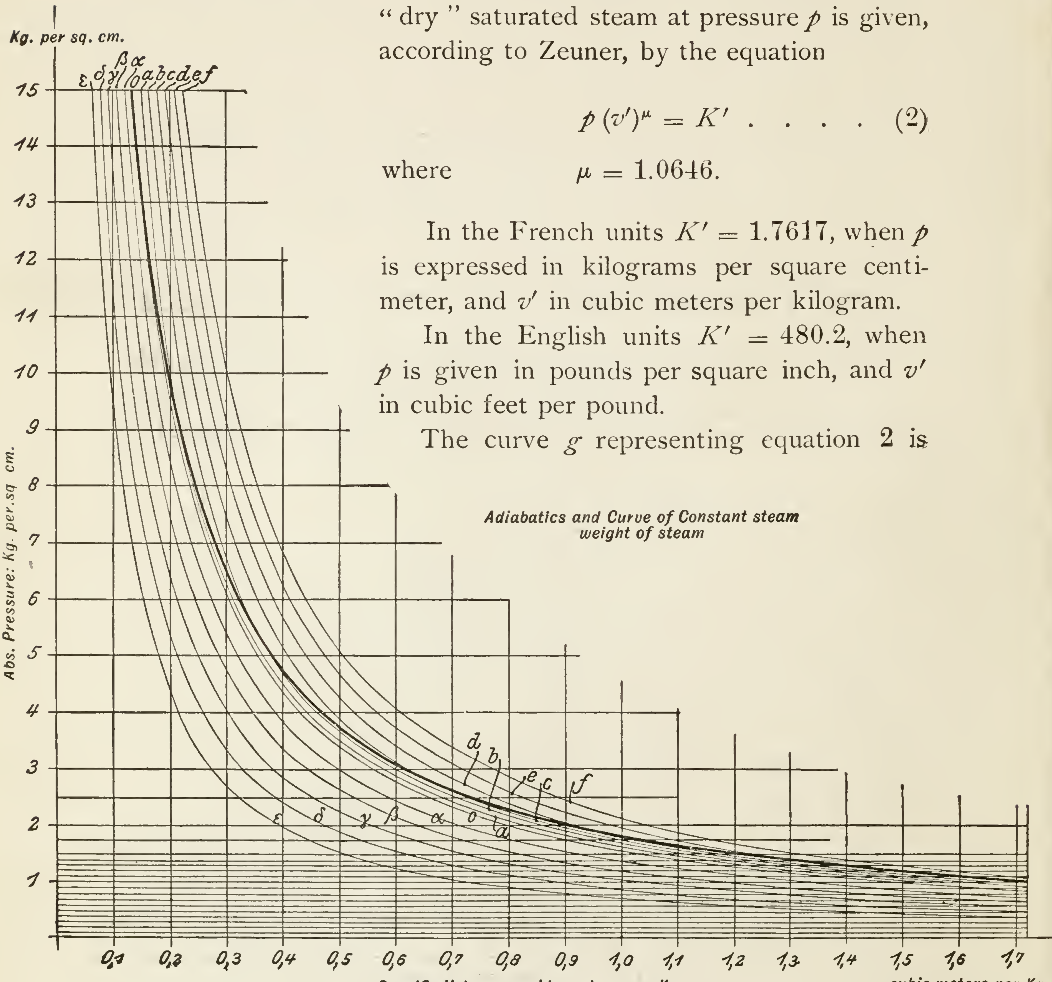

Fig. 2.

called the *limiting curve*, or *curve of constant steam weight.* Below g is found the *moist*, and above it the *superheated* region. In the latter the absolute temperature T is found for pressure p and volume v from the condition equation of *Battelli-Tumlirz:*

$$p\,(v + \alpha) = R\,T \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

where $\alpha = 0.0084,$

and where in the French units $R = 46.7$ when p is in kilograms per square meter, and T the absolute temperature (T = temperature in centigrade degrees + 273).

In the English units $R = 0.591$ when p is in pounds per square inch, and T the absolute temperature (T = temperature in Fahrenheit degrees + 461).

If p is expressed in pounds per square foot, then $R = 85.1$.

In the moist region there exists a mixture of dry steam and water. The volume of one pound of water is σ_0; the increase of volume by complete vaporization at the pressure p is the difference of the values v' and σ_0, and is expressed as σ, that is,

$$\sigma = v' - \sigma_0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

In one pound of the mixture let x be the weight of the steam, and $(1 - x)$ of the water. x is called the *fraction of steam weight in the mixture* or *quality of the steam.*

The volume of one pound of the mixture is then

$$v = x\,v' + (1 - x)\,\sigma_0 = x\,(v' - \sigma_0) + \sigma_0 = x\,\sigma + \sigma_0 \quad . \quad . \quad (5)$$

If on any adiabatic a point A_2 is taken at the pressure p_2 and volume v_2, the quality of steam is found by taking the corresponding volume v' at the pressure p_2 on the curve of constant steam weight and solving for x in equation 5. Generally, $x\,\sigma$ is so large compared to σ_0 that the latter may be neglected, giving the approximate equation

$$v = x\sigma \text{ and } x = \frac{v}{\sigma} \text{ or } \frac{v}{v'} \cdot \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

In Fig. 2 there are drawn a number of adiabatic curves and the curve of constant steam weight, so that with given p and v, x can readily be found.

2. FORMULA OF DE SAINT-VÉNANT.

Consider a frictionless steady flow in the non-conducting channel K, Fig. 3. The particles of steam describe regular paths, called stream lines, by which the elementary channel K' may be supposed to be bounded. At a cross-section A_1 of this channel let the condition of the steam be expressed by the pressure p_1 and volume v_1, at cross-section A_2 by p_2 and v_2. The velocities at these sections are w_1 and w_2 respectively. The relation between these quantities is expressed by the formula first derived by de Saint-Vénant:

$$\frac{w_2^2 - w_1^2}{2g} = \int_{p_2}^{p_1} v\,dp = \text{work represented by area } A_1 A_2 C_2 C_1 \text{ in Fig. 1} \qquad (7)$$

This integral is simply the sum of the infinitely small elements of work, that are absorbed by the increase, or created by the decrease of pressure dp of one pound (or kilogram) of steam; and it is represented in Fig. 1 by the area $A_1A_2C_2C_1$. This area can be found by means of a planimeter and used to calculate w_2. The pressure in any one cross-section of the channel will naturally be greater on the concave side of the channel than on the convex; but if the bend is not too sharp, their difference may be disregarded, and the mean values for the entire cross-sections A_1 and A_2 expressed as $p_1\, v_1\, w_1$ and $p_2\, v_2\, w_2$ respectively.

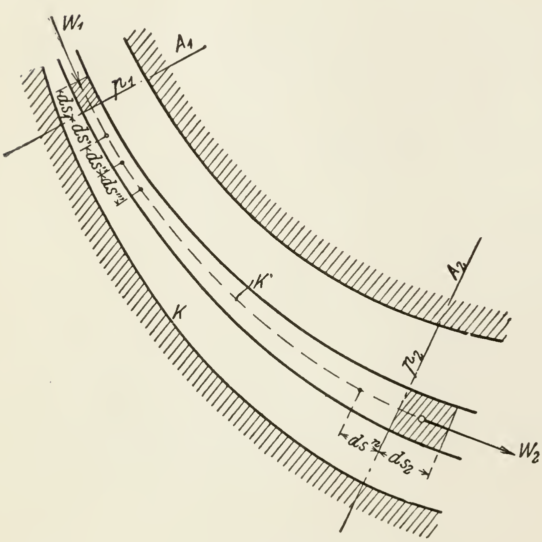

Fig. 3.

The "*Law of Continuity*" says that with steady flow an equal quantity of steam passes any cross-section in a unit of time.

If the weight passing in one second be G_{sc} and the area of the cross-sections be F_1 and F_2, then,

$$G_{sc}v_1 = F_1 w_1, \qquad G_{sc}v_2 = F_2 w_2,$$

from which

$$\frac{F_1 w_1}{v_1} = \frac{F_2 w_2}{v_2}; \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

therefore,

$$w_1 = \frac{F_2 v_1}{F_1 v_2} w_2.$$

Placing this in equation 7 we have

$$\frac{w_2^2}{2g}\left[\left(\frac{F_2 v_1}{F_1 v_2}\right)^2 - 1\right] = \text{Area } A_1 A_2 C_2 C_1.$$

From this equation w_2 can be calculated from the known initial condition of the steam.

DERIVATION OF DE SAINT-VÉNANT'S FORMULA.

Consider the elementary channel K' into which enters at A_1 the elementary weight of steam dG in the elementary time dt, and after flowing through the course $A_1 A_2$, emerges at A_2. To find the increase of velocity, apply to the entering mass element the principle of the conservation of energy. It can be said that the *added external work is equal to the increase of total energy of the particle between the final and the initial conditions.* To consider only the external energy, that is, the kinetic energy, is not sufficient, because we are dealing with an elastic fluid, which, during its motion expands, thus performing internal work.

So far as the *work of external forces* is concerned, weight will always be neglected, as its influence on the steam turbine problem is negligibly small. The work done by the steam pressure on the external surface of the filament of flowing steam considered is zero, for the pressure is at right angles to the line of flow. Only the pressure on the sectional area need be considered. Divide the line of flow into the infinitely small lengths ds_1, ds', ds'', ds''' . . . ds^n, ds_2, in which ds_1 and ds_2 are the length of an

element at the initial and final conditions. Expressing the pressures existing at these cross-sections by p_1, p', p'', p''' . . . and the corresponding areas by f_1, f', f'', f''' . . . the work done on the upper sectional areas is

$$p_1 f_1 ds_1 + p' f' ds' + p'' f'' ds'' + p''' f''' ds''' + \cdots + p^n f^n ds^n.$$

On the lower areas the work done is

$$p' f_1' ds' + p'' f_1'' ds'' + p''' f_1''' ds''' + \cdots + p_2 f_2 ds_2,$$

and is negative because it overcomes the back pressure. In adding the work done, the intermediate quantities disappear, and the sum is

$$dO = p_1 f_1 ds_1 - p_2 f_2 ds_2\,;$$

but since $ds_1 = w_1 dt$ and $ds_2 = w_2 dt$, it follows that

$$dO = (p_1 f_1 w_1 - p_2 f_2 w_2)\, dt \quad . \quad . \quad . \quad . \quad . \quad (9)$$

Let G be the weight of the quantity of steam that flows through the elementary channel per second. Then $dG = G dt$ = weight of the element.

From the law of continuity,

$$G = \frac{f_1 w_1}{v_1} = \frac{f_2 w_2}{v_2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (10)$$

Substituting in equation 9,

$$dO = G\,(p_1 v_1 - p_2 v_2)\, dt \quad . \quad . \quad . \quad . \quad . \quad (11)$$

The *total energy* is composed of kinetic energy and internal energy. The former gains in value

$$dK = \frac{1}{2}\,\delta m\,(w_2^2 - w_1^2) = \frac{1}{2}\,\frac{G}{g}\,dt\,(w_2^2 - w_1^2)\,. \quad . \quad (12)$$

The internal energy of the element is decreased by the equivalent of the work done, which is imparted to its immediate neighborhood during expansion. The volume of the element $G dt$ is

$G\,dt\,v$, and the increase in an infinitely small distance is $G\,dt\,dv$; if the corresponding pressure is p, the work done is $G\,dt\,p\,dv$. The complete work of expansion is, therefore,

$$dE = G\,dt\int_{v_1}^{v_2} p\,dv.$$

In Fig. 1 the integral is represented by the areas $A_1A_2B_2B_1$.

The equation which represents the law of energy may now be written

$$dO = dK - dE.$$

Substituting the values in this equation, and dividing through by $G\,dt$ we have,

$$\frac{w_2^2 - w_1^2}{2g} = p_1 v_1 - p_2 v_2 + \int_{v_1}^{v_2} p\,dv \quad . \quad . \quad . \quad (13)$$

This equation refers to unit weight (one kilogram or one pound) of the flowing steam, and can be easily represented graphically.

In Fig. 1 add to the expansion work, that is area $A_1A_2B_2B_1$, the product p_1v_1 = area $A_1B_1OC_1$ and subtract from the sum the product p_2v_2 = area $A_2B_2OC_2$, then area $A_1A_2C_2C_1$ remains. But this is also represented by the sum of the elements $v\,dp$; therefore equation 13 can be expressed as

$$\frac{w_2^2 - w_1^2}{2g} = \int_{p_2}^{p_1} v\,dp = A_1\,A_2\,C_1\,C_2 \quad . \quad . \quad . \quad (14)$$

in which the direction of integration must be noted, as a negative sign must be used if the limits of the integral are transposed.

From the equation

$$p\,v^k = C \text{ or } p^{\frac{1}{k}}\,v = C^{\frac{1}{k}}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (15)$$

is found the value of the integral in our problem; solving for v, we have

$$\frac{w_2^2 - w_1^2}{2g} = \int_{p_2}^{p_1} v\,dp = \frac{kC^{\frac{1}{k}}}{k-1}\left[p_1^{\frac{k-1}{k}} - p_2^{\frac{k-1}{k}}\right]$$

$$= \frac{k}{k-1} p_1 v_1 \left[1 - \left(\frac{p_2}{p_1}\right)^{\frac{k-1}{k}}\right] \quad . \quad . \quad . \quad . \quad (16)$$

or, multiplying by $C^{\frac{1}{k}}$, substituting for $C^{\frac{1}{k}}$ in the first part of the equation, $p_2^{\frac{1}{k}} v_2$, and in the second part, $p_1^{\frac{1}{k}} v_1$ we have

$$\frac{w_2^2 - w_1^2}{2g} = \frac{k}{k-1}(p_1 v_1 - p_2 v_2) \quad . \quad . \quad . \quad . \quad (16a)$$

Formulæ 16 and 16*a* are the formulæ of *de Saint-Vénant* and *Wantzel* (1839).

In passing from the superheated region to the saturated, the integration must be divided into two parts; in this case the graphical determination of the area $A_1 A_2 C_1 C_2$ will be the simplest. In every case the work area is equal to that representing the work necessary to accelerate one pound (or kilogram) of flowing steam.

3. THE DECREASE OF PRESSURE.

On account of the similarity between the integral expression in equation 7 and the "decrease of head" in hydraulics, we may call

$$\text{Area of work } A_1 A_2 C_2 C_1 = L_0 \quad . \quad . \quad . \quad . \quad (17)$$

the "*drop of head*" between the pressures p_1 and p_2. If the initial velocity w_1 is wholly or approximately negligible (for instance as in flowing out of a very large vessel), then, as in hydraulics, the simple formula

$$w = \sqrt{2gL_0} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (18)$$

is used. Since all losses are being neglected in the above, L_0 may more accurately be called the "theoretical drop or loss of head."

4. THE LAVAL NOZZLE.

If steam (or in general any elastic fluid) flows through a simple orifice from a space of higher into one of lower pressure, the pressure in the orifice will decrease, as will be proved later, to about one-half the initial pressure, and there will occur in the stream after leaving the orifice strong acoustic vibrations. These vibrations cause a loss of efficiency and an effort should be made to avoid them. *De Laval* has accomplished this by adding to the orifice a conically diverging nozzle in which the steam can continuously expand down to the back pressure. The Laval nozzle is simply a pipe of varying cross-section to which *de Saint-Vénant's* formula can be applied. The dimensions of the nozzle for the given initial pressure p_1, the final pressure p_2, and the quantity of steam per second G_{sc} pounds (or kilograms) are best found *graphically* as follows: — We determine the "decrease of head" produced by a drop of pressure from p_1 to any intermediate pressure p_x, and making the allowable assumption that $w_1 = 0$, we find the corresponding velocity

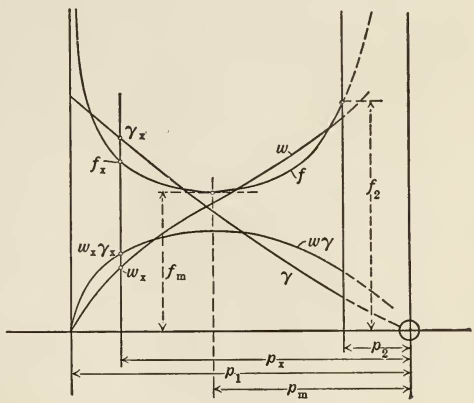

Fig. 4.

$$w_x = \sqrt{2 g L_x}.$$

From the curves of adiabatic expansion, or by computation, the specific volume v_x may be found, and from this the specific weight or density

$$\gamma_x = \frac{1}{v_x}.$$

The equation of continuity then becomes

$$G_{sc} = f_x w_x \gamma_x,$$

whence

$$f_x = \frac{G_{sec}}{w_x \gamma_x} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (19)$$

If we construct the nozzle cross-section f_x as a function of the pressure p_x, Fig. 4 shows that this section has a minimum value f_m for which there is also a corresponding pressure. If $p = p_1$, f_x is infinite, because $w_1 = 0$, *i.e.*, f_x is practically very large, and w_1 very small. The nozzle, Fig. 5, is made very short to the cross

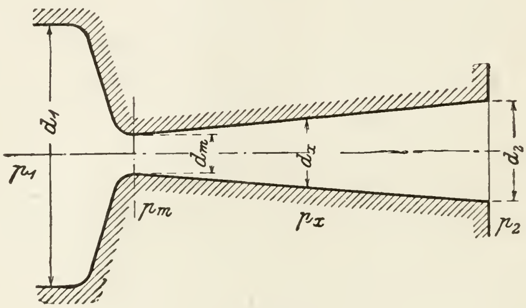

Fig. 5.

section f_m, to avoid frictional losses. From f_m onward the profile remains practically a straight line, with an angle of cone of 10°, as with sharper divergence the steam stream might separate from the wall. The cone is extended until the cross-section f_2 is reached. For any intermediate diameter d_x, the corresponding f_x in Fig. 4 gives, being to the right of f_m, the corresponding pressure p_x.

The nozzle can be easily calculated by Zeuner's method* so long as the steam is superheated or saturated, *i.e.*, so long as k remains constant. Since $w_1 = 0$, formula 16 gives for the pressure p_x the velocity

$$w_x = \sqrt{2g \frac{k}{k-1} p_1 v_1 \left[1 - \left(\frac{p_x}{p_1}\right)^{\frac{k-1}{k}}\right]} \quad . \quad . \quad . \quad (20)$$

The equation of condition gives

* Zeuner's Theorie der Turbinen. Leipsic, 1899, p. 268.

$$v_x = v_1 \left(\frac{p_1}{p_x}\right)^{\frac{1}{k}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (21)$$

and from the law of continuity

$$G_{sc} = \frac{f_x w_x}{v_x} = f_x \sqrt{\frac{2gk}{k-1} \frac{p_1}{v_1} \left[\left(\frac{p_x}{p_1}\right)^{\frac{2}{k}} - \left(\frac{p_x}{p_1}\right)^{\frac{k+1}{k}}\right]} \quad . \quad (22)$$

Determining by the laws of analysis the value of p_x which would make $w_x \div v_x$ a maximum, and therefore f_x a minimum, we get

$$\frac{p_m}{p_1} = \left(\frac{2}{k+1}\right)^{\frac{k}{k-1}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (28)$$

and from this

$$w_m = \sqrt{2g \frac{k}{k+1} p_1 v_1} \quad . \quad . \quad . \quad . \quad . \quad . \quad (29)$$

$$G_{sc} = f_m \sqrt{2g \frac{k}{k+1} \left(\frac{p_m}{p_1}\right) \left(\frac{p_1}{v_1}\right)} \quad . \quad . \quad . \quad . \quad (30)$$

or, for saturated condition with $k = 1.135$, according to Zeuner in French units

$$\left.\begin{aligned} p_m &= 0.5744\, p_1 \\ w_m &= 323 \sqrt{p_1 v_1} \\ G_{sc} &= 199 f_m \sqrt{\frac{p_1}{v_1}} \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (31)$$

in which p is in kilograms per square centimeter, v_1 is in cubic meters per kilogram, f_m is in square meters.
In English units

$$p_m = 0.5744\, p_1,$$

$$w_m = 70.2 \sqrt{p_1 v_1},$$

$$G_{sc} = 43.25 f_m \sqrt{\frac{p_1}{v_1}},$$

in which p is in pounds per square inch, v_1 is in cubic feet per pound, f_m is in square inches.

The final cross-section f_2 is deduced from the equation

$$G_{sc} = \frac{f_2 w_2}{v_2}; \qquad f_2 = \frac{G_{sc} v_2}{w_2};$$

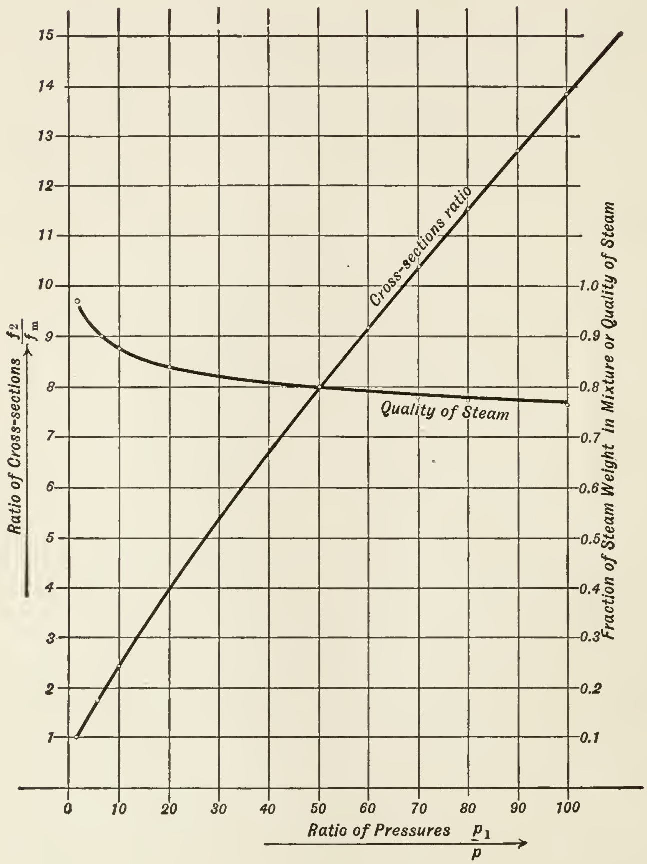

Fig. 6.

or from the relation

$$f_m = \frac{G_{sc} v_m}{w_m},$$

$$\frac{f_2}{f_m} = \frac{v_2}{v_m} \cdot \frac{w_m}{w_2},$$

after w_2 and v_2 have been calculated from equations 20 and 21. *Zeuner* calculates elsewhere the following table:—

$\frac{p_1}{p_2}=$	1.732	2	4	6	8	10	20	50	70	100
$\frac{f_2}{f_m}=$	1	1.015	1.349	1.716	2.069	2.436	3.966	7.980	11.555	13.802

which is graphically represented in Fig. 6. The quality of steam existing at the end of the nozzle is also shown.

The velocity at the narrowest place, w_m, varies only slightly with the initial pressure; for instance, in the French units

for $p_1 = 5$ kilograms per square centimeter, $w_m = 442.4$ meters,
" $p_1 = 12$ kilograms per square centimeter, $w_m = 454.3$ meters.

In the English units

for $p_1 = 71.12$ pounds per square inch, $w_m = 1451.5$ feet,
" $p_1 = 170.68$ pounds per square inch, $w_m = 1490.6$ feet.

If the back pressure p_2 exactly equals p_m, that is

$$\left(\frac{p_2}{p_1}\right) = \left(\frac{p_m}{p_1}\right),$$

then the nozzle can only be laid out as far as the narrowest part. If $p_2 > p_m$, then the pressure p_2 itself exists in the orifice, and w_2 and v_2 are obtained from equations 20 and 21, whence

$$f_2 = \frac{G\,v_2}{w_2},$$

but the nozzle remains *convergent*, or at the very most cylindrical, with a rounded-off entrance.

The complicated phenomena occurring in a too short or too long nozzle for a given ratio $p_1 \div p_2$ will be discussed later.

For the flow through passages of a turbine the derived formulæ are directly applicable, if the curvature be not too great.

5. CLASSIFICATION OF STEAM TURBINES.

There may be distinguished just as many types of steam turbines as of water turbines. The direction of the steam flow

differentiates *Axial* from *Radial Turbines.* In the former the velocity of the steam particles has besides the peripheral component, only a component in the axial direction; in the latter only a peripheral component and a radial component. A more important distinction, however, depends on the pressure existing in the clearance space between the guide and rotating wheels. If this pressure is greater than that at exit from the rotating wheel, we have a *reaction turbine;* if the pressures are equal, an *impulse turbine.* If the channel between the blades of a turbine is not entirely filled by the flowing steam, it may be called a "free jet turbine;" and if the channel be just filled, but without excess pressure, the "limit turbine." If the steam enters around the entire circumference of the rotating wheel it is called *full peripheral admission;* or if only partly, *partial peripheral admission.*

In contrast to hydraulic turbines, there may be in steam turbine design, a combination of two or more successive turbines, which we shall call *few-stage turbines,* or a combination of a large number of successive turbines, which we shall call *many-stage turbines.* Although we cannot distinctly separate these two types, the double nomenclature is necessary, because the turbine with a large number of stages is calculated in a vastly different way from the one with a few stages. In the few-stage turbine the pressure is either utilized in separate stages, or expansion takes place immediately to the final pressure, and the resulting total kinetic energy of the steam is used in several successive turbines. For the latter type *Riedler* has offered the name "*velocity stage*" as distinguished from "*pressure stage.*"

A. AXIAL TURBINES.

6. THE IDEAL SINGLE STAGE IMPULSE TURBINE.

The form of the nozzle is determined by the method derived above, and gives as the absolute velocity of exit, c_1. See Fig. 7. Resolve c_1 into the components w_1 and u, the latter being the peripheral velocity of the wheel, and w_1 the "relative" entrance

velocity at the rotating wheel. w_1 is the resultant of c_1 and the negative velocity $-u$. The direction of w_1 determines the slope α_1 of the first blade element, so that *entrance may be free from shock.* The angle α_1 refers mostly to the back of the blade, as a shock on that side of the blade would be detrimental, and would also cause greater losses than a shock on the front side. With frictionless motion w_1 remains constant in the wheel, and appears as *relative* exit velocity w_2 from the rotating wheel; the resultants of w_2 and u give the *absolute* exit velocity c_2. The angles of c_1, w_1, and w_2 are α, α_1, α_2. Generally, $\alpha_2 = \alpha_1$, whence the cross-sections at entrance to and exit from a rotating wheel will be equal, while α_2 will have a somewhat excessive value. If α_2 is less than α_1 then the passage, keeping constant cross-section, must be widened in a radial direction toward the exit (as in Girard turbines), to avoid eddy currents.

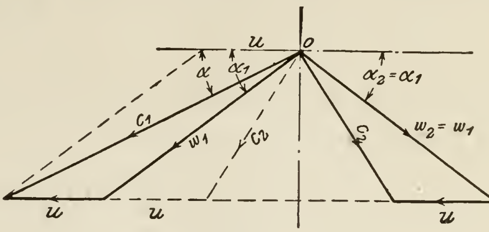

Fig. 7.

The value of the angle α is about 17° to 20°; for α_2 the same value may be used. *De Laval* generally makes $\alpha_1 = \alpha_2 = 30°$.

The capacity for useful work in meter kilograms per kilogram of steam, or in foot-pounds per pound of steam, is

$$L_0 = \frac{c_1^2}{2g}$$

if G_{sc} represents the weight of steam used per second, the useful horse-power in French units is

$$N_0 = \frac{G_{sc} L_0}{75};$$

in English units,

$$N_0 = \frac{G_{sc} L_0}{550}.$$

From L_0 there is lost the work in the steam at exit,

$$L_z = \frac{c_2^2}{2g}.$$

We gain the so-called *indicated steam work* per unit weight of steam,

$$L_i = L_0 - L_z = \frac{c_1^2 - c_2^2}{2g}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

From this the indicated work per second in horse-power is, in French units,

$$N_i = \frac{G_{sc} L_i}{75} \text{ horse-power}; \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

in English units,

$$N_i = \frac{G_{sc} L_i}{550} \text{ horse-power.}$$

The steam consumption per hour per indicated horse-power $= D_i = \dfrac{3600\ G_{sc}}{N_i}$, or, using formula 3, in French units,

$$D_i = \frac{270\,000}{L_i};$$

or in English units,

$$D_i = \frac{1\,980\,000}{L_i}$$

The indicated efficiency is

$$\eta_i = \frac{L_i}{L_0} = \frac{c_1^2 - c_2^2}{c_1^2}.$$

If $\alpha_1 = \alpha_2$, we find by swinging w_2 around the vertical, as in Fig. 7,

$$c_2^2 = c_1^2 + (2\,u)^2 - 2\,c_1\,(2\,u)\cos\alpha,$$

whence,

$$\eta_i = 4\frac{u}{c_1}\left[\cos\alpha - \frac{u}{c_1}\right].$$

If α is *determined, then* η_i *depends only on the ratio* $u \div c_1$, so long as we assume that α_1 is constantly changed, so that the steam always enters without shock. As u increases, η_i first increases to the maximum value

$$\eta_i = \cos^2 \alpha,$$

which is obtained when $\frac{u}{c_1} = \frac{1}{2} \cos \alpha$. Then η_i decreases until $u = c_1 \cos \alpha$, when it becomes 0. As a function of $\frac{u}{c_1}$, η_i will be represented by a parabola.

If, for instance, $\alpha = 17°$, then $\eta_{max} = 0.914$ when $\frac{u}{c_1} = 0.478$. If $c_1 = 1200$ meters (3937 ft.), then $u = 574$ meters (1883.3 ft.), which is impracticable. If u be limited to the experimentally practicable value of 400 meters (1312.3 ft.), that is $\frac{u}{c_1} = 0.333$, then $\eta_i = 0.836$, that is about 8.5% less than in the previous case. However, since the frictional work of the wheel running with no load decreases with decrease of peripheral speed, part of the above loss is regained, and there is no serious loss of economy if we go below the theoretically proper value of $\frac{u}{c_1}$.

7. THE SINGLE STAGE IMPULSE TURBINE, CONSIDERING FRICTION.

The friction in a nozzle has the effect of decreasing the exit velocity to the value

$$c_1 = \phi c_0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

where c_0 denotes the theoretical value

$$c_0 = \sqrt{2\, g L_0} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

The coefficient ϕ can be taken, in long nozzles with condensa-

tion, at 0.95 to 0.90; in short nozzles with free exhaust, at 0.95 to 0.975.* Combining c_1 with $-u$ again gives w_1, but this is de-

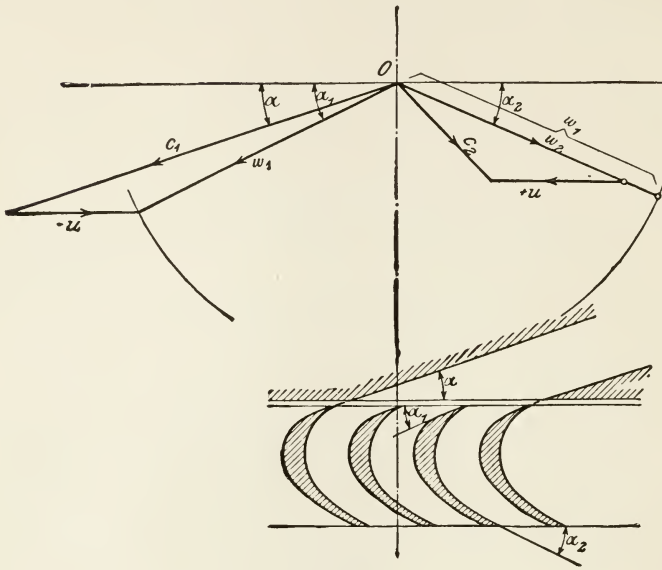

Fig. 8.

creased by friction and eddy currents during exhaust to the smaller value

$$w_2 = \psi w_1 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

in which ψ depends on the velocity w_1, on the form of blades and on other factors. It appears that the smallest value of ψ is 0.7; with smaller values of w_1, ψ would increase, and with $w_1 = 250$ meters (820 feet) might approximately be estimated at from 0.85 to 0.9. Finally, w_2 and $+u$ give the velocity of exit c_2. (Fig. 8.)

These losses by friction expressed as loss of work are,

in the nozzle $\quad \dfrac{c_0^2 - c_1^2}{2g} = (1 - \phi^2)\dfrac{c_0^2}{2g};$

in the blade channel $\quad \dfrac{w_1^2 - w_2^2}{2g} = (1 - \psi^2)\dfrac{w_1^2}{2g}.$

* See Article 16.

The "indicated" work per pound (or kilogram) of steam is

$$L_i = \frac{c_0^2}{2g} - \frac{c_0^2 - c_1^2}{2g} - \frac{w_1^2 - w'^2_2}{2g} - \frac{c_2^2}{2g} \quad . \quad . \quad . \quad (4)$$

The "indicated" efficiency is

$$\eta_i = \frac{L_i}{L_0} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

The "indicated" power in h.p. is in French units,

$$N_i = \frac{G_{sc} L_i}{75}; \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

in English units,

$$N_i = \frac{G_{sc} L_i}{550}.$$

Deducting from N_i the wheel and bearing friction,* we get the effective power at the turbine shaft

$$N_e = N_i - N_r,$$

and the effective efficiency

$$\eta_e = \frac{N_e}{N_0}.$$

7a. DETERMINATION OF THE CROSS-SECTION DIMENSIONS FOR A SINGLE STAGE IMPULSE TURBINE.

Assume as given the power, steam pressure, and vacuum. Assume the peripheral velocity as nearly equal to the most favorable velocity as possible. From the number of revolutions, which depends upon numerous conditions, such as method of propulsion, the accuracy of manufacture, etc., the wheel radius is found. The wheel and bearing frictions N_r can be estimated from the formulæ given in Articles 33 and 53, so that

$$N_i = N_e + N_r$$

* See Articles 33 and 53.

is also given. From the velocity diagram find the "indicated" work L_i per unit weight (kilogram or pound) of steam, and then from equation 6 of the previous article,
in French units,

$$G_{sc} = \frac{75\,N_i}{L_i},$$

or in English units,

$$G_{sc} = \frac{550\,N_i}{L_i}.$$

Divide G_{sc} among an appropriately large number of nozzles, which can be derived as previously explained.

The length of the blades should be such that the jet, even at the widest places (for instance with round nozzles), can enter the wheel without obstruction. At entrance the blades are ground down to nearly a sharp edge; beyond that the blades are of the ordinary constant thickness.

8. THE SINGLE STAGE REACTION TURBINE.

With definite initial and final pressures p_1 and p_2 respectively, there exists in the clearance space between the guide and rotating wheels an intermediate pressure p, which can be chosen at will. From the adiabatic curve find the specific volume v_1, v, v_2 corresponding to the pressure p_1, p, p_2.

The value of p is theoretically, so far as economy is concerned, unimportant, but practically it materially influences on the one hand losses due to leakage, and on the other hand, the peripheral velocity.

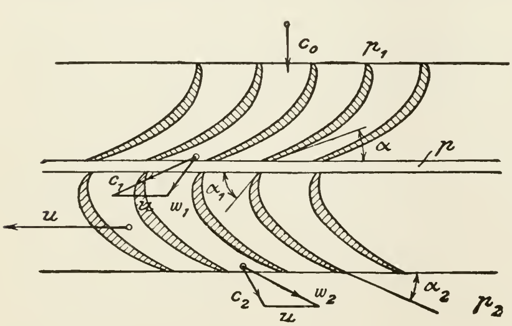

Fig. 9.

If with saturated steam the ratio $\frac{p_1}{p}$ or $\frac{p}{p_2}$ exceeds the limiting value 1.7, the turbine channel, like the conical nozzle,

must be contracted to a minimum cross-section, and widened again from there on.

The reaction turbine generally exists only as a many stage turbine, in which this contraction is unnecessary.

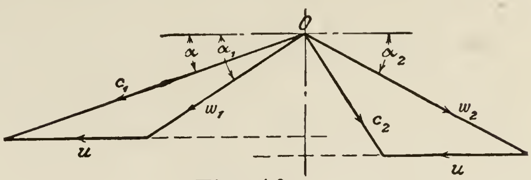

Fig. 10.

Divide the work area according to the pressure p into two parts, L_1 and L_2; then, with the notation in Fig. 9 we have for the guide wheel the equation

$$\frac{c_1^2 - c_0^2}{2g} = L_1 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

where, strictly taken c_0 should be charged to the work L_1, hence c_0 should be calculated as 0.

From c_1 and $-u$ we obtained w_1 (see Fig. 10), which is accelerated in the rotating wheel to w_2 according to the equation

$$\frac{w_2^2 - w_1^2}{2g} = L_2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

and

$$L_0 = L_1 + L_2.$$

The ratio $\frac{L_2}{L_0}$ is called the *degree of reaction.*

The resultant of w_2 and $+u$ gives the velocity of exit c_2, Fig. 10.

The loss of work in a frictionless turbine is $\frac{c_2^2}{2g}$. The theoretically useful work is

$$L_i = L_1 + L_2 - \frac{c_2^2}{2g} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

The indicated horse-power is, in the French units,

$$N_i = \frac{G_{sec} L_i}{75}; \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

in English units,

$$N_i = \frac{G_{sc} L_i}{550};$$

and the indicated efficiency is

$$\eta_i = \frac{L_i}{L_1 + L_2} = \frac{L_i}{L_0} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

To determine how the efficiency varies with peripheral velocity, assume that the axial components of the velocities c_1, c_2, w_1, w_2 are equal and $= c_0$, and also that

$$\alpha = \alpha_2, \quad \alpha_1 = \alpha_2.$$

According to the principle of impulsion explained later on, the indicated power, with the notation of Fig. 11, is

$$L_i = \frac{1}{g}(c_1' + c_2')\, u = (2 c_1 \cos \alpha - u) \frac{u}{g} \quad . \quad . \quad . \quad (6)$$

and the indicated efficiency is

$$\eta_i = \frac{1}{g L_0}(2 c_1 \cos \alpha - u)\, u \quad . \quad . \quad . \quad . \quad . \quad (7)$$

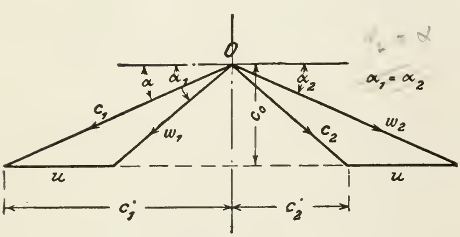

Fig. 11.

Here, too, if it be assumed that the value of α_1 is always such that there is no shock at entrance, and that the degree of reaction is maintained constant, the efficiency as well as the indicated power will increase with the peripheral velocity as the ordinates of a parabola. Both reach their maximum value when

$$u = c_1 \cos \alpha \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

8a. DETERMINATION OF THE CROSS-SECTIONS FOR A SINGLE STAGE REACTION TURBINE.

The indicated power N_i must be estimated from the effective power as in the impulse turbine, and then from equation 4 the weight of steam per second G_{sc} can be calculated.

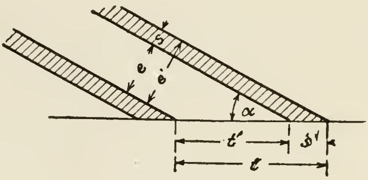
Fig. 12.

With infinitely thin blades the cross-section at exit of a guide wheel with full peripheral admission will be, calling D the mean diameter, a the radial length of the blade, and α the blade angle,

$$F = \pi D a \sin \alpha \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (9)$$

If, with finite thickness of blade, e is the width of the channel, e' the distance between similar blade surfaces at exit, Fig. 12, then $e' - e = s$ the blade thickness, then

$$F = \frac{e}{e'} \pi D a \sin \alpha \quad . \quad . \quad . \quad . \quad . \quad . \quad (10)$$

Similarly, for entrance to and exit from the rotating wheel,

$$F_1 = \frac{e_1}{e_1'} \pi D_1 a_1 \sin \alpha_1 \quad . \quad . \quad . \quad . \quad . \quad (11)$$

$$F_2 = \frac{e_2}{e_2'} \pi D_2 a_2 \sin \alpha_2 \quad . \quad . \quad . \quad . \quad . \quad (12)$$

The increase of the specific volume of steam causes here a considerably greater change of cross-sections than with hydraulic turbines.

From the law of continuity follows the threefold equation

$$G_{sc} = \frac{F c_1}{v} = \frac{F_1 w_1}{v} = \frac{F_2 w_2}{v_2},$$

and these formulæ serve for the calculation of F, F_1, F_2.

For instance, supposing $\alpha = \alpha_2$, $D = D_2$, $e = e_2$, $e' = e_2'$, and $c_1 = w_2$, we get

$$\frac{F_2}{F} = \frac{v_2}{v} = \frac{a_2}{a} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (13)$$

This ratio may become considerable in the low pressure wheels of reaction turbines, in which for instance, expansion occurs from 0.3 to 0.2, or 0.2 to 0.15 atmospheres (4.4 to 3, or 3 to 2.2 pounds per square inch absolute), and the volumes increase approximately in the inverse ratio. If, for constructive reasons, so large a difference of radial lengths is impracticable, because perhaps a must equal a_2, under the assumptions made, and because

$$\frac{F_2}{F} = \frac{a_2 \sin \alpha_2}{a \sin \alpha} = \frac{\sin \alpha_2}{\sin \alpha} = \frac{v_2}{v} \quad . \quad . \quad . \quad . \quad . \quad . \quad (14)$$

the continuity must be satisfied by the angles. α_2 can therefore be given a very much larger value than α, which changes the velocity diagram. The values w_1 and w_2 remain the same, but c_2, that is, the entrance loss, considerably increases.

The calculation is simplified by using the so-called axial clear cross-section, and the axial component of the velocity. If the latter be designated by adding the subscript n as c_{1n}, w_{1n}, w_{2n} and c_{2n}, and if we assume a turbine with infinitely thin blades, then the law of continuity gives

$$G_{sc} = \frac{\pi D_1 a' \sin \alpha c_1}{v} = \frac{\pi D_1 a_1' \sin \alpha_1 w_1}{v} = \frac{\pi D_2 a_2' \sin \alpha_2 w_2}{v_2};$$

but here

$$c_1 \sin \alpha = c_n \,; \quad w_1 \sin \alpha_1 = w_{1n} \,; \quad w_2 \sin \alpha_2 = w_{2n} \quad . \quad (15)$$

We understand by axial clear cross-sections the values

$$F_n = \pi D a' \,; \quad F_{1n} = \pi D_1 a_1' \,; \quad F_{2n} = \pi D_2 a_2' \quad . \quad . \quad (16)$$

Hence,

$$G_{sc} = \frac{F_n c_{1n}}{v} = \frac{F_{1n} w_{1n}}{v} = \frac{F_{2n} w_{2n}}{v_2} \quad . \quad . \quad . \quad (17)$$

that is, *the law of continuity also applies directly for the axial velocities and cross-sections.*

After having found the ideal blade lengths a', a_1', a_2' from equation 17, the effective lengths are found from the equations

$$a = \frac{e'}{e} a'; \qquad a_1 = \frac{e_1'}{e_1} a_1'; \qquad a_2 = \frac{e_2'}{e_2} a_2' \quad . \quad . \quad (18)$$

9. DETERMINATION OF THE POWER AND THE EFFICIENCY BY MEANS OF THE IMPULSE PRINCIPLE.

Since, in the axial turbine, as has been assumed, the direction of the flow of steam is such that there exists nowhere a radial velocity, the power given to the wheel originates in the change of the peripheral component c_u of the absolute velocity, as can be seen from the following:

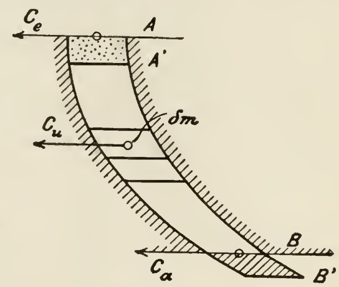

Fig. 13.

Divide the contents of a channel between blades, by parallel planes at right angles to the axis, into a number of infinitely small parts, and denote the mass of one of these by δm (Fig. 13). Call the peripheral component of the pressure in the channel working on this element δP, and apply to the acceleration (or retardation) of the same the fundamental equation of mechanics, that is, the Formula

$$\delta m \frac{dc_u}{dt} = \delta P \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

or

$$\delta m \cdot \delta c_u = \delta P \cdot dt \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

Adding all the similar expressions for the contents of the entire channel, dt may be taken out as a common factor, and $\Sigma dP = P =$ the total peripheral pressure in the channel, that is, the negative of the peripheral driving force

$$P\,dt = \Sigma\,\delta m\,dc_u \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (3)$$

Let c_u' be the value of c_u at the end of time dt, so that

$$dc_u = c_u' - c_u,$$

and equation 2 becomes

$$P\,dt = \Sigma\,\delta m\,c_u' - \Sigma\,\delta m\,c_u \quad . \quad . \quad . \quad . \quad . \quad . \qquad (4)$$

On the left in this equation is the so-called "impulse" of the force P during the time dt; on the right the value of the increase of momentum of the channel contents during this dt time. During the time dt these contents have moved from AB to $A'B'$ (Fig. 13). The momentum of the mass between A' and B is unchanged, the element BB' shows an increase equal to $dm\,c_a$, where c_a is the value of c_u at *exit*. The vanished element AA' shows a decrease equal to $dm\,c_e$, where c_e is the value of c_u at *entrance*. Consequently,

$$\Sigma\,\delta m\,c_u' - \Sigma\,\delta m\,c_u = dm\,(c_a - c_e) \quad . \quad . \quad . \quad . \qquad (5)$$

If M is the mass of steam flowing through in unit time, then

$$dm = M\,dt \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (6)$$

which, substituted in equation 5, finally gives

$$P = M\,(c_a - c_e) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (7)$$

Since this formula holds for one channel, it can be extended to hold for all, so that in equation 7 the letters may be given the following significance:

P = total peripheral force.
M = mass of steam flowing per second.
c_e and c_a = the peripheral components of the *absolute velocities* at entrance to, and exit from, the rotating wheel.

If c_a and c_e are opposite (which is ordinarily the case), the quantity in the bracket is the sum of the absolute velocities, that is

$$P = M([c_a] + [c_e]) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7a)$$

The work per second (in foot pounds or meter kilograms) is the product of the peripheral force and the peripheral velocity

$$Pu = M(c_a - c_e)\,u \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

If we consider the effect on one pound or one kilogram, then

$$M = \frac{1}{g},$$

and Pu equals the indicated work per pound or per kilogram, that is

$$L_i = \frac{1}{g}(c_a - c_e)\,u \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (9)$$

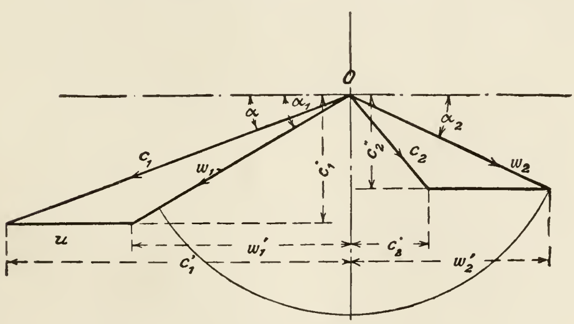

Fig. 14.

which expression, for instance, with the single stage impulse turbine, gives the identical values with equation 4, Article 7.

The proof that this is the case is easily obtained by the following transpositions: According to equation 4, Article 7,

$$L_i = \frac{1}{2g}[(c_1^2 - w_1^2) + (w_2^2 - c_2^2)].$$

Denote the horizontal projections of these velocities by c_1', w_1', w_2', c_2', and the vertical projections by c_1'', c_2''. We then have from Fig. 14,

$$c_1^2 = c_1'^2 + c_1''^2; \qquad w_1^2 = w_1'^2 + c_1''^2,$$

then

$$c_1^2 - w_1^2 = c_1'^2 - w_1'^2,$$

also

$$w_2^2 - c_2^2 = w_2'^2 - c_2'^2,$$

and

$$L_i = \frac{1}{2g}[(c_1'^2 - w_1'^2) + (w_2'^2 - c_2'^2)]$$

$$= \frac{1}{2g}[(c_1' + w_1')(c_1' - w_1') + (w_2' + c_2')(w_2' - c_2')].$$

In this

$$c_1' - w_1' = u; \qquad w_2' - c_2' = u,$$

$$c_1' + w_1' = 2c_1' - u; \qquad w_2' + c_2' = 2c_2' + u,$$

then

$$L_i = \frac{1}{g}[c_1' + c_2']\, u \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (10)$$

agreeing with equation 7*a*.

Mechanics prove that in general the driving turning moment $\mathfrak{M}$ per pound or kilogram of steam is

$$\mathfrak{M} = (a_a c_a' - a_e c_e')\frac{1}{g},$$

in which c_a' and c_e' are algebraically the peripheral components of the absolute exit and entrance velocities; and a_a and a_e their moment arm relative to the shaft. If ω is the angular velocity of the shaft, the formula for the work is

$$L_i = \mathfrak{M}\omega = (a_a c_a' - a_e c_e')\frac{\omega}{g}.$$

The available work per pound or kilogram we have called L_0, then the indicated efficiency is

$$\eta_i = \frac{L_i}{L_0} = \frac{\omega}{g L_0}(a_a c_a' - a_e c_e'),$$

which formula can be advantageously used with radial turbines.

10. THE FEW STAGE IMPULSE TURBINE.

This consists of a succession of impulse turbines. We shall investigate the following case:

a. — ONE PRESSURE STAGE, MULTIPLE VELOCITY STAGES (Fig. 15).

The jet expands in the nozzle to the back pressure, and attains the velocity c_1, which with $-u$ gives the relative velocity w_1. For the ideal turbine $w_2 = w_1$, and it is assumed that $\alpha_2 = \alpha_1$. From w_2 and $+u$ the absolute velocity c_2 is obtained. With this velocity the steam enters a second set of guides and is turned in the direction of the velocity c_1'; so that (theoretically), since the pressure remains constant, c_1' must equal c_2. The angle α_0' which c_2 makes with the circumference of the wheel, is also used as the angle which c_1' makes. The velocities w_1', w_2', c_2' can be applied to the second rotating wheel, and c_2' is changed to c_1'' in a third guide wheel, and then to w_1'' and w_2'' in a third rotating wheel, giving the final exit velocity c_2''. The angle of slope of w_1', w_2' and c_2', c_1'', respectively, and w_1'', w_2'' are alternately equal. The velocity diagram is shown in Fig. 16, and revolving the velocities as shown on the right side of the vertical line around this vertical

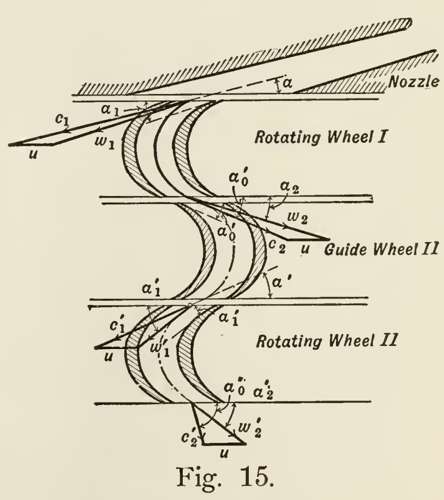

Fig. 15.

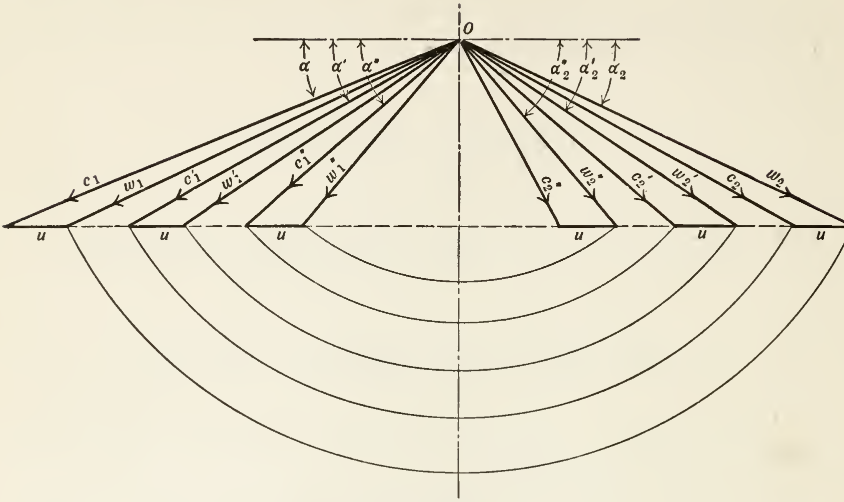

Fig. 16.

as an axis gives the form shown in Fig. 17. The minimum possible value of c_2'' would be c; in this case the peripheral velocity would be

$$u = \frac{c_1 \cos \alpha}{6}.$$

By dividing the velocity into a number of stages, it is possible to reduce the peripheral velocity very considerably.

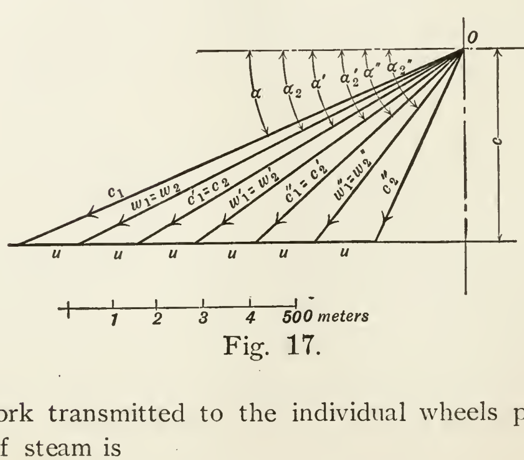

Fig. 17.

The work transmitted to the individual wheels per pound or kilogram of steam is

$$\frac{c_1^2 - c_2^2}{2g}, \quad \frac{c_1'^2 - c_2'^2}{2g}, \quad \frac{c_1''^2 - c_2''^2}{2g} \quad . \quad . \quad . \quad (1)$$

or, $\frac{1}{g}(c_1 \cos \alpha - u)\, u, \ \frac{1}{g}(c_1' \cos \alpha' - u)\, u, \ \frac{1}{g}(c_1'' \cos \alpha'' - u)\, u,$

from which it is evident that these amounts of work decrease rapidly.

If *friction* be considered, then

$$c_1 = \phi\, c_0 = \phi \sqrt{2 g L_0},$$

and $w_2 = \psi w_1$; in the second guide ring $c_1' = \phi' c_2$ and in the second rotating wheel $w_2' = \psi w_1'$; similarly $c_1'' = \phi'' c_2'$ and w_2''

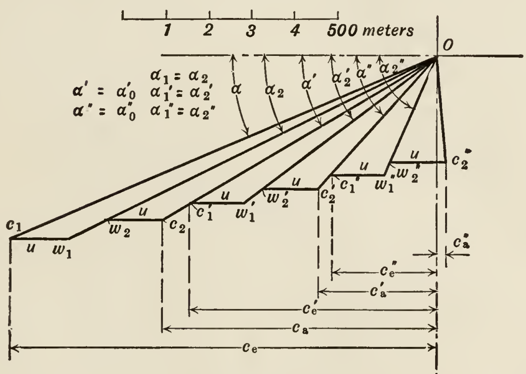

Fig. 18.

$= \psi'' w_1''$, (See Fig. 18), in which the coefficients ϕ, ψ; $\phi'\ \psi'$; $\phi''\ \psi''$ can decrease from 0.7 to 0.85 — 0.9, as mentioned on page 18. The total loss per pound or kilogram of steam is now

$$L_z = \frac{1}{2g}\left[(c_1^2 - c_0^2) + (w_1^2 - w_2^2) + (c_2^2 - c_1'^2) + (w_1'^2 - w_2'^2) + (c_2'^2 - c_1''^2) + (w_1''^2 - w_2''^2) + c_2''^2\right] \quad . \quad . \quad . \quad . \quad . \quad (2)$$

and the indicated power

$$L_i = L_0 - L_z \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

The power can easily be determined by help of Formula 9, Article 9, after measuring for each rotating wheel the peripheral

component of the absolute entrance and exit velocities and denoting them, as in Fig. 18, by c_e, c_a; c_e', c_a'; and c_e'', c_a''. As c_e and c_a are opposite in direction, we have for the first rotating wheel

$$L_i' = \frac{1}{g}(c_a + c_e)\,u,$$

and for all three

$$L_i = \frac{1}{g}[c_a + c_e + c_a' + c_e' + c_a'' + c_e'']\,u \quad . \quad . \quad (4)$$

In Fig. 18 the assumption was made that the entrance and exit angles in the rotating and guide wheels are equal. For the

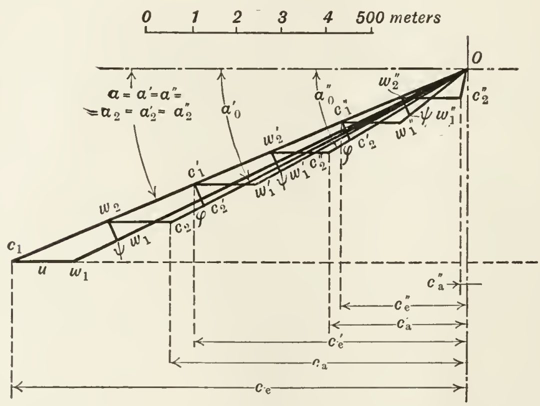

Fig. 19.

latter wheels these angles become too great. To correct this, Fig. 19 was constructed with the assumption that the exit angles for all wheels have the same value, $= \alpha$, while the entrance angles are determined by the velocities w_1, w_1', w_1'' and c_2, c_2', respectively. The expectation that the power is largely increased by the much smaller angle is not fulfilled, because the increase of efficiency according to the correction in Fig. 19 results only to about 5%. In both cases it is questionable whether the gain of power in the last rotating wheel is not absorbed by the increased work of fan resistance.

DETERMINATION OF CROSS-SECTION.

The determination of cross-section follows from the assumption, that in the wheels the specific volume v remains constant. The nozzle width determines the length of the first rotating blades. For the remainder the law of continuity holds, and the axial cross-section had best be constructed as if there were full peripheral admission. If we omit the constant v, and the diameter which is the same for all wheels, then Formula 17, Article 8*a*, becomes

$$a_1 w_{1n} = a_2 w_{2n} = a_0' c_{2n}' = a' c_{1n}' = a_1' w_{1n}' = a_2' w_{2n}' = a_0'' c_{2n}'$$
$$= a'' c_{1n}'' = a_1'' w_{1n}'' = a_2'' w_{2n}''.$$

In the above a_0 and a refer to the entrance to and exit from the guide wheels, and a_1 and a_2 to the same for the rotating wheels, and all are supposed to be the ideal length for infinitely thin blades. Now

$$c_{1n} = w_{1n}, \qquad c_{1n}' = w_{1n}' \cdots$$
$$w_{2n} = c_{2n}, \qquad w_{2n}' = c_{2n}' \cdots$$

whence,

$$a' = a_1', \qquad a'' = a_1'',$$
$$a_0' = a_2, \qquad a_0'' = a_2'.$$

The great decrease in axial velocities due to friction necessitates a correspondingly wider channel. This is especially true of the velocity diagrams of Fig. 19.

b. — FEW PRESSURE STAGES, EACH WITH A VELOCITY STAGE.

The steam leaving the first rotating wheel is led by the guide ring to the next rotating wheel in which it further expands. According to the kind of peripheral admission (partial or full), and according to the direction of the steam path, the exit velocity from the first rotating wheel is transposed into eddy currents, that is, the velocity is destroyed, but is made useful for the second guide wheel.

a. — ASSUME, THAT THE EXIT VELOCITY FOR THE TIME IS ENTIRELY LOST.

We shall neglect the change in thermal condition of the steam which is caused by the loss of velocity c_2. We may select the pressure steps arbitrarily; for the ideal turbine we shall preferably so select these steps that each wheel may perform an equal amount of work. To accomplish this, divide the area of work done (Fig. 20) into as many parts as there are to be steps. For

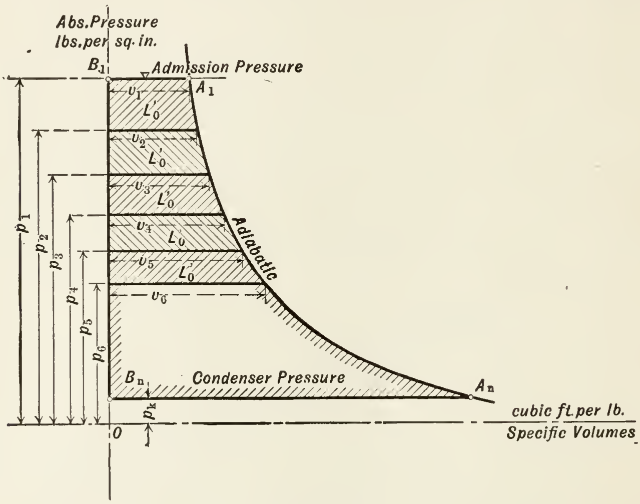

Fig. 20.

each wheel the initial and final pressures are now determined. From these with the peripheral velocity, which may be chosen at will, all dimensions may be calculated as with the simple impulse turbine. Practically we must investigate whether or not the total work of friction could be so decreased by varying the pressure stages as to increase the effective work, even with a resulting lower indicated efficiency. This problem will be discussed later.

Assume that with a single stage turbine having a peripheral velocity u, we attain an indicated efficiency η_i, and let L_0 be the total available work, that is, the contents of the area $A_1 A_n B_n B_1$ in Fig. 20. The theoretical steam velocity at exit from the nozzle, with one stage of expansion, is as before

$$c_1 = \sqrt{2 g L_0} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

If now L_0 be divided into z equal parts, then for a single wheel the available work is

$$L_0' = \frac{L_0}{z},$$

and the corresponding velocity is

$$c_1 = \sqrt{2gL_0'} = \sqrt{2g\frac{L_0}{z}} = \frac{c_1}{\sqrt{z}} \quad . \quad . \quad . \quad . \quad (5a)$$

that is, *the corresponding velocities are inversely proportional to the square root of the number of stages.*

If the same efficiency is desired for each separate wheel of a many stage turbine as for a single stage expansion, then the peripheral velocity should decrease in the same ratio, that is,

$$u' = \frac{u}{\sqrt{z}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

The loss of work of the single stage turbine was theoretically

$$L_z = \frac{c_2^2}{2g} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

The same for single turbines with z stages is

$$L'_{z1} = \frac{c_2'^2}{2g} = \frac{1}{2g}\left(\frac{c_2^2}{z}\right). \quad . \quad . \quad . \quad . \quad . \quad . \quad (7a)$$

and the *total loss* is z times as great, that is,

$$L_z' = \frac{c_2^2}{2g} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7b)$$

therefore *identical with L_z.* If we take into account the change of heat consumption and the friction of the separate wheels, the rela-

tions are considerably changed. The specific volumes shown in Fig. 20 serve for the calculation of the cross-sections for the individual stages of an ideal turbine. It is well to assume at first full peripheral admission. If we then desire, for instance, to double the length of the too short blades of any certain wheel, we can do so by choosing one-half peripheral admission, and so on.

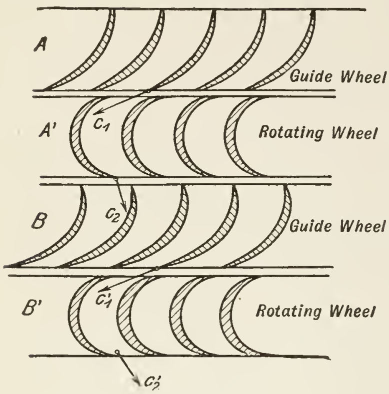

Fig. 21.

β. — *Assuming*, that guide and rotating wheels follow on one another closely, *so that the entire exit velocity* c_2 from a rotating wheel A' *can all be usefully employed in the next guide wheel* B, Fig. 21. The number and divisions of pressure stages are the same as before. For the calculation of the velocity c_1' at exit from the guide wheel B the equation

$$\frac{c_1'^2 - c_2^2}{2g} = L_0' = \frac{L_0}{z} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (8)$$

still holds good; but it will be

$$c_1' = \sqrt{c_2^2 + 2g\frac{L_0}{z}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (9)$$

greater than in the previous case. From c_1' and u we get w_1', w_2' and c_2' which can be used as before for the calculation of the next following c_1'', and so on for the remaining wheels. This method is inconvenient and the result is more quickly reached if *for all turbines the same velocities*, c_1, w_1, w_2, c_2, *and* u *are given.* Here for the first guide wheel a somewhat larger drop of pressure is necessary in order that the steam can be immediately accelerated to the desired velocity. Fig. 7 can be used as the velocity diagram. The steam enters the guide wheel of any intermediate turbine with the exit velocity c_2 of the preceding rotating wheel and is accelerated to the velocity c_1; which necessitates an expenditure of work

$$L' = \frac{c_1^2 - c_2^2}{2g} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (10)$$

In the rotating wheel of an ideal turbine no losses occur ; and the steam again flows away with the velocity c_2. Into the first rotating wheel the steam flows with a negligible velocity. The acceleration to c_1 absorbs, therefore, the work

$$L'' = \frac{c_1^2}{2g}, \text{ or } = \frac{c_1^2 - c_2^2}{2g} + \frac{c_2^2}{2g} = L' + \frac{c_2^2}{2g} \quad . \quad . \quad (11)$$

If in all there are z turbines, then we have

$$L_0 = L'' + L'(z - 1) = zL' + \frac{c_2^2}{2g} \quad . \quad . \quad . \quad (12)$$

from which z can be calculated. The velocities are to be so changed that z becomes a whole number. If we subtract from L_0 the value $c_2^2 \div 2g$, and if we divide the remainder in z equal parts, then the division lines will give the pressures and specific volumes that are necessary for the calculation of the cross-sections. The cross-sections are then calculated as above.

In a frictionless turbine the lost work is only the kinetic energy of the exhaust steam from the last wheel ; that is,

$$l_0 = \frac{c_2^2}{2g} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (13)$$

which is very much smaller than case α (in general for many stage turbines nearly the $1 \div z$ part of the former). Hence theoretically, the previously described utilization of the exit velocity c_2 is advantageous ; but it must be mentioned that in case β, the velocities of flow are greater throughout than in case α, and therefore the friction losses in the channels are likewise increased, and the theoretical gain is decreased. This influence can therefore be exactly noted only in connection with the change of heat condition.

The friction of steam in the blade channels can approximately be allowed for, if we consider the value ζL_0, in which the coefficient of resistance ζ = about 0.25 to 0.40, as lost at the start,

from the total working area L_0. The increase of velocities gives as before L', and in place of equation 12 we now have

$$(1 - \zeta) L_0 = z L' + l_0 \quad . \quad . \quad . \quad . \quad . \quad . \quad (14)$$

from which, with the same L_0, a smaller z results. After l_0 is subtracted from L_0 we would again be able to divide the remainder into z parts in order to get the pressures and steam volumes of the individual stages ; but this process is unreliable because the resistances would change the heat conditions, and therefore the specific volumes of the steam.

We can proceed similarly if the turbine is divided into several groups, each with a constant peripheral velocity.

c. — FEW PRESSURE STAGES, EACH WITH SEVERAL VELOCITY STAGES.

This type of turbine might be explained by a combination of the discussed cases *a* and *b*. It is known that *Curtis* uses such repeated stages, generally with 2 to 3 pressure stages, and each with 2 (formerly also 3) velocity stages.

11. THE MANY STAGE REACTION TURBINE.

These turbines are built only with full peripheral admission and closely succeeding guide and rotating wheels, so that the entire value of the velocity c_2 is made available for the next following stage.

The simplest case is a turbine with equal diameters, and hence with constant peripheral velocity for all wheels. The preliminary calculation of such a turbine is easiest if we also specify c_1, w_1, w_2, and c_2 equal for all wheels, as for instance, according to Fig. 10 ; and otherwise proceed in a similar manner as with the many stage impulse turbine. For any system composed of guide and rotating wheels we get :

For the work in the guide wheel necessary to accelerate the velocity c_2 (exit velocity from the preceding rotating wheel) to c_1, the equation

$$L_1' = \frac{c_1^2 - c_2^2}{2g}; \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

in the rotating wheel for acceleration from w_1 to w_2, the work used is

$$L_2' = \frac{w_2^2 - w_1^2}{2g}; \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

and for each in between system (guide and rotating wheels),

$$L' = L_1' + L_2' \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

For the first guide wheel the velocity is to be increased from the existing small value in the preceding chamber to c_1, and we must apply the work

$$L_1'' = \frac{c_1^2}{2g}; \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

in the first rotating wheel L_2' remains in both together, therefore

$$\begin{aligned} L'' &= L_1'' + L_2' = \frac{c_1^2}{2g} - \frac{c_2^2}{2g} + \frac{c_2^2}{2g} + L_2' \\ &= L_1' + L_2' + \frac{c_2^2}{2g} = L' + \frac{c_2^2}{2g} \quad . \quad . \quad . \quad . \quad . \quad (5) \end{aligned}$$

The entire turbine with z stages consumes the work

$$L_0 = L'' + (z - 1)\, L' = z\, L' + \frac{c_2^2}{2g} \quad . \quad . \quad . \quad . \quad (6)$$

from which z can be calculated. We shall again subtract

$$l_0 = \frac{c_2^2}{2g}$$

from L_0, and divide the remainder in z equal parts, in order to get the pressures and the specific volumes for the calculation of the cross-sections.

The indicated work for the ideal turbine per unit weight (pound or kilogram) of steam is

$$L_i = L_0 - l_0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

Friction can approximately be taken into account as with the impulse turbine.

The design of the turbine in its actual constructive form, with many gradations of peripheral and steam velocities, will be shown in Article 29.

12. COMPARISON OF VELOCITIES AND THE NUMBER OF STAGES IN IMPULSE AND REACTION TURBINES.

For the *single stage* impulse turbine we had

$$c_1 = \sqrt{2\, g\, L_0} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

and with normal exit from the rotating wheel

$$u = \tfrac{1}{2}\, c_1 \cos \alpha \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

For the single stage reaction turbine with *one-half degree of reaction* ($L_1 = \frac{1}{2} L_0$) and $c_0 = 0$, under similar conditions we had

$$c_1 = \sqrt{2 g \frac{L_0}{2}} = \frac{c_1}{\sqrt{2}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

and

$$u' = c_1' \cos \alpha = u \sqrt{2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

That is, *the single stage reaction turbine works with about* 1.4 *times as great a peripheral velocity as the impulse turbine.*

For the many stage impulse turbine, according to system β, and omitting $c_2^2 \div 2g$, we have

$$z = \frac{L_0}{L'} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

We shall assume for simplicity *normal exit from the rotating wheel*, which would for Fig. 7 bring the condition

$$2u = c_1 \cos \alpha \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

Then

$$c_1^2 - c_2^2 = 4u^2$$

and Formula 10, Article 10*b* gives

$$L' = \frac{c_1^2 - c_2^2}{2g} = \frac{2u^2}{g}; \qquad z = \frac{g}{2}\frac{L_0}{u^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

For similar axial exit for the many stage reaction turbine we have (all velocities given a subscript) and with the further simplification that

$$\alpha = \alpha_2, \qquad c_1' = w_2', \qquad w_1' = c_2',$$

$$u' = c_1' \cos \alpha \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

Therefore

$$c_2'^2 = c_1'^2 - u'^2,$$

and according to Formulæ 1 and 2, Article 8,

$$L_1' = \frac{c_1'^2 - c_2'^2}{2g} = \frac{u'^2}{2g}; \qquad L_2' = \frac{w_2'^2 - w_1'^2}{2g} = \frac{u'^2}{2g};$$

therefore

$$L' = L_1' = L_2' = \frac{u'^2}{2g}.$$

From which

$$z' = \frac{L_0}{L'} = g' \frac{L_0}{u'^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (9)$$

If we make $u = u'$, then

$$z = \frac{z'}{2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (10)$$

at the same time Formulæ 6 and 8 give with equal α:

$$\frac{c_1}{c_1'} = 2;$$

that is, *with the same peripheral velocity the impulse turbine has only one-half as many stages as the reaction motor, still the steam velocities are about twice as large.*

Let at another time $z = z'$, then follows from Formulæ 7 and 9,

$$u = \frac{u'}{\sqrt{2}} \quad \text{and} \quad c_1 = c_1' \sqrt{2},$$

that is, with *the same number of stages the peripheral velocity of the impulse turbine is about* 1.4 *times as small, and the steam velocity* 1.4 *times as large, as by using the one-half reaction degree.*

B. THE RADIAL TURBINES.

The radial turbines can, in the first approximation, be judged according to the formulæ of axial steam admission, as the action of centrifugal force is negligible with the generally very short blades. Only with many stage turbines can this effect enter into the problem as an important correction, which will be discussed further on.

II.

THEORY OF THE STEAM TURBINE THERMODYNAMICALLY CONSIDERED.

A. THE STEADY FLOW OF STEAM.

13. NOTATIONS AND DEFINITIONS.

THE specific heat of water is, according to *Regnault*,

$$c = 1 + 0.00004\,t + 0.0000009\,t^2 \quad . \quad . \quad . \quad . \quad (1)$$

in which t is the temperature in degrees Centigrade. Or

$$c = 1 + 0.000022\,(t - 32) + 0.0000005\,(t - 32)^2,$$

when t is the temperature in degrees Fahrenheit.

To heat 1 pound (or kilogram) of water from 0° to $t°$, we need the heat

$$q = \int_0^t c\,dt \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

When the water reaches the boiling point at pressure p and temperature t, the "external heat of vaporization" necessary for total vaporization at constant pressure is,

$$r = 607 - 0.708\,t \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

in the French units;

$$r = 1092 - 0.708\,(t - 32)$$

in the English units.

To change 1 pound (or kilogram) of water of 0° temperature to a mixture of x pounds (or kilograms) of steam, and $(1 - x)$

pounds (or kilograms) of water of the temperature $t°$ at a constant kept pressure, requires the addition of the heat

$$\lambda_x = q + xr. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

For $x = 1$; in French units

$$\lambda = 606.5 + 0.305\, t \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

in English units

$$\lambda = 1091.7 + 0.305\, (t - 32)$$

With complete vaporization at constant pressure one pound (or kilogram) of steam, increasing its volume from σ_0 to $v' = \sigma + \sigma_0$ performs the work

$$p\, (v' - \sigma_0) = p\,\sigma \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

which was taken from the heat of vaporization. As "internal energy" of the steam, there remains in latent form the heat quantity

$$u - u_0 = \lambda - Ap\,\sigma = q + \rho \quad . \quad . \quad . \quad . \quad . \quad (7)$$

in which $\rho = r - Ap\,\sigma$ and is called the "internal heat of vaporization," and u_0 is the unknown energy of the water at 0° C. With incomplete vaporization to the quality of steam x, we have per pound (or kilogram),

$$u - u_0 = q + x\rho \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

In the superheated territory we take the specific heat at constant pressure and constant volume c_p and c_v as unchanged; that is

$$c_p = 0.48, \qquad c_v = 0.369, \qquad \frac{c_p}{c_v} = 1.3 \quad . \quad . \quad . \quad (9)$$

in spite of the fact that late observations have shown without doubt an increase with rising temperature. As, therefore, the obtained values differ very greatly from one another, a dependable value is impossible at this time.

As condition equation, we shall choose the formula derived on page 3,

$$p\, (v + 0.0084) = 46.7\, T \quad . \quad . \quad . \quad . \quad . \quad . \quad (10)$$

expressed in meter kilogram units, or

$$p\,(v + 0.0084) = 85.1\,T$$

expressed in foot-pound units.

If we heat superheated steam at constant volume, then the added heat increases the internal energy; that is,

$$u - u' = c_v\,(T - T') \quad . \quad . \quad . \quad . \quad . \quad (11)$$

in which u' and T' stand for the condition of saturated steam.

To change 1 pound (or kilogram) of water at 0°C. (32° F.) into superheated steam at the temperature $t°$ at constant pressure, it is necessary to add the heat

$$\lambda = q + r + c_p\,(t - t') \quad . \quad . \quad . \quad . \quad . \quad (12)$$

in which t' is the temperature of saturated steam.

14. FUNDAMENTAL EQUATIONS OF THERMODYNAMICS.

ZEUNER'S FORMULA.

We shall take, in Fig. 22, any two cross-sections A_1 and A_2 of the steam flow in a steady working turbine, and let p_1 and p_2 be the existing pressures in A_1 and A_2, w_1 and w_2 the velocities, u_1 and u_2 the (internal) energies or capacity of doing work per pound (or kilogram), v_1 and v_2 the volumes per pound (or kilogram), F_1 and F_2 the cross-sections. During the time element dt, between the places A_1 and A_2, the external "useful" work $G\,E\,dt$ is performed and the heat quantity $G\,Q_s\,dt$ (through conduction and radiation) is carried away to the outside. The cross-section $A_1\,A_2$ moved during this time to $B_1\,B_2$, and a steam mass of $G\,dt$ pounds (or kilograms) flows through them. The total energy of the enclosed quantity of steam between A_1 and A_2 at the beginning of the time element is found again in the total energy at the end of the time element and in the work given up to the outside as well as in the heat quantity carried away. The total energy of the quantity of steam enclosed

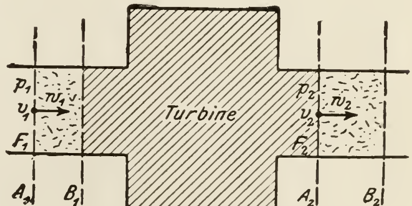

Fig. 22.

between A_2 and B_1 is, at the beginning and at the end, equally large, and can be omitted from the equation. We shall add to the useful work that amount which is performed by the surface pressure in the moved cross-sections A_1 and A_2, either positively or negatively, and get, *neglecting* the always small work against gravity due to the existing differences of heights, the equation

$$G\,dt\,u_1 + A\frac{G}{g}\frac{w_1^2}{2}dt = A\,G\,E\,dt + G\,Q_s\,dt + G\,dt\,u_2$$
$$+ A\frac{G}{g}\frac{w_2^2}{2}dt + A\,F_2 p_2 w_2\,dt - A\,F_1 p_1 w_1\,dt,$$

in which $A = \frac{1}{424}$ of the mechanical equivalent of heat, and g stands for the acceleration due to gravity. If we take care that

$$G = \frac{F_1 w_1}{v_1} = \frac{F_2 w_2}{v_2},$$

and substituting for $F_1 w_1$ and $F_2 w_2$ in these equations, we get

$$[u_1 + A p_1 v_1] - [u_2 + A p_2 v_2] = A\,E + Q_s + A\left[\frac{w_2^2}{2g} - \frac{w_1^2}{2g}\right]. \quad (1)$$

The steam can, at A_1 and A_2, be in the wet, saturated or superheated condition ; in all cases

$$\lambda = u + A\,p\,v,$$

is the heat which 1 pound (or kilogram) of water at 0° C. temperature must have brought to it to change it at constant pressure p into steam at condition $p\,v$. If this steam is just dry "saturated," then λ would correspond with the *total heat of vaporization* in the formula of Zeuner, if we neglect, what is allowable in all steam turbine problems, the specific volume of water as compared to that of saturated steam.

For moist steam, if we denote with σ the increase of volume of saturated steam, the following approximately holds good :

$$\lambda = u + A\,p\,x\,\sigma = q + x\,\rho + A\,p\,x\,\sigma = q + x\,r \quad . \quad (1a)$$

For superheated steam we write

$$A\,p\,v = A\,p\,(v - v') + A\,p\,v',$$

and get

$$u + A\,p\,v = u' + c_v\,(T - T') + A\,p\,(v - v') + A\,p\,v',$$

in which the letters with primes refer to the limit curve or curve of constant steam weight. From the condition equation follows

$$A p (v - v') = A R (T - T') = (c_p - c_v)(T - T')$$

and

$$u' + A p v' \text{ is also equal to } q + r,$$

so that finally

$$\lambda = u + A p v = q + r + c_p (T' - T) \quad . \quad . \quad . \quad (1a)$$

which also satisfies, in fact, the above definition for superheated steam.

The value λ we shall call, as *Mollier* has done, *the heat contents per pound (or kilogram) at constant pressure;* also, sometimes the *total heat of steam.*

The fundamental law is then,

$$\lambda_1 - \lambda_2 = A E + Q_s + A\left[\frac{w_2^2}{2g} - \frac{w_1^2}{2g}\right] \quad . \quad . \quad . \quad (1b)$$

or in words:

*The decrease of the heat contents is equal to the heat value of the gained "useful work," plus the heat carried away to the outside plus the increase of kinetic energy per pound (or kilogram) of steam.**

If steam flow takes place *without heat loss* and *without giving up useful work,* we get

$$\frac{w_2^2}{2g} - \frac{w_1^2}{2g} = \frac{1}{A}(\lambda_1 - \lambda_2) \quad . \quad . \quad . \quad . \quad . \quad (2)$$

or in words: The increase of flow energy is, with adiabatic flow without doing work, equal to the work of the decrease of heat contents per pound (or kilogram) of steam.

Equation 2 becomes (approximately) applicable for the flow in a nozzle and a single guide wheel or rotating wheel channel.

The second fundamental formula is obtained when we apply the energy equation to the quantity of steam in an infinitely small volume element of the steam stream, that is, on the relative movement of

* Formula $1b$ was first derived by Zeuner; and we in technical literature must thank Prof. Mollier for the very useful introduction of Gibbs' "Heat function for constant pressure," λ. Mollier gave it the term "heat contents."

its mass particles towards the center of gravity of the element. We must, to accomplish this, imagine the so-called rejuvenating forces of the relative motion (centrifugal forces, etc.) acting upon the mass particles, but the center of gravity remaining at rest. The internal energy experiences in a time element the increase $dG \,.\, du$, the work of the surface forces is $-dG\,p\,dv$, corresponding to the expansion $dG \,.\, dv$ of the element. The additional forces mentioned do not assist the work, because the center of gravity remains in relative rest. The increase of kinetic energy (for the movement relative to the center of gravity) is of the infinitely small higher order and can be omitted. The added heat consists of dQ, which was taken from the surrounding, and the value dR, which appears as heat caused by the change of the friction work against the walls, or by the inner eddy current work. (See the unusually clear demonstration by Grashof, Theoret. Maschinenlehre, Vol. I, p. 61). If we use the law of energy somewhat in the form: The added heat serves to increase the internal energy and to overcome the surface forces, we would get

$$dQ + dR = du + A\,p\,dv \quad . \quad . \quad . \quad . \quad . \quad (3)$$

If $dQ = 0$ as well as $dR = 0$, then the steam goes through a *frictionless adiabatic change of condition.* If only $dQ = 0$, then there will be no heat added from the outside, but the change of condition is nevertheless not, as in the former sense, adiabatic.

14a. THE WORK OF FRICTION AND THE LOSS OF KINETIC ENERGY.

Let us now consider an adiabatic flow without resistance, with the initial condition $p_1 v_1$ and the final condition $p_2 v_2'$, Fig. 23. Let the final velocity be w_2', the heat contents λ_2'; these values are related according to the formula

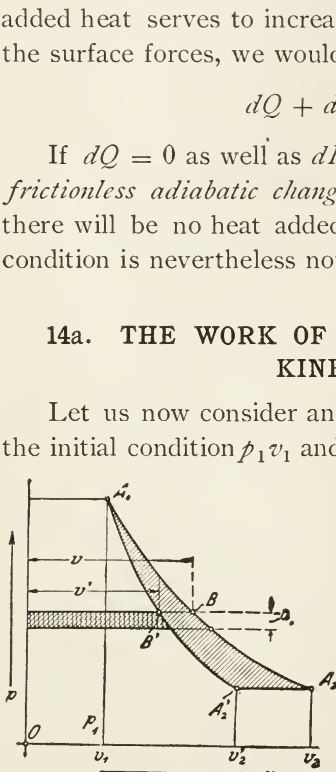

Fig. 23.

$$\frac{w_2'^2}{2g} = \frac{w_1^2}{2g} + \frac{1}{A}(\lambda_1 - \lambda_2').$$

With this we shall compare a flow with the same initial condition, but experiencing resistances which, with the *same final pressure* p_2, will give

a different volume v_2, a different and smaller velocity w_2, and a different steam contents λ_2, for which the formula

$$\frac{w_2^2}{2g} = \frac{w_1^2}{2g} + \frac{1}{A}(\lambda_1 - \lambda_2)$$

holds good. *The loss of kinetic energy* $\frac{Z}{A}$, which alone is of importance is

$$\frac{1}{A}Z = \frac{w_2'^2}{2g} - \frac{w_2^2}{2g} = \frac{1}{A}(\lambda_2 - \lambda_2'),$$

or

$$Z = \lambda_2 - \lambda_2' \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

that is, *the heat value of the loss of energy* (Z) *is that quantity of heat necessary to change one pound* (*or kilogram*) *of steam from the final condition of frictionless adiabatic expansion into the actual final condition.*

Equation 3 can also be written in the form

$$dQ + dR = du + A\,dpv - A\,v\,dp = d\lambda - A\,v\,dp \quad . \quad (3a)$$

If $dQ = 0$, and $dR = 0$, that is, the flow is without resistance, then the integration between A_1 and A_2' gives

$$0 = \lambda_2' - \lambda_2 - \int_1^{2'} A\,v'\,dp,$$

in which v' is a corresponding volume to p on the curve A_1A_2'. When $dQ = 0$ but $dR > 0$, then

$$R = \lambda_2 - \lambda_1 - \int_1^2 A\,v\,dp,$$

in which v refers to A_1A_2, that is, the actual expansion line. After subtracting we have

$$R = \lambda_2 - \lambda_2' - A\left[\int_1^2 v\,dp - \int_1^{2'} v'\,dp\right],$$

or if the integration order is reversed to get rid of the negative sign,

$$R = \lambda_2 - \lambda_2' + A\int_2^1 (v - v')\,dp.$$

Referring to Fig. 23, it follows:

$$R = Z + \text{heat value of the work-area } A_1A_2A_2' \quad . \quad . \quad . \quad (3b)$$

that is, R and Z are not at all identical; but rather *the effective loss of kinetic energy compared to the frictionless adiabatic expansion is the contents of the work-area* $A_1 A_2 A_2'$ *less than the value of the work of friction (and eddy currents).* These phenomena can be explained by the fact that the work of friction is always immediately changed into heat, and for this reason can still give in the following time element an addition to the useful work.

From equation 3a in combination with equation 1, taking $E = 0$ and $Q_s = 0$, results the known formula,

$$\lambda_1 - \lambda_2 = A\left(\frac{w_2^2}{2g} - \frac{w_1^2}{2g}\right) = -A\int_1^2 v\,dp - R \quad . \quad . \quad (3c)$$

which expresses the generalization of the formula of *de Saint-Vénant.*

15. THE ENTROPY TABLE.

In order to simplify the calculations of the change of condition of steam in the turbine, the entropy of steam is drawn in the well-

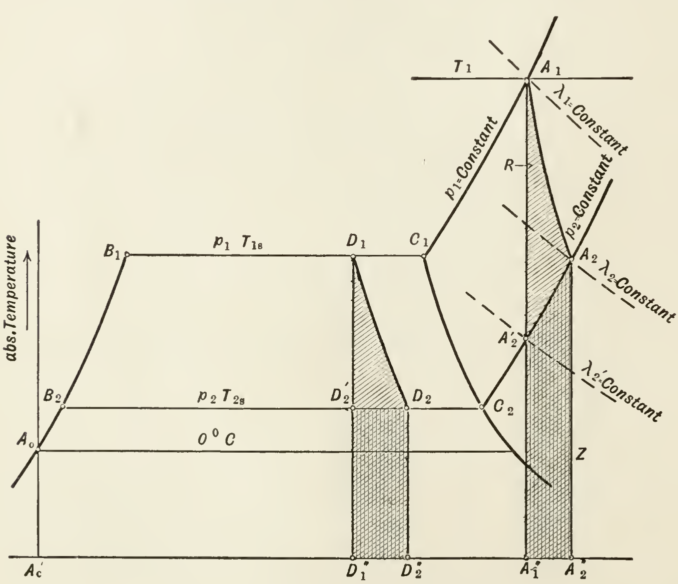

Fig. 24.

known manner in Table I as a function of the absolute temperature, and calculated for the superheated territory with the constant

value $c_p = 0.48$ of the specific heat for constant pressure. The table is made more complete by adding the lines v = constant and λ = constant, so that any condition given in terms of p and x or by p and T, the entropy, the volumes and the heat of steam can immediately be read. The line v = constant shows at the limit curve or curve of constant steam weight a break, because the values which were until now considered reliable do not agree with the values of *Tumlirz-Battelli*, which were used in the superheated territory, and it appeared advisable without making any correction experiments to let this difference simply stand as it is.

In the entropy table the entire changes of condition of the flowing steam, but especially the "heat of friction" R and the "heat loss" Z, can now be represented as follows, Fig. 24.

Let the initial condition in the superheated territory be fixed at A_1; the adiabatic frictionless expansion to the prescribed final pressure p_2 leads to the point A_2', which lies vertically beneath A_1, while the true final condition is represented by A_2. According to our explanation, A_0 represents the "normal condition," water at 0° C. (32° F.), we have

$$\begin{aligned}\lambda_1 &= \text{Area } A_0' A_0 B_1 C_1 A_1 A_1'' A_0', \\ \lambda_2 &= \text{" } A_0' A_0 B_2 C_2 A_2 A_2'' A_0', \\ \lambda_2' &= \text{" } A_0' A_0 B_2 C_2 A_2' A_1'' A_0',\end{aligned}$$

and it follows from what has been said, that with adiabatic (frictionless) flow

the *"available" steam heat at* $\lambda_1 - \lambda_2'$ = Area $B_2 B_1 C_1 A_1 A_2' C_2 B_2$,
the loss of kinetic energy (in heat measure) for the true change of condition $Z = \lambda_2 - \lambda_2'$ = vertically sectioned area $A_2' A_2 A_2'' A_1''$,
the actual *work of friction* (in heat measure) R = slanting sectioned area $A_1 A_2 A_2'' A_1'' A_1$.

The same holds good, if A is exchanged with D, for a change of condition in the saturated territory.

With this representation the reader should become well acquainted, because future developments depend upon it.

In order to calculate the velocity in A_2, we take it from the diagram, on whichever of the curves plotted for λ_2 = constant, A_2 is situated; the difference $\lambda_1 - \lambda_2$ gives the increase of kinetic energy, therefore, for instance, we have if w_1 equal 0, the value $\frac{1}{A}\frac{w_2^2}{2g}$.

16. EXPERIMENTS WITH STEAM IN NOZZLES.

THE EXPERIMENTAL APPARATUS.

Fig. 25 consists of the actual nozzle with a centrally located thin measuring tube, one end of which is closed, the other connected with a manometer or vacuum gauge; in the middle the tube has a 1 to 1.5 mm. (0.04 to 0.06 in.) diameter hole at right angles to the axis. By means of the micrometer screw this tube can be moved in one direction or the other, and the measuring opening brought to any position of the nozzle axis. We also find along the cone nozzle, holes bored at right angles to the wall of the nozzle; these are also connected with manometers.

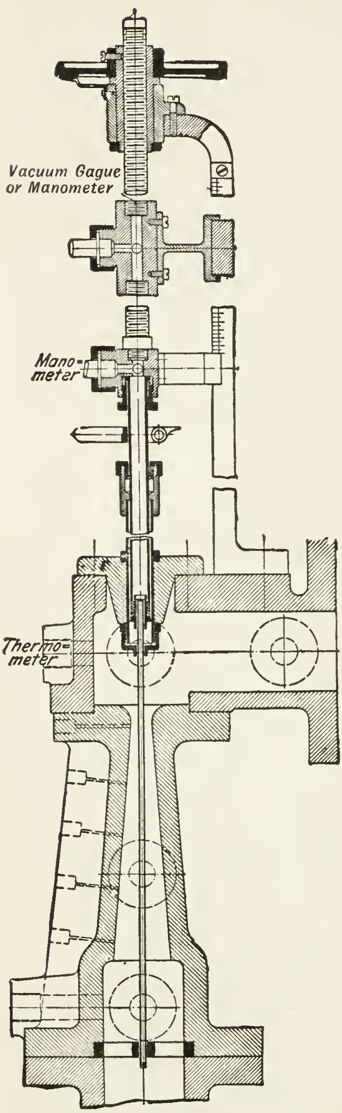

Fig. 25.

THE MEASUREMENT OF PRESSURE.

The measurement of pressure must be tested as to its reliability, for as can be seen, it is not sufficient to place a measuring opening tangentially to the flow of steam, but the position and the condition of the edges of the opening may be responsible for a disturbing effect. There were, among others, two measuring tubes used of 5 mm. (0.2 in.) outside diameter which had in a middle thick walled part, holes about 1.2 mm. (0.047 in.) diameter drilled at 45° to the axis as in Fig. 26. As was to be expected, a whirl developed with back pressure due to the sharp corners of the holes as shown by *a*, and they gave on this account a

higher pressure reading than the holes as shown by b. It can hardly be doubted that the readings of a were as much higher than the true existing pressure at the orifice as those of b were lower. The intermediate readings of the ordinary (thin walled) tube, with holes bored normally, could not materially differ from the true pressures. The difference between the measured pressures of the tubes with a and b holes was within the limits of the vacuum 5 to 10 mm. (0.2 to 0.4 in.) mercury column, and increased at about 2 to 3 atmospheres absolute (29 to 40 lb. per sq. in. absolute) to 0.15 kg. per sq. cm. (2.13 lb. per sq. in.), and with higher pressures (and correspondingly smaller steam velocities) decreased again. Nearly the same results were had by the introduction of sloping holes in the wall of the wide end of the nozzle. These figures show how exact the following reported experiments were.

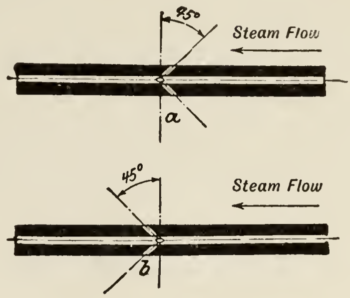

Fig. 26.

THE RESISTANCE TO FLOW.

The resistance to flow, especially the loss of flow energy up to any certain cross-section f_x, can be mathematically found with the assumption that the pressures and the velocities at the individual points of a cross-section are of sufficiently little varying values, to allow the introduction of an average value. This is proved so far as the pressure goes for the case of unopposed expansion in the experiments on the nozzle I used, in which the observed pressure in the centrally located tube varied only slightly from the observed pressures at the walls of the nozzle at the points where holes were bored.

Let p_1, t_1, x_1, be the pressure, temperature, quality of steam at the beginning of the nozzle (observed).

p_x be the observed pressure in cross-section f_x.

G be the steam weight flowing through in pounds per second (or kilograms per second).

λ_1 be the total heat of steam at the beginning of the nozzle.

w_1 be the velocity at the beginning of the nozzle.

In the cross-section f_x the steam is wet, with the unknown quality of steam x:

$$\lambda_x = q + x\,r \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

The energy equation gives

$$A\frac{w_x^2}{2g} = A\frac{w_1^2}{2g} + \lambda_1 - (q + x\,r) \quad . \quad . \quad . \quad . \quad (5)$$

Equilibrium requires

$$G = \frac{f_x w_x}{v_x}, \text{ or approximately } = \frac{f_x w_x}{x\,\sigma} \quad . \quad . \quad . \quad (6)$$

where σ is the difference of volume of 1 pound (or kilogram) of steam from 1 pound (or kilogram) of water under similar conditions. We place x from equation 6 in equation 5 and get

$$A\frac{w_x^2}{2g} = A\,\frac{w_1^2}{2g} + (\lambda_1 - q) - \frac{f_x r}{G\sigma}w_x \quad . \quad . \quad . \quad (7)$$

from which w_x can be deduced. w_1 is determined by the initial conditions and by G; the value $\frac{w_1^2}{2g}$ gives for the experiments an unimportant correction.

From equation 6 we find

$$x = \frac{f_x w_x}{G\sigma},$$

and finally

$$\lambda_x = q + x\,r.$$

Now we can calculate in the well-known way or take from the entropy table the quality of steam x' by adiabatic expansion from the initial conditions to the pressure p_x, and obtain

$$\lambda_x' = q + x'\,r.$$

K.

The loss of energy is according to equation 4

$$Z = \lambda_x - \lambda_x' = (x - x')\, r.$$

We show in Fig. 27 curves of pressure in the investigated nozzle with adiabatic, and with changes of condition of 10% and 20% energy loss. The small end of the nozzle was somewhat irregular, therefore less useful for measurements with high pressure. The observations * are on this account only given for the enlarged parts and correspond to the initial value $p_1 = 10.48$ kg. per sq. cm. (149 lb. per sq. in.); $t_1 = 198°$ C. (388.4° F.), that is,

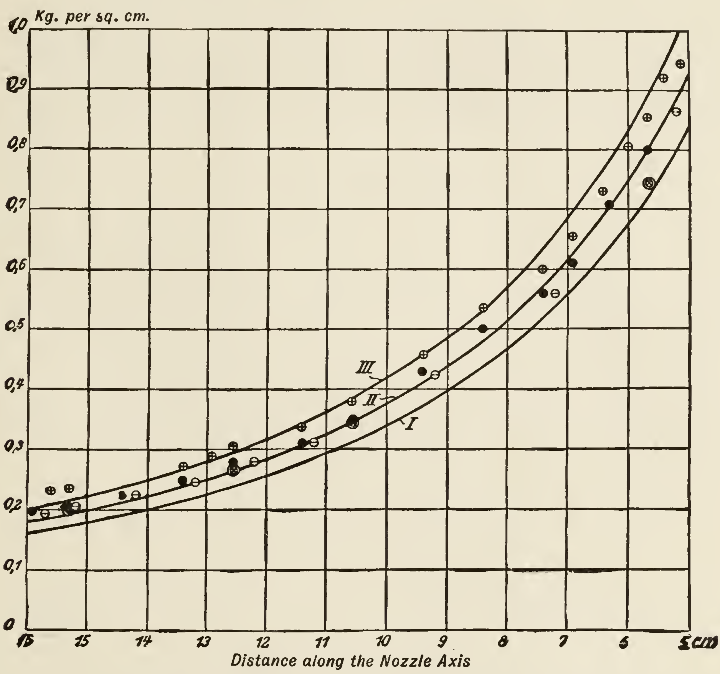

Fig. 27. Fall of Pressure in the Nozzle.

⊕ Holes in the measuring tube sloping against the stream (pressure readings too large).
● Holes in the measuring tube at right angles.
⊖ Holes in the measuring tube sloping with the stream (pressure readings too small).
◉ Pressure at the rim of the stream (holes at right angles).

Curve I: Adiabatic flow without resistance.
" II: Flow with 10% energy loss.
" III: " " 20% " "

* In these and the following experiments I was assisted by Messrs. Keller and Merenda.

a slight superheating to do away with the doubt that moisture existed in the steam. The measuring tube had a diameter of 5 mm. (0.2 in.), and the outside end towards the place where the steam entered was cooled by having cold water poured upon it. Still, it may have expanded more or less during the course of the experiment, so that we find here a further source of error.

An intermediate diameter of the nozzle can be represented by the formula

$$d = 12.19 + \frac{L}{6.485} \text{ mm.,}$$

where L is the distance of the cross-section from the head end of the nozzle (in mm.), between the limits $L = 60$ and 160 mm. Or

$$d = 0.48 + \frac{L}{6.485} \text{ in.}$$

when L is expressed in inches between the limits $L = 2.4$ and 6.3 in. For smaller values of L the meridian lines were not true, straight lines.

The steam weight flowing through per second was $G = 0.153$ kg. (0.34 lb.). The narrowest part of the nozzle had a diameter of 12.5 mm. (0.5 in.). From this we have by applying Zeuner's formula for saturated steam,

$$G = 199 f \sqrt{\frac{p_1}{v_1}} = 0.151 \text{ kg. per sec.,}$$

or in English units,

$$G = 43.25 f \sqrt{\frac{p_1}{v_1}} = 0.33 \text{ lb. per sec.}$$

The slight superheating causes, therefore, an increase of the constant factor; but only of about 1.5 per cent., while *Lewieki* found 6 per cent. for high degree of superheating.

To represent the course of pressures of adiabatic flow without resistance, we calculate at any pressure p_x the quality of steam x of the adiabatic expansion, the heat of steam

$$\lambda_x' = q + xr,$$

and we get with the initial heat of steam λ_1, the velocity w from the formula

$$A\frac{w^2}{2g}=\lambda_1-\lambda_x',$$

by omitting the very small initial steam velocity. The specific volume v is very nearly $x\sigma$, and the "continuity equation" $Gv=fw$ gives the cross-section f, from which the corresponding distance along the nozzle axis (taking into account the cross-section of the measuring tube) can be found.

In order to represent the change of pressure when, for instance, ζ per cent. is taken as energy loss, we calculate as above. The quality of steam increases for the intermediate pressure p_x the value

$$\Delta x=\frac{\zeta(\lambda_1-\lambda_x')}{r}$$

so that

$$x_\zeta=x+\Delta x,$$

and the velocity is taken from the equation

$$A\frac{(w)^2}{2g}=(1-\zeta)(\lambda_1-\lambda_x').$$

So we get for the nozzle used the following values:—*

I. ADIABATIC FLOW WITHOUT RESISTANCE.

In the French Units.

Pressure	$p_x=$	2	1.5	1	0.7	kg. per sq. cm.
Quality of Steam . .	$x=$	0.9172	0.9025	0.8828	0.8668	
Velocity	$w=$	764.2	823.0	894.5	950.2	m.
Distance along the nozzle axis	$L=$	19.8	28.2	42.7	58.2	mm.
Pressure	$p_x=$	0.5	0.3	0.2	0.1	kg. per sq. cm.
Quality of Steam . .	$x=$	0.8532	0.8320	0.8175	0.7935	
Velocity	$w=$	997.2	1070	1111	1184	m.
Distance along the nozzle axis	$L=$	75.9	107.6	140.0	209.0	mm.

* For these and further calculations a slide rule was used; as because of the uncertainty of the steam tables it was useless to work to a greater degree of accuracy.

In the English Units.

Pressure	$p_x =$	28.44	21.33	14.22	9.954	lb. per sq. in.
Quality of Steam . .	$x =$	0.9172	0.9025	0.8828	0.8668	
Velocity	$w =$	2507	2700	2935	3117	ft.
Distance along the nozzle axis	$L =$	0.780	1.110	1.681	2.291	in.

Pressure	$p_x =$	7.110	4.266	2.844	1.422	lb. per sq. in.
Quality of Steam . .	$x =$	0.8532	0.8320	0.8175	0.7935	
Velocity	$w =$	3272	3511	3645	3885	ft.
Distance along the nozzle axis	$L =$	2.988	4.236	5.512	8.228	in.

II. FLOW WITH TEN PER CENT. LOSS OF ENERGY.

In the French Units.

Pressure	$p_x =$	1	0.7	0.5	0.3	0.2	kg. per sq. cm
Quality of Steam . .	$x =$	0.9007	0.8868	0.8750	0.8564	0.8438	
Velocity	$w =$	848.8	901.5	946.2	1010	1054	m.
Distance along the nozzle axis	$L =$	46.6	63.2	81.7	115.6	149.0	mm.

In the English Units.

Pressure	$p_x =$	14.22	9.954	7.110	4.266	2.844	lb. per sq. in.
Quality of Steam . .	$x =$	0.9007	0.8868	0.8750	0.8564	0.8438	
Velocity	$w =$	2785	2958	3105	3314	3458	ft.
Distance along the nozzle axis	$L =$	1.835	2.488	3.217	4.552	5.867	in.

III. FLOW WITH TWENTY PER CENT. LOSS OF ENERGY.

In the French Units.

Pressure	$p_x =$	1	0.7	0.5	0.3	0.2	kg. per sq.cm.
Quality of Steam . .	$x =$	0.9186	0.9068	0.8968	0.8808	0.8701	
Velocity	$w =$	800.3	850.0	892.2	953.2	994.2	m.
Distance along the nozzle axis	$L =$	51.5	68.8	88.3	113.9	159.4	mm.

In the English Units.

Pressure	$p_x =$	14.22	9.954	7.110	4.266	2.844	lb. per sq. in.
Quality of Steam . .	$x =$	0.9186	0.9068	0.8968	0.8808	0.8701	
Velocity	$w =$	2626	2789	2928	3127	3262	ft.
Distance along the nozzle axis	$L =$	2.028	2.709	3.477	4.484	6.276	in.

The experiments gave the following values of the pressures in the observed parts of the nozzle:

A. MEASURING TUBE WITH HOLES SLOPING AGAINST THE STREAM.

In the French Units.

Distance in the nozzle axis	$L=$ 51	54	57	60	64	69	mm.
Pressure	$p_x=$ 0.945	0.922	0.857	0.804	0.728	0.654	kg. per sq. cm.
Distance in the nozzle axis	$L=$ 74	84	94	106	114	125.5	mm.
Pressure	$p_x=$ 0.599	0.536	0.462	0.355	0.337	0.306	kg. per sq. cm.
Distance in the nozzle axis	$L=$ 129	134	144	153	156	164	mm.
Pressure	$p_x=$ 0.289	0.272	0.257	0.235	0.231	0.222	kg. per. sq. cm.

In the English Units.

Distance in the nozzle axis	$L=$ 2.008	2.126	2.244	2.363	2.520	2.716	in.
Pressure	$p_x=$ 13.44	13.11	12.19	11.44	10.35	9.445	lb. per. sq. in.
Distance in the nozzle axis	$L=$ 2.913	3.308	3.701	4.174	4.489	4.941	in.
Pressure	$p_x=$ 8.519	7.624	6.570	5.049	4.793	4.352	lb. per sq. in.
Distance in the nozzle axis	$L=$ 5.079	5.276	5.670	6.024	6.142	6.457	in.
Pressure	$p_x=$ 4.111	3.869	3.655	3.343	3.285	3.158	lb. per sq. in.

B. NORMAL MEASURING TUBE WITH HOLES AT RIGHT ANGLES TO THE AXIS.

In the French Units.

Distance in the nozzle axis	$L=$ 56.7	63	74	84	94	105.5	mm.
Pressure	$p_x=$ 0.797	0.708	0.558	0.501	0.428	0.348	kg. per sq. cm.
Distance in the nozzle axis	$L=$ 114	125.5	134	144	153	159	mm.
Pressure	$p_x=$ 0.312	0.278	0.248	0.223	0.202	0.196	kg. per sq. cm.

In the English Units.

Distance in the nozzle axis	$L=$ 2.233	2.480	2.913	3.308	3.701	4.154	in.
Pressure	$p_x=$ 11.34	10.07	7.936	7.125	6.087	4.950	lb. per sq. in.
Distance in the nozzle axis	$L=$ 4.489	4.941	5.276	5.670	6.024	5.260	in.
Pressure	$p_x=$ 4.438	3.954	3.528	3.172	2.873	2.788	lb. per sq. in.

C. MEASURING TUBE WITH HOLES SLOPING WITH THE STREAM.

In the French Units.

Distance in the nozzle axis	$L =$	52	56.7	72	92	105.5	112	mm.
Pressure	$p_x =$	0.866	0.791	0.560	0.424	0.347	0.311	kg. per sq. cm.

Distance in the nozzle axis	$L =$	122	125.5	132	142	153	157	162	mm.
Pressure	$p_x =$	0.281	0.269	0.245	0.225	0.204	0.193	0.185	kg. per sq. cm.

In the English Units.

Distance in the nozzle axis	$L =$	2.047	2.233	2.835	3.622	4.154	4.410	in.
Pressure	$p_x =$	12.32	11.25	7.965	6.031	4.935	4.424	lb. per sq. in.

Distance in the nozzle axis	$L =$	4.804	4.941	5.198	5.591	6.024	6.182	6.378	in.
Pressure	$p_x =$	3.997	3.826	3.485	3.200	2.901	2.745	2.631	lb. per sq. in.

Finally the pressures at the rim of the stream measured at holes drilled at right angles in the nozzle wall were

In the French Units.

In the Distance	$L =$	56.7	105.5	125.5	153	mm.
Pressure	$p_x =$	0.742	0.349	0.272	0.202	kg. per sq. cm.

In the English Units.

In the Distance	$L =$	2.233	4.154	4.941	6.024	in.
Pressure	$p_x =$	10.55	4.964	3.869	2.873	lb. per sq. in.

The graphical representation, Fig. 27, shows that the observations B are nearer to the values C than those of A. I lean to the opinion that this is not due to the increase of suction of the normal measuring tube, but to the increase of back pressure due to the too sharp edges of the tube A which was directed against the stream. From the curves we can see that the loss of energy at about one atmosphere pressure (14.7 lb. per sq. in. absolute) had reached about 10 per cent., and that it gradually increases to nearly 20 per cent. towards the end of the nozzle (with $L = 160$ mm. [6.3 in.]). But should we wish to consider the evidently too high pressure readings of the tube A as correct, then the energy loss would be something like 25 per cent., which would mean *that the flow in the*

nozzle is accompanied by extraordinary high resistances, and this alone would be proof that the pressure readings of A are too high.

Here, of course, we must note that the steam in our experiment expands to only 0.2 kg. per sq. cm. (2.84 lb. per sq. in.) and that the continuation of the expansion to about 0.1 kg. per sq. cm. (1.42 lb. per sq. in.) must cause additional losses. Still, later experiments lead me to the opinion, that the total loss for the investigated nozzle does not exceed 15 per cent.

Experiments to find the nozzle resistance were also conducted by *Delaporte* and *Lewicki.* The former used a nozzle of 6 to 9 mm. (0.24 to 0.35 in.) diameter, and with a length that was estimated from a sketch to be about 50 mm. (1.97 in.), *Delaporte* states in "Revue de Mécanique," May, 1902, that with exit into the atmosphere the loss of kinetic energy can be fixed at 5.2 per cent. by measuring the exerted pressure of the stream. The very small length of the pipe in conjunction with the low-flow velocity would decrease the losses, but still they appear somewhat too small.

Lewicki's experiments in the Zeitschr. d. Ver. deutsch. Ing., 1903, p. 49, are made under similar conditions. The pressure ratio was about 6.86, the nozzle diameter equalled 6.06 to 6.75 mm. (0.24 to 0.27 in.), nozzle length about 30 mm. (1.18 in.), and the ratio of exit cross-section to the narrowest cross-section 1.62 (therefore somewhat too small). The results with slightly superheated steam showed a mean loss of energy of 8% ; therefore more than with *Delaporte.*

If we wish to get the "coefficients of resistance" as we do in hydraulics, we must, according to equation 3*b* change the loss of kinetic energy to total friction work. For technical problems, however, only the former has any significance, and it is advisable to *transpose the coefficients of resistance to this loss of kinetic energy.*

This is approximately for a cylindrical tube

$$Z = A \zeta \frac{l}{d} \frac{w^2}{2g} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

For the conical nozzle we must add element after element, and place

instead of $\frac{dl}{2r}$ the value $dl\,\frac{U}{4F}$, as we are dealing with the area of a ring, in which U is the sum of the circumference of the nozzle and the measuring tube, and F the area of the ring. A graphical integration gives for ζ, with 29.7 calories as total loss and with 5 and 160 mm. (0.2 and 6.3 in.) as limits for l, the value

$$\zeta = 0.039.$$

The nozzle with the inner measuring tube, taking into consideration friction, would be equal to a simple cylindrical tube of about 17 mm. (0.67 in.) internal diameter, for which, according to *Darcy*, referring to the actual friction work, a coefficient of friction $\zeta = 0.049$ would be given. For this nozzle also, referring to the actual friction work ζ, would the values R and Z be proportionately greater. The above comparison now shows that it is allowable to consider the resistance to flow in the diverging nozzle as simple tube friction. So long as a free expansion is possible, there is no reason for assuming any particular resistances (such as shock or eddy currents, etc.). The resistances in these experiments are probably somewhat too large, as the vacuum in the jet condenser used was about 0.43 kg. per sq. cm. (6.1 lb. per sq. in.) absolute; directly behind the nozzle the pressure increased from 0.2 to 0.4 atmospheres (2.9 to 5.9 lb. per sq. in.) and the back pressure due to this increase could have partly influenced the pressure at the nozzle end.*

All in all, we will, until we have more information, calculate for

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* On the whole, it is clear that the calculation with a constant mean condition in a cross-section gives only a first approximation. If we observe a stream flowing into the open air we will note a lighter outer layer and a clouded milky core, which shows that at the rim the friction of the wall caused a temporary superheating, while in the undisturbed interior part of the stream the adiabatic expansion goes on with a considerably higher degree of condensation. On the other hand, there is a possibility that in the limited time which is available for the expansion of the steam, the condensation corresponding to the drop of pressure has not entirely occurred, that is, that the steam has not given up the entire latent heat that we calculate it should. For the outflow of hot water this phenomenon has been experimentally proven by Prof. Knoblauch of Munich. With steam this deviation could only be minimum *because otherwise the exit quantity would not coincide so nearly with the theoretical values.* Experiments made by the author, by introducing a mercury thermometer in place of the measuring tube, also gave a negative result.

nozzles with small divergence and less than 50 mm. (1.97 in.) length and with about 5 to 8 mm. (0.2 to 0.3 in.) diameter, or with nozzles with greater divergence and of 100 to 150 mm. (3.94 to 5.91 in.) length and with diameters at the narrowest place of from 6 to 10 mm. (0.24 to 0.39 in.), an energy loss of 10 to 15%. The decrease of velocity is about half as great.

THE PRESSURE AT THE RIM OF THE STREAM.

The pressure at the rim of the stream was found to be of nearly the same value as the corresponding pressure in the middle of the stream. This also proves that the stream, in a nozzle *with the conical shape of wall as here used*, does not lose contact with the wall. An isolated stream could penetrate the surrounding steam layers that are at rest only with immense losses. The pressure at the rim seems to be slightly lower than that in the middle of the stream and would therefore point to the probable *higher pressure in the axis;* still the differences, with the exception of the point $L = 56.7$ mm. (2.23 in.), are too small to decide these questions with certainty.

17. ARTIFICIALLY INCREASED BACK-PRESSURE.

THE ACTION OF A DIFFUSER.

Through partly closing a valve that is placed between a nozzle and a condenser we can produce any desired back-pressure. The hereby resulting pressures are shown in the curves in Fig. 29. We notice that the pressure first follows the line of free expansion, and then increases according to the value of the back-pressure in more or less sudden leaps. At some places, as for example in curve E, the increase of pressure equals $1\frac{1}{2}$ atmospheres (22 lb. per sq. in.) in a tube length of 3 mm. (0.12 in.). I see in these extraordinary heavy increases of pressure, a realization of the theoretically derived "Compression Shock" of *von Riemann;* * because steam particles possessed of great velocity strike against a slower moving steam mass and are therefore compressed to a higher degree.

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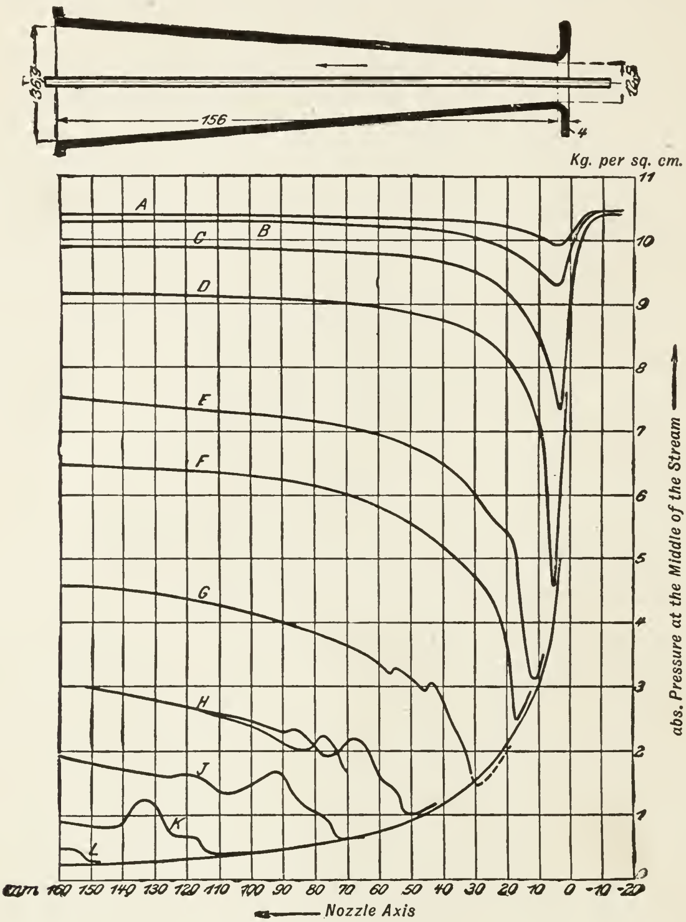
* Riemann-Weber. — The partial differential equations of Mathematical Physics, 1901, p. 469.

Such compression shocks will always occur when the nozzle has a greater length; that is, a larger cross-section divergence than the initial and the final pressures demand.

Especially noticeable are the wave-shaped pressure variations occurring at the low pressures following the sudden leap, and



Dimensions given are in mm.

Figs. 28 and 29.

which are to be considered as initial acoustic vibrations, but which are soon used up through friction (and also are prevented from spreading by the conical shape of the nozzle). This will be more fully discussed later. The place at which the leap occurs varies very easily when the initial conditions (for instance, the tempera-

ture) receive the slightest change before entering the nozzle; and with it the acoustic waves also change, as is shown in Curve *H*. In these curves we only wanted their shape and not their exit values, so we did not try to maintain any exact initial temperature, and variations within the limits of 194° to 200° C. (381° to 392° F.) were allowed. In Fig. 29 the observations were made with measuring tubes of 3 and 5 mm. (0.12 and 0.2 in.) diameters; and for this reason the curves do not all entirely join the constructed expansion line.

THEORY OF STEAM SHOCK.

Let *C*, in the space shown in Fig. 30, be a stationary shock plane. From the right hand steam is flowing against it with a velocity w_1, the pressure p_1, and the specific weight γ_1; to the left the corresponding values are w_2, p_2, γ_2. We shall assume the pipe cylindrical; but for infinitely small zones of shock the following will nevertheless also be true for conical pipes. We shall take about *C* an infinitely small element A_1B_1. *Riemann's* Theory will be applied to this simple case as follows: In the time element dt the cross-sections A_1B_1 will move to A_2B_2; the increase of velocity is, according to the law of propulsion,

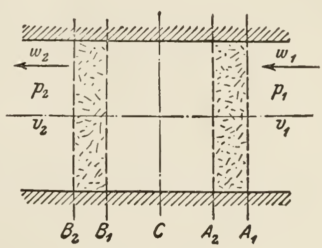


Fig. 30.

$$\left(f w_2 dt \frac{\gamma_2}{g}\right) w_2 - \left(f w_1 dt \frac{\gamma_1}{g}\right) w_1 = f(p_1 - p_2)\,dt;$$

or

$$w_2^2\gamma_2 - w_1^2\gamma_1 = g\,(p_1 - p_2) \quad . \quad . \quad . \quad . \quad (9)$$

In this the equation of the law of continuity for unaccelerated bodies holds good,

$$w_1\gamma_1 = w_2\gamma_2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (10)$$

and the equation gives

$$\left.\begin{aligned} w_1 &= \sqrt{\frac{p_1 - p_2}{\gamma_1 - \gamma_2} \cdot \left(\frac{\gamma_2}{\gamma_1}\right) g} \\ w_2 &= \sqrt{\frac{p_1 - p_2}{\gamma_1 - \gamma_2} \cdot \left(\frac{\gamma_1}{\gamma_2}\right) g} \end{aligned}\right\} \quad . \; . \; . \; . \; . \quad (11)$$

From this it would seem as if p_1, p_2, γ_1, γ_2 could be arbitrarily chosen, and the occurrence of the shock depended only on the retardation of the velocities w_1, w_2.

Lord Rayleigh * has denied the possibility of any such compression shocks, basing his denial on the following considerations. He writes

$$\frac{w_2^2 - w_1^2}{2g} = -\int_{p_1}^{p_2} v\,dp \quad . \; . \; . \; . \; . \quad (12)$$

or with Equation 10,

$$w_1^2\left(\frac{\gamma_1^2}{\gamma_2^2} - 1\right) = w_1^2\left(\frac{v_2^2}{v_1^2} - 1\right) = -2g\int_{p_1}^{p_2} v\,dp.$$

If we take w_1, p_1 as given, and p_2, v_2 as variable, and if we express the latter as p and v, we get by differentiating the above equation,

$$\frac{w_1^2}{v_1^2}dv = -g\,dp,$$

and from this

$$p = \text{constant} - \frac{w_1^2}{g v_1^2} v \quad . \; . \; . \; . \; . \quad (13)$$

This is the law which, according to *Rayleigh*, must exist between p and v, if both a compression shock and a conservation of energy shall exist. As this law does not correspond to the facts, *Rayleigh* states that a shock could not actually occur.

Rayleigh has here overlooked that the exit Equation 12 holds good only for occurrences without internal shock losses; but, as the steam shock naturally demands a considerable internal loss of kinetic energy, the equation used must be

$$\frac{w_2^2 - w_1^2}{2g} = -\int_{p_1}^{p_2} v\,dp - R \quad . \; . \; . \; . \quad (14)$$

* "Theory of Sound," 1896, Vol. II, p. 32.

or simplified

$$\frac{w_2^2 - w_1^2}{2g} = \lambda_1 - \lambda_2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (15)$$

Equations 9, 10, and 15 determine then, *three* variables. We have then, for instance, with *assumed initial conditions* of p_1, x_1, w_1 (and the help of p_2, x_2, w_2, from which are obtained λ_2 and R) *fully determined the final condition.* This point is also left unexplained by *H. Weber*,* and we could, according to his determination, say that, with all values of p_1, λ_1, w_1, which satisfy Equations 9 and 10, a compression shock is possible, and the law of energy is not contradicted. Actually, the conditions after the shock are entirely determined by the given initial conditions before the shock, and the loss of kinetic energy is also fully determined.

In actual calculation we would, for example, assume p_2 for trial, eliminate x_2 from Equations 10 and 15, calculate w_2 and substitute in Equation 9, repeating this until finally the latter is correct.

Examination of Fig. 29 also shows that the re-conversion of the assembled flow energy of the steam into pressure also takes place with considerable losses where there is no actual shock but a gradual transposition. If we compare two points at equal pressures on the up and the down running curves, the kinetic energy at the latter is found to be much smaller than at the former.

The re-compression of the entering steam particles of high velocity when conically diverging shows the same occurrences as that which takes place in a so-called *diffuser.* From our experiments we see that a diffuser works with poor efficiency, and that we must limit ourselves to small differences of pressures (Curves A to D).

From Article 19 it appears to follow that in back of the place of shock a compression occurs very much like the adiabatic, and from this assumption we could derive a clear calculation of the shock curves.†

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* Pages 489 and 497.

† See Prandth and Proell, "Additions to the Theory of Steam Flow." Zeitschr. d. Vereins deutsch, Ing., 1904, p. 348.

SMALL PRESSURE DIFFERENCES BEFORE AND AFTER PASSING THROUGH THE NOZZLE.

Small pressure differences before and after passing through the nozzle lead to an interesting phenomenon, which was not provided for in the older theory. It shows, namely, that the pressure at the narrowest part of the nozzle drops very considerably with the slightest drop of pressure behind the nozzle, and does not in any way reach the value of the back pressure.* The nozzle exerts, to a certain degree, a strong suction, and the quantity of steam flowing increases exceedingly fast, as the following table shows :

In French Units.

Pressure in front of the nozzle	$p_1 =$ 10.45	10.48	10.45	10.40	kg. per sq. cm.
Pressure in back of the nozzle	$p_2 =$ 10.40	10.36	10.30	9.90	kg. per sq. cm.
Difference of pressure	$p_1 - p_2 =$ 0.05	0.12	0.15	0.50	kg. per sq. cm.
Pressure at the narrowest place	$p_x =$ 9.89	9.74	9.17	7.32	kg. per sq. cm.
Weight of steam flowing through per second	$G =$ 0.073	0.109	0.113	0.152	kg. per sq. cm.

In English Units.

Pressure in front of the nozzle	$p_1 =$ 148.6	149	148.6	147.89	lb. per sq. in.
Pressure in back of the nozzle	$p_2 =$ 147.89	147.32	146.47	140.78	lb. per sq. in.
Difference of pressure	$p_1 - p_2 =$ 0.71	1.68	2.13	7.11	lb. per sq. in.
Pressure at the narrowest place	$p_x =$ 140.64	138.5	130.4	103.09	lb. per sq. in.
Weight of steam flowing through per second	$G =$ 1.03	1.55	1.6	2.16	lb. per sq. in.

It can be seen that, by aid of the pressure observations at the narrowest place, a steam meter could be constructed similar to the Venturi water meter. An experiment in the laboratory at Zurich shows a steady variation of pressure reading, and it should be further investigated to determine whether a point of still greater rest cannot be found by changing the measuring orifice.

From *Zeuner's* Formula (Article 4, Equation 22) it would appear, so long as the quantity of steam flowing through is equally large

* This same observation has also been made by A. Fliegner. (See Schweiz. Bauzeitung, Vol. XXXI, Nos. 10 to 12.)

in all cross-sections, that the pressure at the narrowest place must always reach the value $p_m = 0.57 p_1$ (for saturated steam). That this is not so is explained as follows: If through a nozzle there is steadily flowing at one time G, at another time G' pounds (or kilograms) of steam, then for flow without resistance the following relations hold:

$$G = f \phi p \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (16)$$

$$G' = f' \phi (p) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (16a)$$

in which $\phi (p)$ represents *Zeuner's* root factor in the above mentioned equation, f and f' are the variable cross-sections corresponding to p. If G' is smaller than G, then with equal p, f' must be smaller than f; the flow then demands a narrower nozzle as shown dotted in Fig. 31, which agrees with the true nozzle only at the entrance, and whose narrowest place is located at about C'. At B', before C' has been reached, occurs an enlargement which decreases the velocity and introduces important resistances. From this point equation 16a does not hold good, and herein we find the explanation of this interesting apparent contradiction.

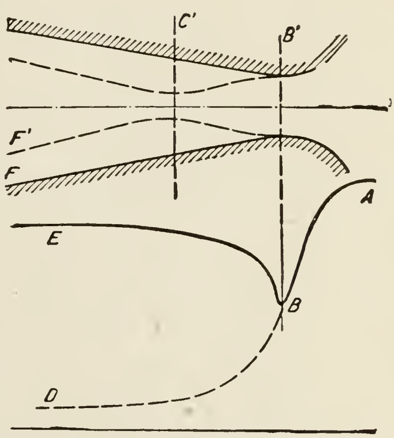
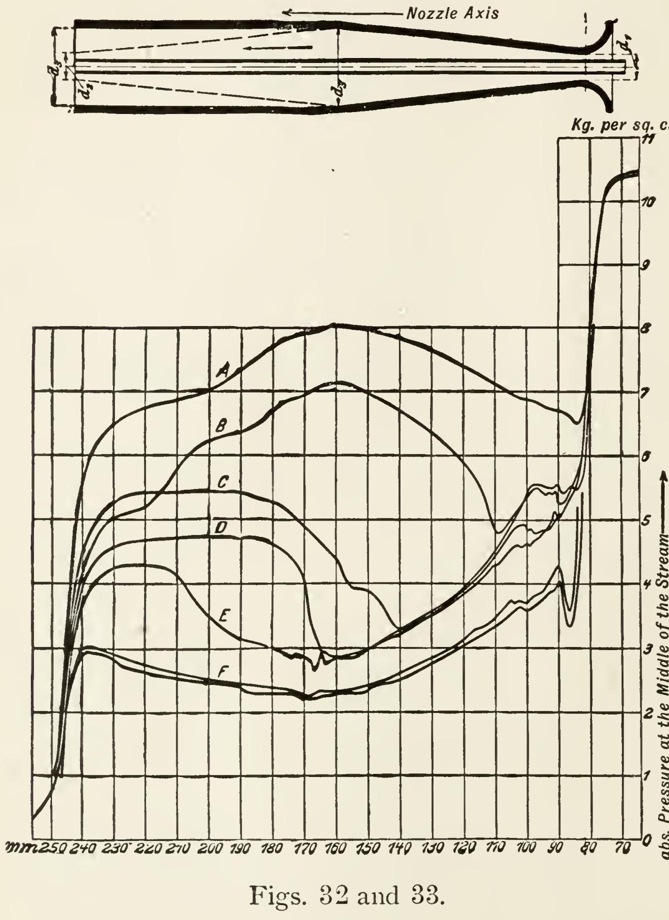


Fig. 31.

18. THE INFLUENCE OF ENLARGEMENT OF CROSS-SECTION.

The influence of a cross-section enlargement was investigated for steam flow by placing the wide ends of two nozzles together. In Fig. 32 the line A shows the variations of pressure in the case where the end of the second nozzle has the same cross-section as the narrowest cross-section at the inflowing side. The

pressure drops at entrance in the narrowest place from 10.5 to about 6.5 kg. per sq. cm. absolute (149.3 to 92.4 lb. per sq. in. absolute), and rises in the conical enlargement to about 8 kg. per sq. cm. (113.8 lb. per sq. in.). In the second nozzle the pressure again drops towards the opening and beyond it quickly to the vacuum



Figs. 32 and 33.

pressure. We now change the second nozzle so that the exit orifice diameter $d_2 = 10.8$ mm. (0.425 in.), while the wide end remains unchanged at a diameter of $d_3 = 12.1$ mm. (0.48 in.) and the entrance diameter remained at $d_1 = 10.3$ mm. (0.406 in.). The effect of these dimensions is shown by the line B. In a

similar way the lines C and D correspond to a widening of the orifice to 11.4 and 12.0 mm. (0.449 in. and 0.471 in.) respectively. Finally, the second nozzle was bored cylindrical to 12.1 mm. (0.48 in.) diameter and gave the line E, in which the pressure at entrance dropped at the narrowest place to about 5.5 kg. per sq. cm. (78.2 lb. per sq. in.), and from there on to the end of the conical nozzle it dropped further to about 3 kg. per sq. cm. (42.7 lb. per sq. in.). In the *cylindrical pipe we get the seemingly inconsistent fact* that the pressure does not drop but *increases more than one atmosphere;* and only at about 10 mm. (0.4 in.) from the end of the tube does the vacuum make itself felt and starts to decrease the pressure.

Line F was obtained after the rounded off entrance at d_1 had been removed, so that a sharp cornered entrance was offered. This caused a stream contraction at entrance which we shall discuss later. The result is a sudden drop in the pressure curve and a decrease of the through flowing steam volume (because of the decrease of the narrowest cross-section) which held the pressure lower throughout. The increase of pressure in the cylindrical tube is here also present.*

19. ISENTROPIC LINES.

An investigation into the laws of the experimentally found phenomena is made easier by a discussion of the so-called *isentropic lines* as below.† We shall assume, that the steam expands adiabatically and without friction, that is, with constant entropy from the initial condition A_1 to the final pressure corresponding to A_2 in Fig. 34, in which the coördinates are entropy and absolute temperature. By means of the process already given, we find, as in Fig. 4, for the exhaust velocity $w_1 = 0$ the cross-section f_x that

* The irregularities at the beginning of the lines B to E are caused by the slight porosity of the casting at the place in question, that is, caused by the influence of the friction coefficient.

† The suggestion for the construction of the isentropic curves is due to *Professor Prandtl of Hanover.* These are better for the investigation of the true condition changes than the lines of constant loss of kinetic energy, shown in Fig. 27, because they hold good as well for compression as for expansion in the nozzle.

is necessary, for instance, per one pound (or kilogram) of steam at the pressure p_x.

We shall now imagine that the steam before entrance into the nozzle is brought to the condition B_1, Fig. 34, by being throttled to a lower pressure. The total amount of heat does not change, that is, B_1 lies on the curve λ_1 = constant that passes through A_1, but the entropy has increased. If the steam expands from here adiabatically to B_2, we can, in similar manner, find a new cross-section as function of p, also the corresponding curves to C_1, D_1, etc. Instead of one pound (or kilogram) of steam, we shall now take the actual quantity of steam flowing through the nozzle per second as unity, and instead of the cross-section, we take the distance of this cross-section from a fixed point in the axis of the nozzle. In this way we get the curves I, II, III. . . . in Fig. 34*a*, which give for every point in the nozzle axis two possible pressures. A frictionless flow would be reversible, that is, the steam could as well flow through the nozzle, expanding or compressing. The upper parts of the isentropic curves correspond to compressed steam and low velocities, the lower parts to a condition of strong expansion and greater velocities. But the true flow always takes place with an increase of the entropy (see Table), and therefore the true condition curve, whether descending or ascending, must cut the adiabatics in the order I, II, III. . . . In order to prove this absolutely we have placed in the figure the curves of Fig. 28 dotted and we find, for instance, at curve D, a nice agreement with the adiabatic during the increase of pressure in the conical enlargement. With all curves we prove, that during the period of actual steam shock several isentropics are crossed, thus proving that large shock losses occur here. Further on, the lines F, G, and H show a too steep ascent, which, on the one hand, can be accounted for in the varia-

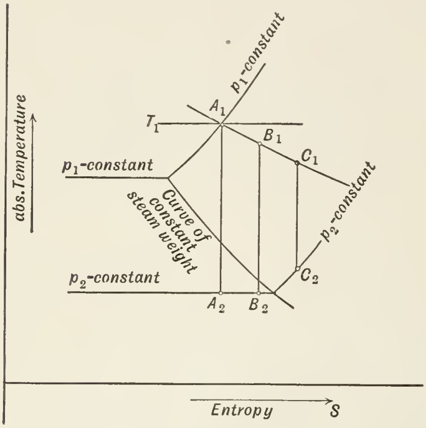


Fig. 34.

tion of the superheating of the steam during the experiment, because, as mentioned before, we were more concerned as to the qualitative. Under certain conditions, the carrying over of heat from the nozzle to the slow-flowing steam plays a part. From all curves we see

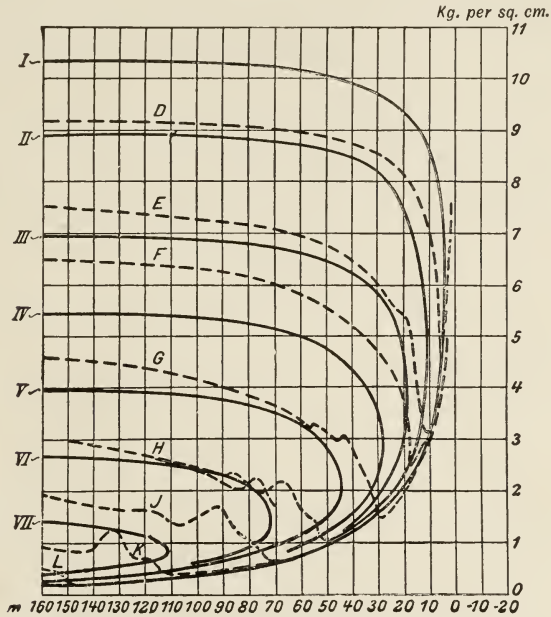


Fig. 34*a*.

clearly that the main part of the resistance occurs during the period of steam shock, and with quiet flow the retransposition of velocity into pressure causes much smaller losses. The analogy with the *Bidon* "water jump" is evident.

20. METHOD OF CALCULATION.

As can easily be seen, we have to do without a general integration of the equations of motion, and nothing else remains but to confine ourselves to an investigation of an elementary occurrence.

If we apply "the equation of kinetic energy," that is, equation 3*c*, to an infinitely short division of the nozzle, we have

$$\frac{w\,dw}{g} = -v\,dp - \frac{dR}{A}. \qquad (17)$$

The elementary work of friction we express as in hydraulical resistances, by the equation

$$\frac{dR}{A} = \zeta_r \frac{dz}{2r} \frac{w^2}{2g} \qquad (17a)$$

in which ζ_r is the assumed constant coefficient of resistance, r the radius, and dz the elementary axial length of the nozzle. From equation 2, which refers in like manner to an element, we get

$$\frac{w\,dw}{g} = -d\lambda \qquad (18)$$

and the "law of continuity" gives

$$Gv = fw \qquad (19)$$

From these equations, by eliminating from dw and dv the value of the differential quotients $dp \div dz$ at their instantaneous values, we can determine the remaining variables and so at least gain some idea of the fall or rise of a pressure line. If we carry out the calculation, the expression would not yield any more comprehensive results, but it is possible by means of an approximate assumption to simplify the investigation of the changes of condition. This can be accomplished with the assumption of the law

$$pv^k = \text{constant} \qquad (20)$$

or also

$$(p + \alpha)\, v^k = \text{constant} \qquad (20a)$$

from which we are certain, that for small intervals the curve representing the change of condition will agree with the true curve. Equation 20 contains two constants; we can, therefore, allow a point and the tangent at that point to correspond with the true condition curve. In equation $20a$ we deal with three constants; we can therefore let a point, the corresponding tangent, and its radius

of curvature be equal. Both approximations do not hold good for places of instability; for instance, at apexes. On this account equation 18 becomes superfluous; and we can differentiate equations 19 and 20 or 20*a*; from these, and from equation 17 we can eliminate dv and dw, from which is shown that we can introduce into the formulæ the expression for *acoustic velocity of the steam* corresponding to the condition p, v,

$$w_s = \sqrt{g\,k\,p\,v} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (21)$$

We get for the change of pressure in the direction of the nozzle axis

$$\frac{dp}{dz} = \frac{\left[\frac{\zeta_r}{2\,r} - \frac{2}{f}\frac{df}{dz}\right]}{[w^2 - w_s^2]}\frac{w^2}{2}k\,p \quad . \quad . \quad . \quad . \quad . \quad . \qquad (22)$$

For the nozzle with circular cross-section without the internal measuring tube, the numerator takes the value, with r as radius, of

$$\left[\frac{\zeta_r}{2\,r} - \frac{4}{r}\frac{dr}{dz}\right];$$

or we have, when $\phi = 2\frac{dr}{dz}$ of the cone angle of the nozzle,

$$\frac{dp}{dz} = \left(\frac{\zeta_r - 4\,\phi}{w^2 - w_s^2}\right)\frac{w^2\,k\,p}{4\,r} \quad . \quad . \quad . \quad . \quad . \quad . \qquad (23)$$

With Formula 20*a* we would only have to replace p with $(p + a)$.

The pressure rises or falls in relation to the flow accordingly as the value of $dp \div dz$ is positive or negative. As now the actual velocity w is initially nearly equal to nothing, but later reaches or exceeds w_s, we have an initially negative denominator. With rounded away entrance $dr \div dz$, that is, "the conical angle" ϕ is initially negative, therefore the numerator is positive. If at the beginning $dp \div dz$ is negative, the pressure drops. The further course of the pressure depends on whether and how soon a change of signs occurs. For the line A, Fig. 33, a change of sign occurs

first in the numerator, as the conical divergence makes ϕ positive, and the numerator negative. The gradual convergence in the second nozzle again makes ϕ negative and the numerator positive, the pressure again decreases.

The curious combination of the values of the coefficients of friction, the conical angles, and the true and acoustic velocities, determines therefore the rise and fall of the pressure. The case in which *w gradually increases so as to reach and exceed w_s, is especially interesting, because $dp \div dz$ passes through the value ∞ from negative to positive, and therefore an apex with perpendicular tangents is to be expected,* unless at this same time a change of sign occurs in the numerator. Still we must note that at this point of change ζ_r as well as k varies greatly, so that for these critical points our equation no longer fully holds.

Referring especially to the cylindrical tube, ϕ would equal 0, and the sign would depend only upon the denominator. We can, therefore, say: *In a cylindrical tube the pressure referring to the flow (not depending upon the value of the back pressure) increases or decreases accordingly as the actual steam velocity is larger or smaller than the acoustic velocity.**

---

* It is evident that an integration of the equations of motion, if that were generally possible, and if the laws of resistances were accurately known, must give the same representation of the variation of pressures. Practically useful results are obtained for the cylindrical tube with the assumption that ζ_r is constant. This case *Grashof* has solved in his Theoret. Machinenlehre, Vol. I, p. 658, also with the assumption of the law.

$$p v^k = C.$$

This calculation gives with comparative ease the simplified equation

$$\log \xi - \alpha^2 (\xi - 1) = \beta z \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (24)$$

in which is to be placed

$$\xi = \left(\frac{p}{p_0}\right)^{\frac{k+1}{k}}; \quad \alpha = \frac{w_{s0}}{w_0}; \quad \beta = \frac{\zeta_r (k+1)}{4 r},$$

and p_0, w_0, are the pressure and velocity at entrance ($z = 0$); w_{s0} stands for the acoustic velocity for the existing conditions at entrance. If *Grashof* had undertaken a discussion of his equation, he would have found the improbable rise of pressure for the case $\alpha > 1$. Because of the probable variability of ζ_r (as internal losses also occur with the compression), equation 24 would not correctly represent the entire change of pressure in the latter case, as in fact is also shown by a comparison of the observed lines.

With equation 23 we could also approximately answer the question, how a nozzle must be made so that, with expansion, the pressure will remain constant. This means that $dp = 0$, that is

$$\phi = \tfrac{1}{4}\zeta_r \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (23a)$$

The nozzle would simply become conical with the given conical angle, provided ζ_r be constant.

For air the introduction of an approximate equation is superfluous, as *Lorenz* stated in his noteworthy article in the *Physikalischen Zeitschr.*, 4th year, p. 333. While equations 17 and 17*a* remain unchanged, we must introduce in equation 18 in the place of heat of steam λ, as can easily be seen, the value $c_p T$ + constant and obtain

$$\frac{w\,dw}{g} = -\,c_p\,dT = -\,\frac{k}{k-1}\,d\,(p\,v) \quad . \quad . \quad . \quad (18a)$$

in which $k = c_p \div c_v$. If we introduce the acoustic velocity which corresponds to adiabatic change of condition without resistance, that is, the value

$$w_k = \sqrt{k\,g\,p\,v} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (21a)$$

we get from the equations 17, 17*a*, 18*a*, 19, with the abbreviations

$$\zeta = \frac{\zeta_r}{4\,r}, \qquad \alpha = \frac{w^2}{w_k^2}(k-1) + 1 \quad . \quad . \quad . \quad . \quad (21b)$$

the formula

$$\frac{dp}{dz} = \frac{\alpha\zeta - \dfrac{df}{f\,dz}}{w^2 - w_k^2}\,k\,p\,w^2 \quad . \quad . \quad . \quad . \quad . \quad . \quad (22a)$$

Lorenz with the exact Formula 22*a*, affirms our law that when the acoustic velocities are reached $\frac{dp}{dz}$ will generally $= \infty$; that is, the pressure line must have a vertical tangent, provided that the

numerator in equation 22*a* does not disappear. From his further very interesting conclusions the following is here given :

The velocities can reach a maximum in general only with decreasing pressures ; and these maximums are possible only within a tube of increasing cross-section.

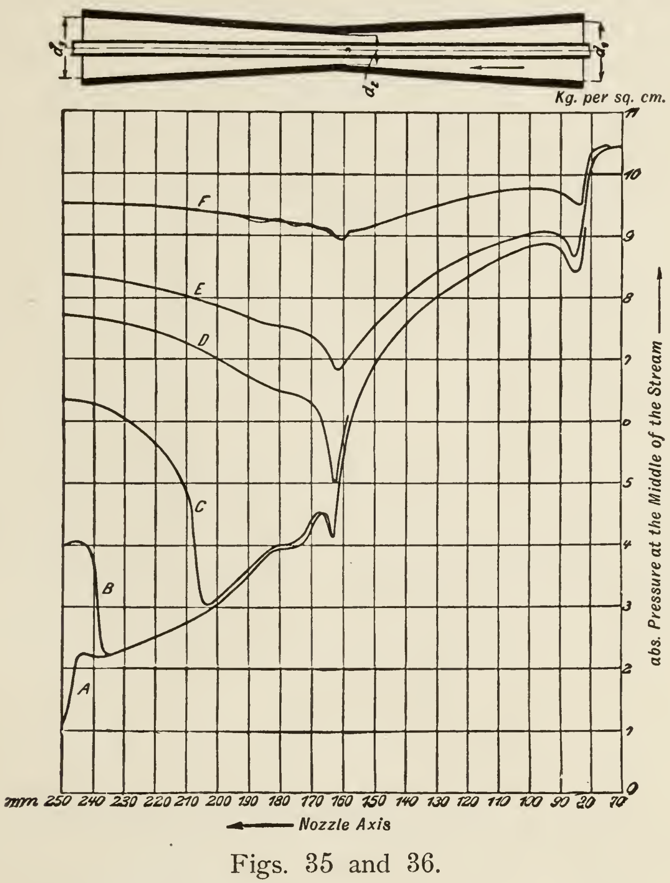
For the remaining deductions of *Lorenz* we must refer to the original work.*

21. THE NOZZLE WITH ELONGATED ENTRANCE.

The nozzle with elongated entrance, Fig. 35, constructed by laying together two converging nozzles, is to be used to bring out more clearly, by increasing the horizontal scale, the occurrences that take place at the narrowest part of the nozzle, and which, with ordinary nozzles, occur at a length of a few millimeters from the entrance. Curve *A*, Fig. 36, shows the change of pressure with free expansion; curve B, with a back pressure of 4 atm. absolute (58.8 lb. per sq. in. abs.). In the latter case the compression shock occurred just at the orifice, and showed a highly drawn-out rise of pressure. Curves *C*, *D*, *E*, and *F* are drawn with more and more back pressure. The most singular of these experiments are the waves which the curves show at the place of change from the converging to the diverging nozzle. Initially the first nozzle was about 0.1 mm. (0.004 inch) wider at its narrowest place than the other nozzle, so that a ridge was formed which was hardly noticeable. But after the nozzles were brought to an equal diameter by means of a reamer, the waves did not disappear. Only after polishing, and then only for certain degrees of superheat, were the middle waves finally gotten rid of, while they, for other degrees of superheat, still appeared. This phenomenon is explained by a change of sign of the numerator and denominator of equation 23, brought about by the quality of steam and the smoothness of the walls at various places of the nozzle. If the pressure curves run smooth, then the change occurs in one and the same section of the tube; in all others are the places

* The important discussion of Prandtl and Proell, mentioned on page 71, cannot be here placed into the text.

for numerator and denominator separate. A further complication is caused by the steam stream in the middle varying from those at the rim, which are exposed to the greatest friction.

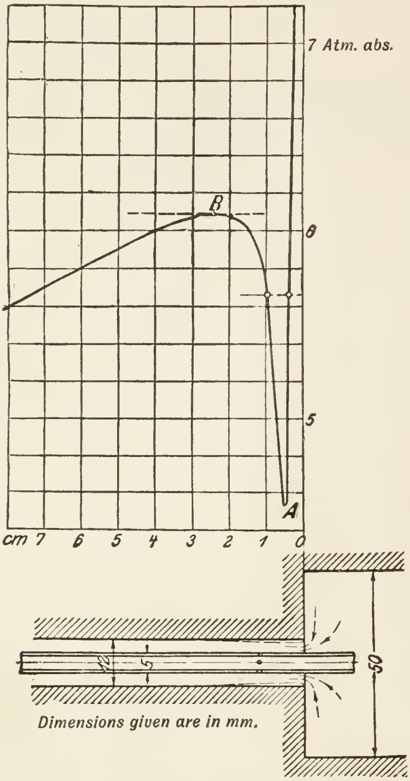


Figs. 35 and 36.

22. THE CONTRACTION OF THE STREAM.

The contraction of the stream at the entrance occurs always when the tube entrance has sharp edges, and can be seen in curve *F*, Fig. 33. It also is shown in the curves of pressure *A* to *E* in Fig. 35, but is especially noticeable with a cylindrical tube of 12 mm. (0.47 in.) diameter with sharp edges, having a meas-

uring tube of 5 mm. (0.2 in.) diameter, and arranged as in the experiment shown in Fig. 25, the tube being used instead of the nozzle. In Fig. 37*a* is drawn the outline of the stream, and above the observed pressure curve, Fig. 37. It is probable that the acoustic velocity has not yet been reached in this territory, therefore the denominator of Formula 23 remains negative. The numerator, on the other hand, has without doubt two changes of signs. It changes at *A*, Fig. 37, from positive to negative, and at *B* again to positive. *A* as well as *B*, therefore, represent zero points of the numerator, that is, vertical tangents; but at *A* a change of pressure occurs suddenly, as the velocity may have nearly approached the acoustic velocity, and therefore the denominator shows a small value.



Figs. 37 and 37*a*.

The Formula $\frac{dp}{dz}$ can be used for the calculation of the friction coefficients ζ_r from the slope of the tangent of the pressure curve. We can, in each case, calculate the experimental value of G, and from it w; and if w is known, the energy losses can be calculated without going back to $\frac{dp}{dz}$.

23. EXPERIMENTS ON THE FLOW OF STEAM FROM ORIFICES.

The orifices had a bore of about 12 mm. (0.47 in.), and were so built that in place of the nozzle in the measuring apparatus, Fig. 25, there was first used a 50 mm. (1.97 in.) diameter

Fig. 39.

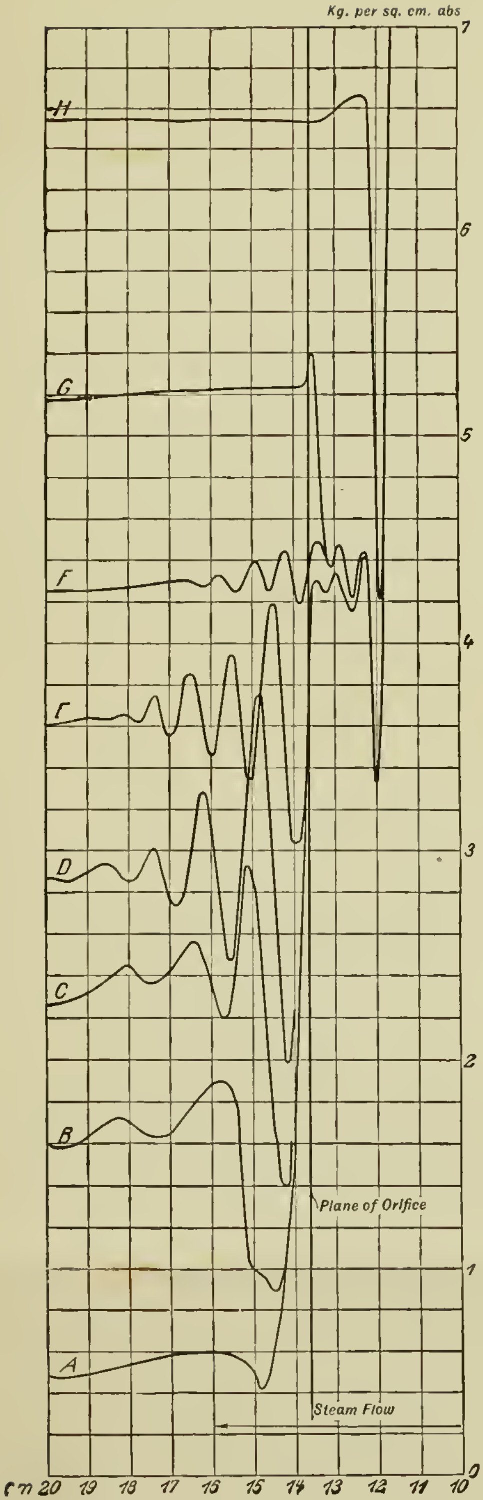


Fig. 41.

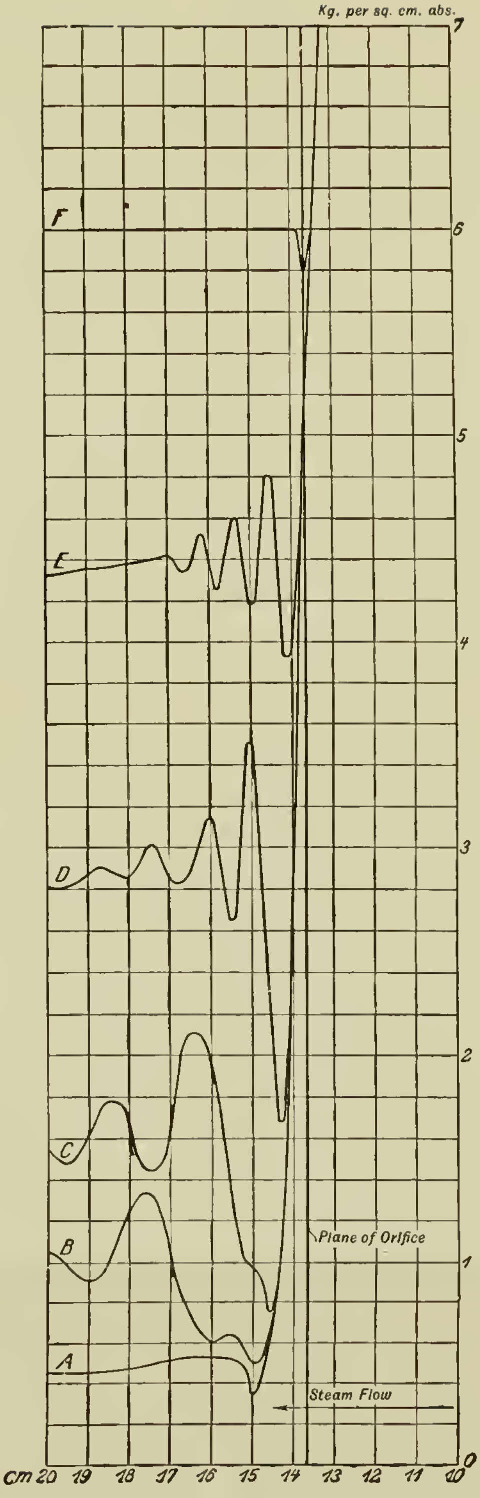


Fig. 43.

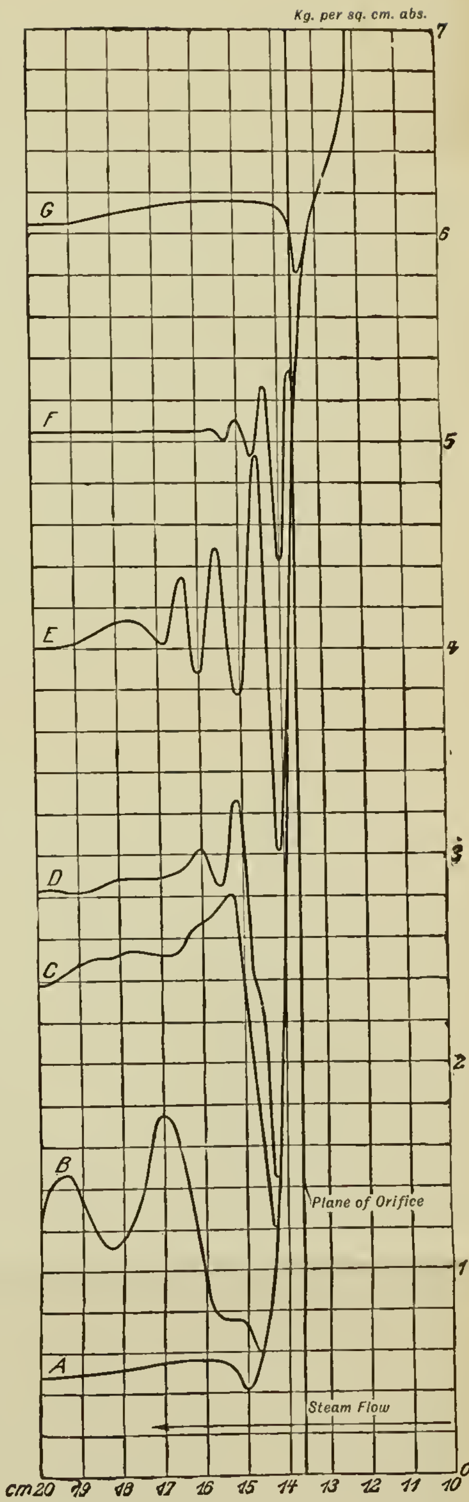


Fig. 38.

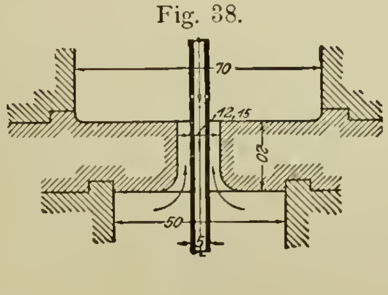
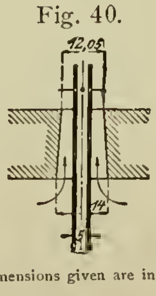
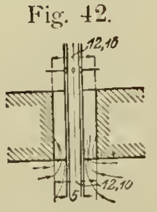


Fig. 40.



Dimensions given are in mm.

Fig. 42.



entrance pipe, then an orifice in the form of a hole in a bronze plate of 20 mm. (0.79 in.) thickness, and finally a 70 mm. (2.76 in.) diameter exit tube, while the connection to the condenser was again formed by tubes of 50 mm. (1.97 in.) diameter. The measuring tube had a diameter of 5 mm. (0.2 in.), and had holes of 1.5 mm. (0.06 in.) diameter at right angles to its outer surface.

Fig. 39 represents the change of pressure when an *orifice with rounded entrance* is used, as shown in Fig. 38. At outflow into a vacuum of about 0.4 kg. per sq. cm. abs. (5.69 lb. per sq. in. abs.), there appears, as far as we can see, a periodic variation of the condition of the steam; and it is highly probable that, only on account of the insufficient length of the measuring-tube it was not possible to observe the re-occurrence of the pressure variations. The very next curve *B*, at about 1.3 atm. (18.5 lb. per sq. in.) absolute back pressure, shows plainly the regular increase and decrease of pressure. The curves *C* and *D* peculiarly show (in spite of unchanged condition of the flow) only a very slight variation period. Exceedingly violent and fully regular are, on the other hand, the "*dampened*" *oscillations* that are shown in curve *E*; in curve *F* they decrease, and in curve *G* they entirely cease.

Totally similar pressure curves are obtained with the *conical orifice* as shown in Fig. 40, which has at both ends sharp edges. At entrance there is a small drop which cannot be represented in Fig. 41; at exit the fall of pressure is still more regular than with the rounded-off orifice. The periodic vibrations are questionable in curve *A*, but at *F*, on the other hand, they are without doubt not present any more.

There is a large difference when we use a *sharp-edged cylindrical orifice*, as shown in Fig. 42, on account of the unavoidable stream contraction which occurs at entrance. As can be seen from Fig. 43, first an expansion takes place to about 3.3 kg. per sq. cm. (46.9 lb. per sq. in.). From here the pressure quickly rises to 4.4 kg. per sq. cm. (62.6 lb. per sq. in.), and after several small oscillations, drops to the vacuum pressure. The contraction causes the same effect as if the orifice at entrance was a conically divergent nozzle, and thus decreases the pressure at the orifice as compared with the former experiments. The change of

pressures behind the orifice is again the same, and shows especially at curve *D* beautifully drawn-out oscillations. In curve *G*, with about 5.2 kg. per sq. cm. (73.9 lb. per sq. in.) back pressure, there occurs just before exit a very noticeable compression shock. In *H* we have only the deep pressure drop due to the stream contraction.

The experiments give the desired explanation of the so often spoken of phenomena of flow from orifice. As is known, *Mach* * and *Emden* † have proven by means of photographs the presence of regular successive light and dark lines in the outflowing stream which cannot be said to be anything else but acoustic waves; but as to the value of the existing pressures in this stream we were totally in the dark.

Emden assumes that at the places of compression the same condition exists as in the orifice (p. 440). But he says (p. 436) in contradiction to himself, that there exists the same pressure at every place in the stream as exists in the surrounding atmosphere, and claims that there is a change of density. According to this, for instance, for air, places of smallest velocity, that is smallest kinetic energy, must coincide with places of smallest temperature, that is, smallest potential energy, which is obviously impossible. By his calculations he further believed to have proved that the difference between the initial and the orifice pressures could only be used to produce the additional velocity of flow; the remainder of the available work is transposed into "acoustic energy." Our experiments contradict these opinions; they prove *that the steam at first expands to the existing pressure before the orifice, that therefore during the first rush* (as in the case of suddenly releasing a compressed spring) *too much potential energy is changed into kinetic energy.* Only this excess changes into acoustic vibrations and is rechanged into heat at the rim of the stream by friction and eddy currents.

The oscillations occur in the axial direction as well as in the radial. The stream flows at the pressure of the orifice into a region of much smaller pressure, begins therefore to expand in the radial direction. The resulting drop of pressure accelerates the particles also in the axial direction.

* E. Mach and P. Salcher, Wiedemanns Annalen, 1890, Vol. 41, p. 144.

† R. Emden, Wiedemanns Annalen, 1899, Vol. 69, p. 264.

24. FLOW FROM A CONICALLY DIVERGING NOZZLE INTO THE ATMOSPHERE.

The stream flowing from a diverging nozzle shows the same phenomena as from the simple orifice. Fig. 44 represents the change of pressure of a stream flowing from a nozzle of about 7 to 12 mm. (0.276 to 0.472 in.) diameter. The pressure at

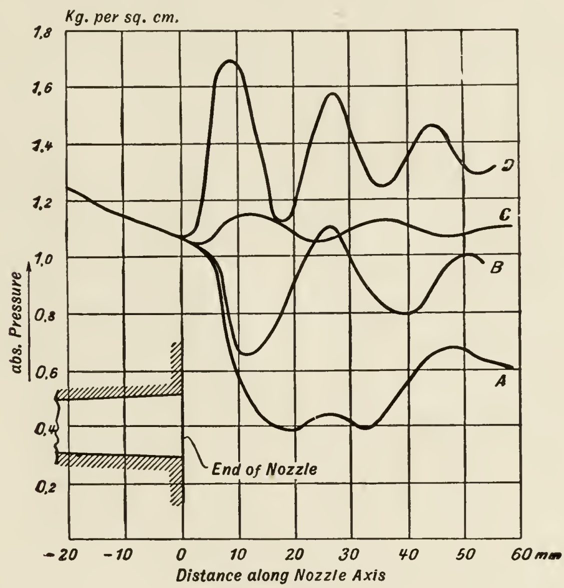


Fig. 44.

the end cross-section of the nozzle reached about 1.05 kg. per sq. cm. (14.93 lb. per sq. in.) absolute. The stream exhausted into an experimental chamber as described below, in which the pressure could be varied. If vacuum exists, the steam expands according to curve *A;* with only partial vacuum according to *B.* The very smallest pressure above atmosphere, as in curve *C,* already shows small oscillations; with somewhat larger pressures we obtained the well drawn out acoustic oscillations according to curve *D.* Fig. 44*a* represents a second series of experiments in which the steam in the nozzle expanded to about 0.7 kg. per sq. cm. (9.95 lb. per sq. in.) absolute. Exhausting into a higher

vacuum gave very regular acoustic vibrations, as in curve *A*. At *B* it was possible to so fix the back pressure that *every trace of a vibration disappeared.* As soon as the back pressure increased, the vibrations again occurred as curve *C* shows. Curve *D* finally represents such a high back pressure, that the rise of pressure is extended into the interior of the nozzle, and on account of these conditions, the vibrations became considerably less intense than in the first experiment. The course of the regular expansion

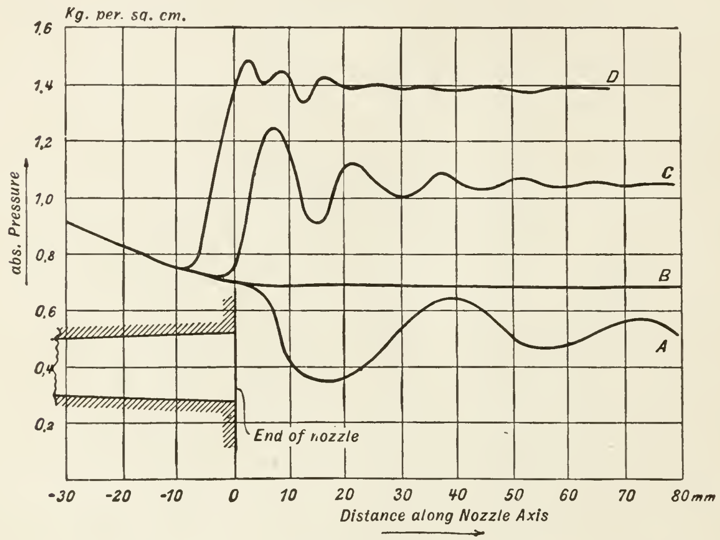


Fig. 44*a*.

lines in the interior of the nozzle is not influenced by the back pressure, and at this place no vibrations occur.

From these experiments may be said that the steam in a nozzle expands nearly adiabatically independent of the back pressure. *If the stream flows into a space in which a back pressure exists exactly equal to the final pressure of expansion, then the pressure in the stream is not changed at all. If the back pressure is lower, acoustic vibrations occur, as with a simple orifice; if the back pressure is too high, a steam shock occurs with more or less drawn-out oscillations.* By entirely filling out the cross-section of a diverging nozzle, a vibration is made difficult, if not entirely impossible. We can hardly go wrong if we say that the drawn-out vibrations occurring

in a simple orifice are caused primarily by the sudden difference of pressures at the stream's rim and the surrounding atmosphere, which the stream quickly spreads. There is, therefore, the possible assumption, *that when vibrations still occur in the interior of the nozzle, the stream has detached itself at such places from the wall.* The absence of any pressure vibrations in the observed regular expansion line, is inversely a further proof that the stream entirely fills out the cross-section.*

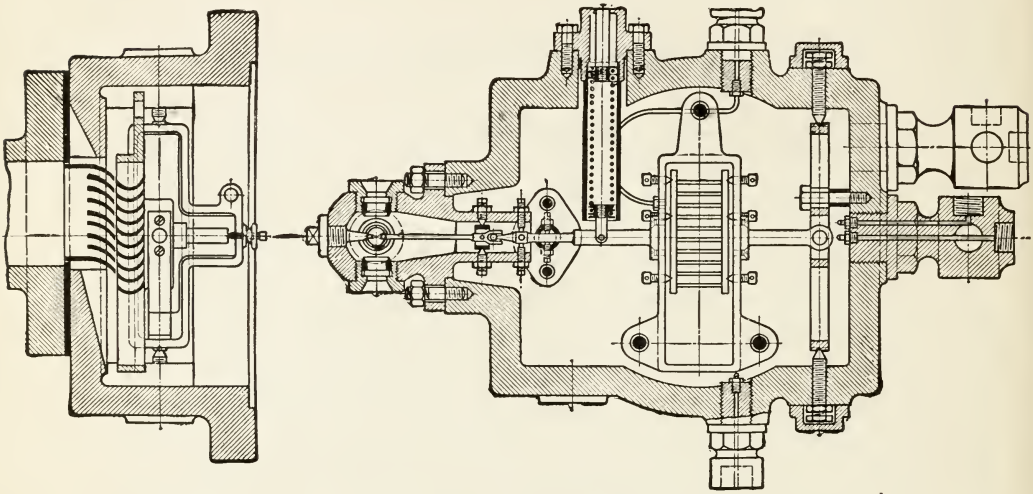
25. EXPERIMENTS WITH TURBINE BLADES.

It is known that the resistance of rotating wheels in motion may be very much different from those in a position of rest, because of the constantly changing influences, due to the narrowing of the channels in the rotating wheels by the blades of the guide wheels; still experiments with stationary blades, although representing an imperfect condition, may give much desired information. In order to carry out such experiments the arrangement shown in Figs. 45 and 45*a* was devised and used. It consists of an enclosed space in which a frame was hung as shown, for the support of the rotating blades. The friction caused by the supporting center points was found to be harmless, as was to be expected; because the flow of steam occurred with so much vibration that the

* The presence of oscillations at the exit of steam from nozzles has already been observed by Chief Engineer *Kienast*, *Prof. Gutermuth*, and *P. Emden*. The experiments of the latter were published by *A. Fliegner* in Schweiz. Bauzeitung, 1903, Vol. XLI, p. 173. *Emden's* nozzle had a diameter of 5.5 to 11 mm., with about 30 mm. length. It was, therefore, too divergent for an initial pressure of 5 kg. per sq. cm. and atmospheric back pressure, and on account of this, and also on account of the sharp edge at entrance, we can easily see that the stream detached itself from the walls of the nozzle. Any results that would be derived from these unfavorable circumstances cannot be taken as correct as our experiments show. Contradicting the opinion held by many that with the Laval turbine the steam cannot reach any higher velocity than the acoustic velocity, that is, only about 450 m. (1 476 4 ft.), we draw attention to the fact that these turbines could not show a steam consumption of only 7 kg. per h. p.$_e$ hour (15.6 lb. per English h. p.$_e$ hour). The theoretical velocity is higher than 1 000 m. (3 280 ft.); if only so much energy remains as would correspond to the kinetic energy of 450 m. (1 476.4 ft.) velocity and should the remainder be transposed into heat through acoustic vibrations, then even an ideal turbine could not transpose more than 25 per cent. of the available energy into work, while actually more than 50 per cent. is produced. Facts lay aside, therefore, these incorrect opinions.

very small friction could not cause any jamming. The two turning axes at right angles to each other were for the determination of the circumferential and the axial components of the pressures of the steam reaction.

In order to accomplish this, a vertical and horizontal spring balance were attached to the frame, which could be balanced by means of weights, and which would register the pressure by means



Figs. 45 and 45*a*.

of a micrometer screw. An extension of the frame moves a light pointer which registers every movement tenfold, and by means of a fixed mark which can be observed through two glass windows, the frame can be brought exactly to the same point in the horizontal as well as in the vertical position. After we note the existing spring pressures of the unloaded frame in its zero position, we allow the steam to enter and then adjust the frame again to its zero position. The difference of the spring pressures gives the exerted forces, and in this way we measure the tangential and the axial components T and A of the total reaction of the steam, Fig. 46.

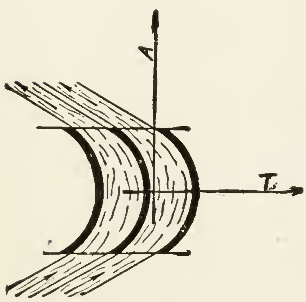
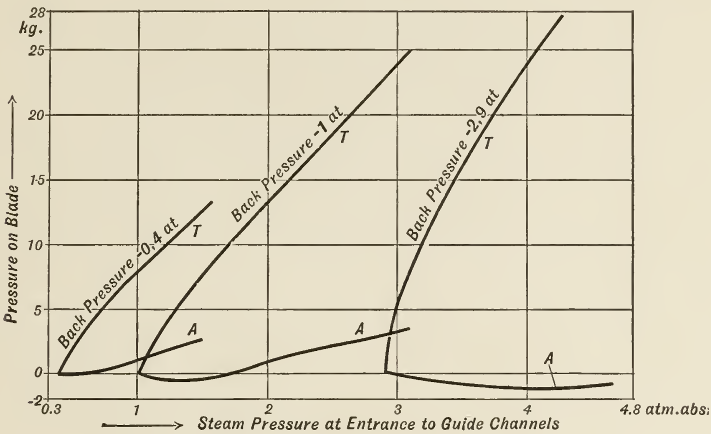
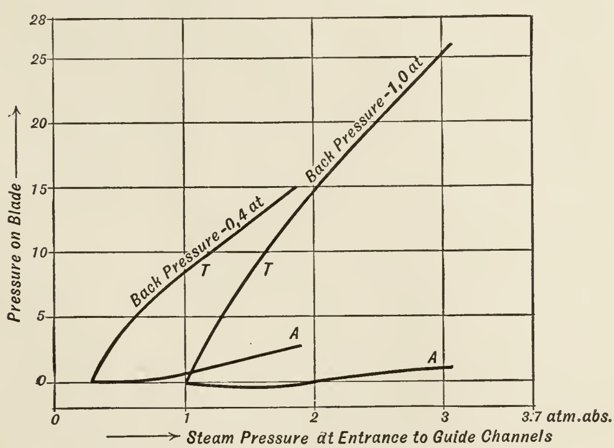


Fig. 46.

The blade model was made of sheet bronze with blades having everywhere the same thickness. There were tested: 1. — Guide and rotating wheels, each 30 mm. (1.18 in.) wide with a clearance



Breadth of the guide wheel, 30 mm. (1.18 in.).
Breadth of the rotating wheel, 30 mm. (1.18 in.).
Clearance, 0.8 mm. (0.031 in.).

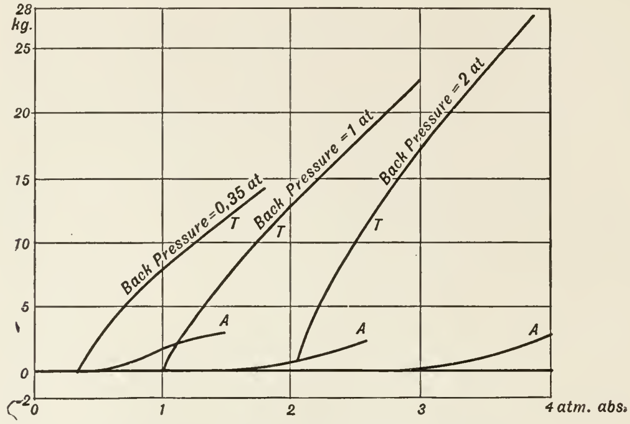


Breadth of the guide wheel, 30 mm. (1.18 in.).
Breadth of the rotating wheel, 30 mm. (1.18 in.).
Clearance, 4.5 mm. (0.177 in.).

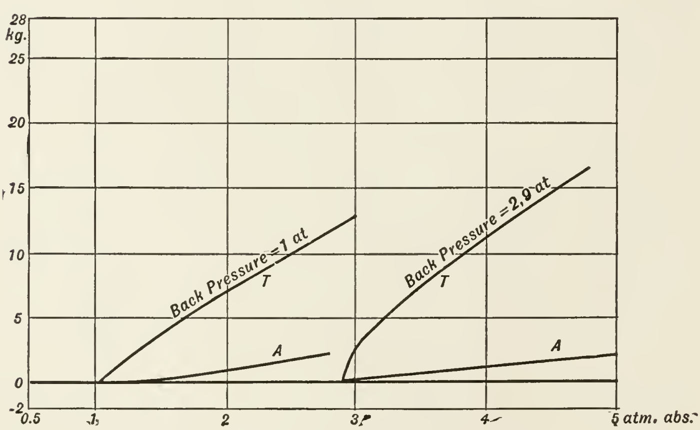
Figs. 47 and 48. Circumferential Pressure *T* and Axial Pressure *A*.

of about 0.8 mm. (0.031 in.); 2. — The same with a clearance of about 4.5 mm. (0.177 in.); 3.—The same rotating wheel with a guide wheel of 25 mm. (0.98 in.) and clearance of 1 mm.

(0.039 in.); 4. — The same rotating wheel with a guide wheel of 15.5 mm. (0.61 in.) and a clearance of 1 mm. (0.039 in.). The exit angle of the guide wheel and the entrance and exit angles



Breadth of the guide wheel, 25 mm. (0.98 in.).
Breadth of the rotating wheel, 30 mm. (1.18 in.).
Clearance, 1 mm. (0.039 in.).



Breadth of the guide wheel, 15.5 mm. (0.61 in.).
Breadth of the rotating wheel, 30 mm. (1.18 in.).
Clearance, 1 mm. (0.039 in.).

Figs. 49 and 50. Circumferential Pressure *T* and Axial Pressure *A*.

of the rotating wheel were all equal to 30°. Figs. 47 to 50 represent the derived results in the above order. The abscissas represent the pressures at the guide wheels; the pressure in back of the

rotating wheels is noted on each curve. The ordinates are the blade pressures in kilograms. The steep curves represent the circumferential components, the less steep ones the axial pressure. Both reach the value naught when the pressure in front of the rotating blades is equal to the back pressure. As the boiler pressure remained unchanged at about 10 atmospheres (147 lb. per sq. in.), the steam was somewhat superheated by throttling.

Most noteworthy in these curves is that in cases 1 and 2 the axial force, in spite of the undoubtedly existing blade friction, becomes negative at small pressures above atmosphere, and the more so, the greater the pressures. The reason of this is perhaps that by the existing equal number of guide and rotating wheel channels the cross-section at the end of the guide channel forms the narrowest section of the entire passage formed by the guide and rotating wheel blades. At small pressures above atmosphere an expansion occurs below the pressure of the surroundings, so that the outer or surrounding pressure receives the overweight and the blades press against the guide wheel. Case 2 shows how great this influence is; in spite of a clearance of 4.5 mm., a negative pressure exists. Of course the change of pressure in the clearance space must still be more closely investigated experimentally.

From the steam mass per second M, which is observed, and from the condition of the steam before and after the blades, there can also be calculated, with the theoretical velocity w, the theoretical blade pressure

$$P_0 = 2\,M w \cos \alpha$$

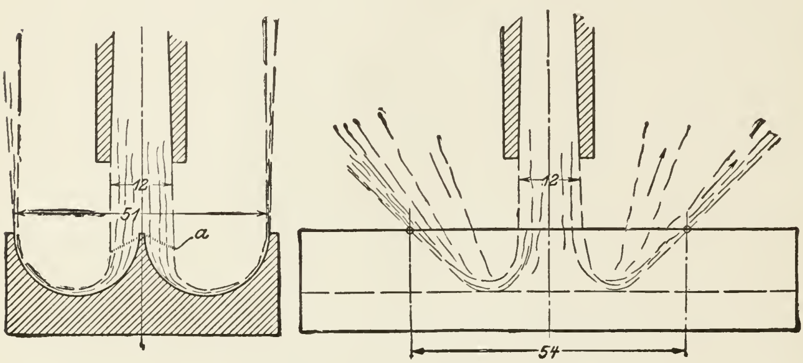
in tangential direction for frictionless flow (in which α is the entrance and exit angle of the rotating blades). As the losses in the guide blades are small because of the gradual increase of velocity, we may say, as a first assumption, that at exit from the guide wheel the theoretical velocity exists, which drops to a smaller exit value w' in the rotating blades on account of friction. The effective tangential pressure would then be

$$P_e = M\,(w + w') \cos \alpha,$$

which allows w' to be found. The loss of kinetic energy in the blades in terms of the available energy is

$$\zeta = \frac{w^2 - w'^2}{w^2}.$$

A further experiment with rotating blades was made with "The Limit Turbine" having everywhere equal cross-section, therefore constructed with an enlargement in the middle of the blades. There was obtained with a velocity of about 400 m. (1 312.3 ft.) the coefficient of loss of kinetic energy (as above figured) $\zeta = 0.30 - 0.40$, and, as was natural, the smallest values when the blades (with equal divisions in the guide and rotating wheels) stood oppo-



Dimensions are given in mm.

Figs. 51 and 52.

site to one another, and was largest when they were transposed. It could not be determined whether the losses decreased with smaller velocities. The experiments will be continued.

A peculiar picture is presented by the reflection of a steam stream against an open bucket of *Pelton* form, as in Figs. 51 and 52. The stream flowing from a 7 by 12 mm. (0.276 by 0.472 in.) nozzle, on striking the bucket spreads in an unusual degree. The somewhat compressed rim of the stream leaves the bucket at a breadth of about 54 mm. (2 13 in.), that is, $4\frac{1}{2}$ times the nozzle width. A small part of the steam spreads still further. On account of this great spreading the stream appears as a veil of only paper thickness. At *a* there is a noticeable point of compression that is doubtless caused by the steam striking the edge.

These facts teach us to be careful; especially is it to be noted that in designing a turbine we can promise success only when the designer, by making numerous experiments, has become acquainted with the phenomena of steam flow.

B. THE LAWS OF ENERGY IN THE STEAM TURBINE.

26. THERMODYNAMIC EFFICIENCY.

We compare the effective power L_e of a turbine, taking into account the steam and bearing frictions for a given initial condition of the steam and a given condenser pressure, with the power L_0 of an ideal turbine in which no friction occurs and in which the energy of the steam is completely utilized; that is, so that the exit velocity drops to 0. The same is obtained from one pound (or kilogram) of steam in a frictionless reciprocating engine without steam throttling, and with non-conducting cylinder walls, which has no clearance spaces and expands its steam to the condenser pressure.

The ratio

$$\eta_e = \frac{L_e}{L_0} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

we call the *thermodynamic efficiency, referred to the effective power.*

If we designate the heat contents or total heat of steam at the initial condition as λ_1, and that, after adiabatic expansion to the condenser pressure, as λ_2', then the theoretical power is, according to what has been said before, in meter kilograms for one kilogram of steam, or in foot-pounds for one pound of steam

$$L_0 = \frac{(\lambda_1 - \lambda_2')}{A} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

The total change of heat Q_0 is much greater than $A\,L_0$, and approaches more or less closely to λ_1, according to the feed water temperature. The "total efficiency" is the ratio

$$\eta_0 = \frac{A L_e}{Q_0} \quad \ldots \ldots \ldots \quad (3)$$

The determination of L_0 is simplified by means of the entropy table; *Rateau* (Annales des Mines, 1897) and *Mollier* (Z. 1898) have also given empirical formulæ from which we can calculate L_0. *Rateau* gave for saturated steam,

$$D_0 = 0.85 + \frac{6.95 - 0.92 \log p_1}{\log \left(\frac{p_1}{p_2}\right)} \quad \ldots \ldots \quad (4)$$

Mollier gave for saturated steam,

$$D_0 = \frac{6.87 - 0.9 \log p_2}{\log \left(\frac{p_1}{p_2}\right)} \quad \ldots \ldots \quad (5)$$

for superheated steam,

$$D_0' = \frac{D_0}{1 + 0.000755 \left[(T' - T) - T_1 \text{ nat log } \frac{T'}{T} \right] D_0} \quad . \quad (6)$$

In which,

D_0 and D_0' = the steam consumption for the entire turbine in kg. per h. p. per hour.
p_1 = the initial pressure in kg. per sq. cm.
p_2 = the final pressure in kg. per sq. cm.
T = the absolute temperature of saturation.
T' = the absolute temperature of superheat.
T_1 = the absolute final temperature (exhaust).

In the English units, *Rateau's* formula for saturated steam is

$$D_0 = 1.9 + \frac{17.91 - 2.056 \log p_1}{\log \left(\frac{p_1}{p_2}\right)};$$

Mollier's formula for saturated steam

$$D_0 = \frac{17.68 - 2.011 \log p_2}{\log \left(\frac{p_1}{p_2}\right)};$$

and for superheated steam,

$$D_0' = \frac{D_0}{1 + 0.0001876\left[(T' - T) - T_1 \text{nat} \log \frac{T'}{T}\right] D_0}.$$

In which

D_0 and D_0' = steam consumption for the entire turbine in lb. per h. p. per hour.
p_1 = the initial pressure in lb. per sq. in.
p_2 = the final pressure in lb. per sq. in.
T = the absolute temperature of saturation.
T' = the absolute temperature of superheat.
T_1 = the absolute final temperature (exhaust).

In the French system a horse power for one hour gives 270 000 meter kilograms, or 637 heat units*; if D_0 kilograms of steam were used, then the work exerted on one kilogram is

$$L_0 = \frac{270\ 000}{D_0} \text{ m. kg.,}$$

or the useful transposed heat is

$$\lambda_1 - \lambda'_2 = A\ L_0 = \frac{637}{D_0} \text{ heat units} \quad . \quad . \quad . \quad . \quad (7)$$

In the English units

$$L_0 = \frac{1\ 980\ 000}{D_0} \text{ ft. lb.}$$

The useful transposed heat is

$$\lambda_1 - \lambda_2' = A\ L_0 = \frac{2\ 544.65}{D_0} \text{ B. t. u.}$$

---

* This number was taken to correspond with the value of $A = \frac{1}{424}$ meter kilograms which is used in the steam tables.

In practical steam turbine problems the translator has taken the privilege of correcting Formula 7, placing for 637 the value 632.47 as the number of calories in one metric horse power hour. This was done to bring the results deduced from the French units into agreement with those in English units.

26a. FINAL CONDITION OF THE STEAM IN ANY STEAM TURBINE.

We shall assume that a certain turbine is selected and its steam consumption D_e per h. p.$_e$ hour experimentally determined. Also that we have given the bearing friction, the power needed to drive an air pump, etc.; in fact, *all those outside consumptions of power, whose values do not again return to the steam as heat.* The sum of these powers is added to the effective power and gives the total external work that the steam performs, equal to N_e'. In N_e' *is not included,* for instance, the steam friction on the turbine wheels, because this is entirely changed into heat, and again appears as a part of the heat contents of the steam. From N_e' we get the total steam consumption $D_e' = D_e \frac{N_e'}{N_e}$ in kilograms per hour, or in pounds per hour, which must not be confused with D_e. From D_e we get finally the absolute delivered external work in meter-kilograms, or foot-pounds, that is furnished by 1 kilogram or 1 pound of steam.

In the French units,

$$L_e' = \frac{1}{A} \cdot \frac{637}{D_e'}.$$

In the English units,

$$L_e' = \frac{1}{A} \cdot \frac{2544.65}{D_e'}.$$

By experiment is determined the initial condition A_1 in Fig. 24, and the final pressure of the expansion, p_2. Further we can estimate the losses due to conduction and convection, which we will designate as Q_s per kilogram or pound of steam. If c_2 is the exit velocity (from the last rotating wheel of the turbine), then the steam carries away the kinetic energy $\frac{c_2^2}{2g}$. According to Formula 1, Article 14, we have

$$\lambda_1 - \lambda_2 = AL_e' + Q_s + \frac{Ac_2^2}{2g} \quad . \quad . \quad . \quad . \quad (1)$$

and from this λ_2 and also x_2 can be determined. At a more distant point in the exhaust pipe c_2 will have changed into the smaller value c_2', upon which depends the corresponding increase of λ_2.

As c_2 itself depends on the final condition, we can, by determining p_2 and x_2, assume c_2 for trial, and calculate the specific volumes, and the relative and the absolute exit velocities. If we place c_2 in equation 1, we must, if the assumption was correctly made, get the assumed λ_2. In general, Q_s is negligibly small, and the third part of the equation of small value, so that a very approximate calculation of the same suffices.

In any new design of a turbine the values of the losses, as we shall explain later, must be estimated; c_2 chosen at will and from this λ_2 determined.

AXIAL TURBINES.

27. THE SINGLE STAGE IMPULSE TURBINE.

A. LARGE PRESSURE DIFFERENCES; DESIGN OF NOZZLE.

If we construct the nozzle according to the formula of *Zeuner*, the final pressure will be greater than the prescribed back pressure, because friction transposes the kinetic energy into heat, and the expansion line will rise above the adiabatic. A nozzle calculated according to *Zeuner* would actually be somewhat too short. *Rateau*, and after him *Delaporte*, have proved by experiments that the pressure of a stream on blades or suitable plates decreases very little if the nozzles are made short; that is, the stream is allowed to flow out at a slight overpressure. But our experiments on page 83 show, that violent acoustic vibrations then take place in the stream, which we should always try to avoid. It is easy to make the nozzle dimensions suitable to the assumed back pressure; We will choose in the entropy diagram as condition curve starting at point A_1, Fig. 24, not the adiabatic $A_1 A_2'$, but the curve $A_1 A_2$, in which to be sure, the intermediate path remains yet undetermined; but A_2 must be so chosen on the prescribed curve $p_2 =$ constant, that area $A_1' A_2 A_2'' A_1''$ must be equal to the losses of kinetic energy; that is

$$\lambda_2 - \lambda_2' \text{ must equal } \zeta\,(\lambda_1 - \lambda_2') \quad . \quad . \quad . \quad . \quad (8)$$

in which ζ is equal to the values as given in Article 16. Now proceed as in Article 4; that is, read at a few intermediate points the values of the corresponding pressures p_x, volume v_x and the heat of

steam λ_x. With permissible negligence of the inflowing velocity w_1, we get the corresponding velocity from the equation

$$A \frac{w_x^2}{2g} = \lambda_1 - \lambda_x,$$

which is shown in Fig. 4 as a function of the pressure p_x represented from right to left. With $\gamma_x = \frac{1}{v_x}$ as the specific weight at that point, we get from the "equation of continuity"

$$G_{sc} = f_x w_x \gamma_x.$$

The nozzle cross-section corresponding to p_x is

$$f_x = \frac{G_{sc}}{w_x \gamma_x}.$$

We find graphically the minimum cross-section f_m with its corresponding pressure as well as the final cross-section f_2 and the ratio of divergence

$$\frac{f_2}{f_m}.$$

From this the nozzle can be drawn as in Article 4.

B. OCCURRENCES IN THE BLADE CHANNEL.

The steam, on leaving the nozzle, flows out, as we shall assume, in parallel steam threads, and is compelled to take a more or less sharp bend and thereby exerts a "centrifugal pressure," which greatly compresses the steam mass on the concave side of the channel. On the convex side, conversely, an expansion occurs, and the steam particles will approximately describe the diverging stream lines shown in Fig. 53. Besides this first unequal pressure in the direction of the radius of the channel, we must take care of a second in the direction of the stream, which is caused by friction. The occurrences are perhaps, all in all, the same as in a cylindrical straight tube, if we assume as constant the cross-section for the entire blade length, measured at right angles to the flow. If the entrance velocity had been smaller than the acoustic velocity of the corresponding condition, then the pressure in the direction

of the stream would have decreased. This takes place when the pressure ratio (for saturated steam) is smaller than 1.7. Here we would expect a very slight over-pressure in the clearance space, also a slight loss due to leakage, so long as there is no tendency for expansion and abnormal phenomena caused by the finite thickness of the blades, which was discussed in Article 25.

Should the pressure ratio be greater than 1.7, then the acoustic velocity is exceeded, the pressure in the blades at first increases in the direction of the stream-flow, and only at the end of the blade channels does it again decrease to the pressure of the surroundings.

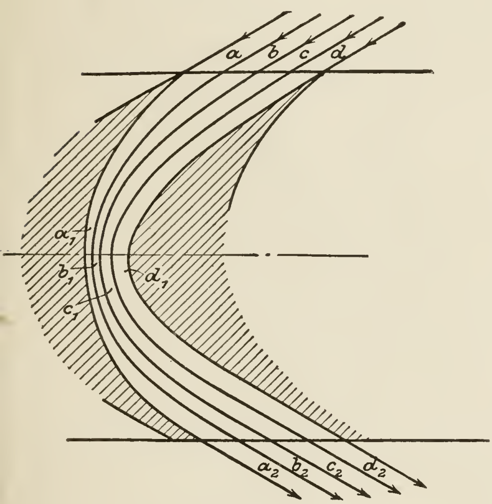


Fig. 53.

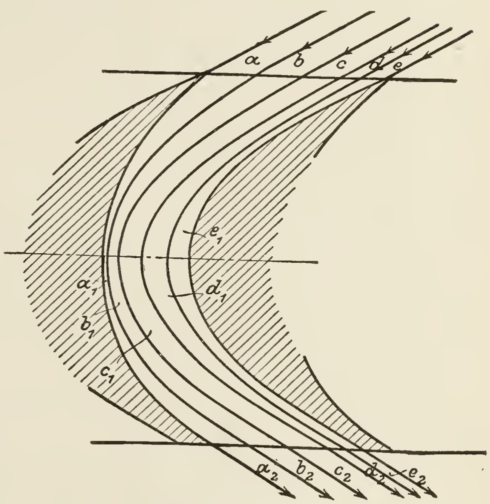


Fig. 54.

The velocity will decrease to the point of greatest compression, then again increase so that the final value (w_2) always remains smaller than the initial value (w_1).

If the blades are made of tin bent to shape (of constant wall strength), then the channel forms in the middle, an enlargement of section, and we must expect an expansion and then a compression shock, as was noticed in the experiments in Article 18, Fig. 33. Even with an uniformly equal width of channel, we have in the boundary stream thread on the convex side an expansion as shown in Fig. 53, at d_1 and e_1; therefore here also is a probability of steam-shock taking place.

These disadvantages *Curtis*, in his patents, tries to avoid by narrowing the channel at the place of greatest bending (Fig. 54), so that the outer stream thread (shown in Fig. 54 at d) experiences *no* expansion, hence no *pressure change*, and gives no cause for a steam-shock.

Experiments investigating these phenomena have evidently great value.

C. THE CONSTRUCTION OF THE VELOCITY DIAGRAM AND THE DETERMINATION OF THE POWER.

This follows exactly as we have explained in Article 7.

For the impulse turbine with radial Pelton buckets the velocity diagram shown in Fig. 55 is used. From c_1 we get through com-

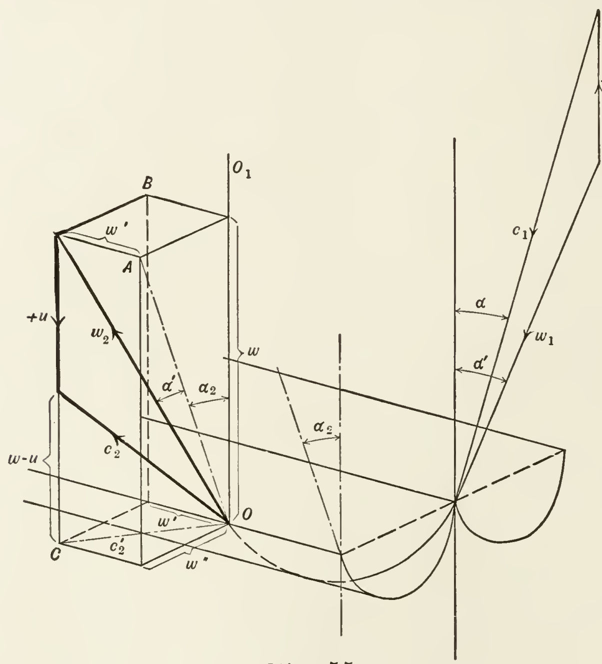


Fig. 55.

bination with $(-u)$ the velocity w_1. The stream divides to both sides, so that the center line of the stream describes approximately a helix with an angle of slope α'. We can easily convince ourselves that the additional forces (such as centrifugal, etc.,) of relative movement will give a negligibly small deviation. The projection

OA of the relative velocity w_2 then forms with the circumferential tangent $O\,O_1$ the angle α_2, while the slope of w_2 towards OA gives the entrance angle α'. If now again $w_2 = \psi w_1$, then follows, using the notations in the figure:

$$OA = w_2 \cos \alpha',$$

$$w = OA \cos \alpha_2 = w_2 \cos \alpha' \cos \alpha_2,$$

$$w' = w_2 \sin \alpha',$$

$$w'' = OA \sin \alpha_2 = w_2 \cos \alpha' \sin \alpha_2.$$

From this we get

$$c_2'^2 = w'^2 + w''^2,$$

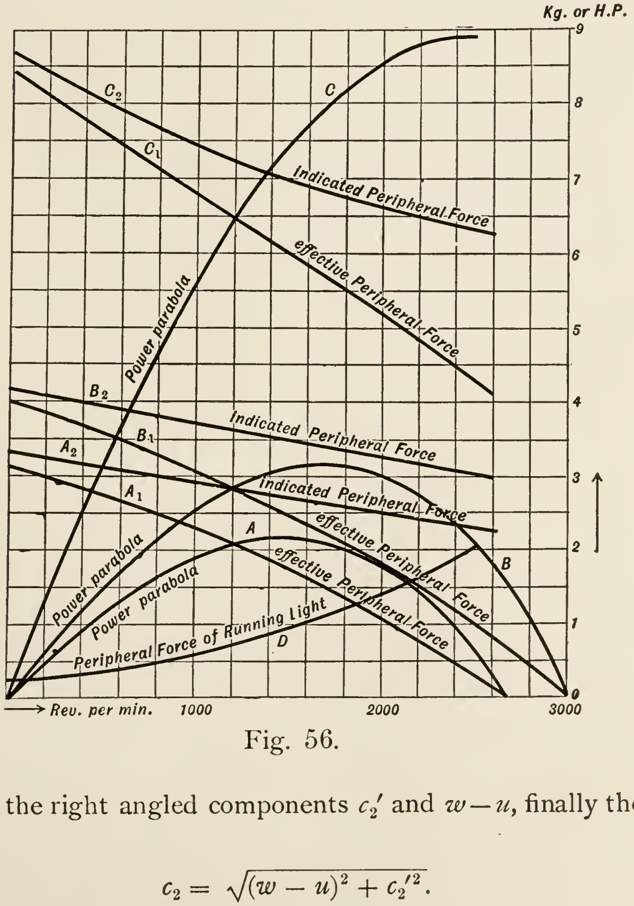


Fig. 56.

and from the right angled components c_2' and $w-u$, finally the exit velocity

$$c_2 = \sqrt{(w-u)^2 + c_2'^2}.$$

With the decided spreading of the stream the value of c_2 is of course only to be taken as a rough approximation. The inferiority of this blade form is proved by the experiments on page 90, and its application cannot be recommended.

In Fig. 56 are the well-known parabolas, which represent the power of an impulse turbine as a function of the circumferential velocity, for a 10 h. p. Laval turbine at the mechanical laboratory of the Polytechnicum in Zürich. The circumferential forces vary little from straight lines. There is also added the circumferential force corresponding to the total friction, but it could not be found with sufficient accuracy, and on this account we omit from the calculation the thermodynamic efficiency of this experiment.

D. SMALL PRESSURE DIFFERENCES.

If we have to design an impulse wheel as a part of a many-stage turbine for small differences of pressure, in which the ordinary blades are used as a guide wheel instead of nozzles, we would proceed with our calculations in a similar manner. We would hardly make a great error if the gradually retarded movement through the guide channels is assumed as without resistance, and imagine the entire losses as concentrated in the rotating wheel.

28. THE FEW STAGE IMPULSE TURBINE.

A. ONE PRESSURE AND SEVERAL VELOCITY STAGES.

In regard to the nozzle we refer to what has been previously said. λ_2 is the heat contents at the nozzle end. The change of condition in the blades of the rotating and guide wheels is, as we have above described, a complicated occurrence. Still we must assume for the impulse turbine, that in each clearance there exists the pressure of the surroundings. This assumption allows us, in the entropy diagram, to enter on the curve of the condenser pressure $p_2 =$ constant, the corresponding points, and then get the correct specific volumes. To accomplish this we calculate the resulting loss per pound (or kilogram) of steam up to the clearance space in front of the second guide wheel, which is

$$L_z = \frac{c_0^2 - c_1^2}{2g} + \frac{w_1^2 - w_2^2}{2g} \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

and which is changed into heat. The corresponding steam condition requires, therefore, a point on the curve $p_2 =$ constant, with the heat of steam

$$\lambda_3 = \lambda_2 + A\,L_z \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

In the second guide wheel there is added to λ_3 also

$$A\,L_z' = A\frac{c_2^2 - c_1'^2}{2g} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

that is, the point referred to has been moved more to the right, and so on.

The determination of the cross-section is now obtained by aid of the law of continuity, by letting v_1, v_2, v_1', v_2' . . . be the existing specific volumes in the successive clearance spaces, counting from the nozzle end, and figuring with the axial velocities w_{1n}, w_{2n}, c_{2n}, c_{1n}' . . . as though full circumferential admission existed. We then have as the diameters are the same for all wheels, and for infinitely thin blades, the same designation as in Article 10, in which we should observe that

$$\left.\begin{array}{ll} a' = a_1' & a'' = a_1'' \\ a_0' = a_2 & a_0'' = a_2' \end{array}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

$$\frac{a_1 w_{1n}}{v_1} = \frac{a_2 w_{2n}}{v_2} = \frac{a_1' c_{1n}'}{v_1'} = \frac{a_2' w_{2n}'}{v_2'} \quad . \quad . \quad . \quad . \quad . \quad (5)$$

a_1 is determined by the nozzle; equation 5 gives $a_2\, a_0'\, a_1'\, a_2'$ . . . which must be corrected in the design because of the thicknesses of the blades.

We could ask, if there would not be an improvement in making the blades of such form, that the pressure is forced to remain constant. For a straight cylindrical tube this question has been answered in the affirmative on page 77 by equation 23*a*. We had to construct the tube with a slight enlargement of section as an expansion nozzle. Unfortunately the necessary enlargement is so slight that for a blade, we can expect nothing from this because of the great interference caused by its bending.

B. SEVERAL PRESSURE STAGES, EACH WITH A VELOCITY STAGE.

We start with the assumption that the outflow velocity c_2 of any wheel is used up (to a negligibly small value) by eddy currents, that change their corresponding kinetic energy into heat.

If only five to ten wheels are to be used, mark approximately in the entropy diagram the pressure stages to which the steam shall expand for each individual wheel. Then construct the first wheel, that should work between the pressures p_1 and p_2 as was shown in Article 27. To calculate the next wheel, find for the same, the correct initial condition of the steam. The initial pressure will be p_2; the quality of steam or the superheat is raised according to adiabatic expansion. The total work of resistance and the exit losses for one pound (or kilogram) of steam in the first wheel is

$$L_z = \frac{c_0^2 - c_1^2}{2g} + \frac{w_1^2 - w_2^2}{2g} + \frac{c_2^2}{2g} + \frac{L_r}{G} \quad . \quad . \quad . \quad (6)$$

foot-pounds (or meter kilograms), and is changed into heat. In the work of resistance we must also include the *friction and work of fan resistance* $\frac{L_r}{G}$ *of the wheel referred to, for one pound* (or kilogram) *of steam;* which must be assumed from values derived from already constructed machines. Practically, the heat value of the wheel friction is so slight that we could omit it in the entropy diagram and subtract the entire wheel friction at the end from the indicated steam work. If, for instance, the adiabatic expansion of pressure p_1 and temperature T_1 led to the temperature T_2' at a pressure p_2, then in the superheated territory there would be an increase to T_2 according to the equation

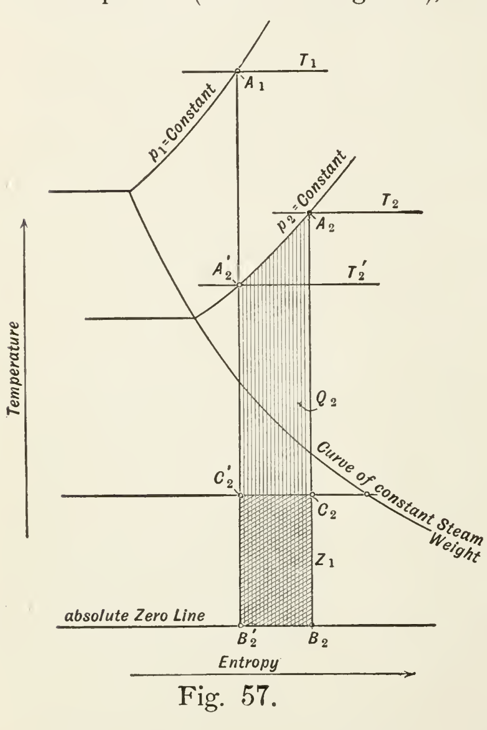


Fig. 57.

$$Q_z = A\,L_z = c_p\,(T_2 - T_2') \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

For the second wheel the initial conditions are p_2, T_2 and the initial heat contents

$$\lambda_2 = q_2 + r_2 + c_p\,(T_2 - T_{2s}) \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

If the steam is wet, then the quality of the steam x_2' would be raised to x_z according to the equation

$$Q_z = r_2\,(x_2 - x_2'), \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (9)$$

and then

$$\lambda_2 = q_2 + x_2 r_2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (10)$$

In Fig. 57, which represents the entropy curves, the heat

$$Q_z = \text{Area } A_2'A_2B_2B_2',$$

is shown sectioned. Q_z *is not totally lost,* because the steam still works in the wheels following. By the change of the work of resistance into heat, the entropy for one pound (or kilogram) of steam has been increased by the value $\Delta s = B_2'\,B_2$. If $C_2\,B_2 = T$ be the temperature which corresponds to the pressure of the condenser, then only $\Delta s \cdot T_k =$ Area $C_2'\,C_2B_2B_2'$, represents *the loss of work Z_1 in heat units* which the described non-reversible change has caused.

The quantity of heat per pound (or kilogram) of steam that is changed into work in the first wheel is

$$Q_1 = \lambda_1 - \lambda_2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (11)$$

The work $L_1 = \dfrac{1}{A}\,Q_1$ is to be taken as the effective, if the wheel friction is included in L_z. As was said above, we will not in general do this, and we shall then take $\dfrac{1}{A}\,Q_1$ as the "indicated work."

After we have calculated all wheels in this manner, we shall figure the steam condition at the pressure of the condenser, so that the exhaust energy $\dfrac{c_2^2}{2g}$ is supposed to have been changed into

heat. If the corresponding heat of steam is λ_k, then we would gain in all per pound (or kilogram) of steam the indicated work

$$L_i = \frac{1}{A}(\lambda_1 - \lambda_k) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (12)$$

From this, in the French units,

$$N_i = \frac{G_{sc} L_i}{75} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (13)$$

and in the English units

$$N_i = \frac{G_{sc} L_i}{550}$$

and

$$N_e = N_i - N_r \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (14)$$

Finally a trial calculation is made from wheel to wheel, to see if, by decreasing the diameter or by changing the drop of pressure, we could save enough of friction and fan resistance work of the wheel under consideration, that even under the circumstances of poorer utilization of steam energy we could possibly obtain a gain of effective work.

It is evident, that *in place of pressure stages* we could just as well specify *temperature stages*, still the representation of the heat of friction in the superheated territory will then not be so simple as until now.

b. — It is assumed, that the outflow velocity c_2 of a wheel could be made partially or wholly useful at the entrance to the next guide wheel.

The treatment of this case is, according to the above given explanation easily accomplished by means of the entropy diagram.

29. THE MANY STAGE REACTION TURBINE.

The reaction turbine gives the simplest case for the many stage system ; therefore we shall begin with it.

We start with the assumption that the wheels follow one another closely, Fig. 58, so that the outflow velocity of each wheel is utilized. We shall observe as reliable steam condition

that which exists in the end cross-section of a guide or rotating channel.*

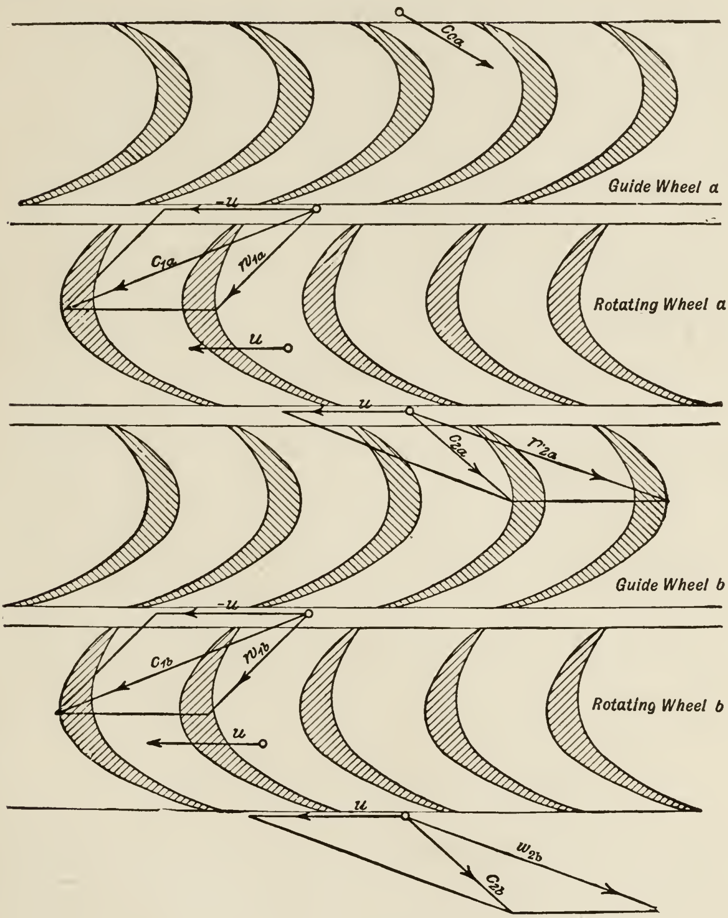


Fig. 58.

The velocities belonging to a certain guide and rotating wheel set shall be represented with the same letters; as, for instance, for the successive wheel sets, a, b, c, the velocities are as follows:

$$c_{1a},\ w_{1a},\ w_{2a},\ c_{2a};$$

$$c_{1b},\ w_{1b},\ w_{2b},\ c_{2b};$$

$$c_{1c},\ w_{1c},\ w_{2c},\ c_{2c};\ \ldots\ldots$$

* In Fig. 58 the points are placed in the clearance spaces only for the sake of clearness.

Let, furthermore, the heat contents be represented

In the steam chamber . . . . λ_1
At entrance to the guide channel a . . . λ_1'
At exit from the guide channel a . . . λ_a
At exit from the rotating wheel channel a λ_a'
At exit from the guide channel b . . . λ_b
At exit from the rotating wheel channel b λ_b', etc.

In the steam chamber the steam has only an unimportant velocity.

According to the laws of steam flow we get, for entrance from the chamber into the guide wheel a

$$A\frac{c_{0a}^2}{2g} = \lambda_1 - \lambda_1' \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

for the flow in guide wheel a,

$$A\frac{c_{1a}^2 - c_{0a}^2}{2g} = \lambda_1' - \lambda_a \quad . \quad . \quad . \quad . \quad . \quad . \quad (1a)$$

for the rotating wheel a, referring to the relative velocities,

$$A\left(\frac{w_{2a}^2 - w_{1a}^2}{2g}\right) = \lambda_a - \lambda_a' \quad . \quad . \quad . \quad . \quad . \quad (1a')$$

For the guide wheel b the "entrance velocity" is c_{2a}; therefore, we have

$$A\left(\frac{c_{1b}^2 - c_{2a}^2}{2g}\right) = \lambda_a' - \lambda_b \quad . \quad . \quad . \quad . \quad . \quad . \quad (1b)$$

for the rotating wheel b again

$$A\left(\frac{w_{2b}^2 - w_{1b}^2}{2g}\right) = \lambda_b - \lambda_b' \quad . \quad . \quad . \quad . \quad . \quad (1b')$$

and so on. For the calculation we recommend working in heat units as we then deal with small figures. The expressions on the

left sides can each be written in French units with $A = \frac{1}{424}$ heat units, as

$$A \frac{c_x^2}{2g} = \left(\frac{c_x}{91.2}\right)^2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

or in English units with $A = \frac{1}{778}$ heat units,

$$A \frac{c_x^2}{2g} = \left(\frac{c_x}{223.7}\right)^2$$

which simplifies the calculation.

The design of a new turbine is most easily accomplished, when we choose the velocity c_1, the angle α, the variable peripheral velocity u, and the exit angle α_2 from wheel to wheel, according to a certain plan, so that by means of simple triangulation w_1, α_1, w_2, and c_2 can be found. Equation 1 then gives the differences $\lambda_a' - \lambda_b$, $\lambda_b - \lambda_b'$, which we will designate by h_b', h_b'' and call it in short, the "drop of heat" (analogous to a hydraulic drop).
Then

$$h_x = h_x' + h_x'' \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

represents for the *turbine x utilized single "drops."*

THE AVAILABLE "TOTAL DROP."

The available total drop is determined from the following data: the known results of steam consumption of existing turbines indicate, that in general, with full load on the turbine blades, we must expect a loss of energy from 20 to 30%. To this loss is added the kinetic energy of exhaust steam $= \frac{c_2^2}{2g}$ (where c_2 is the outflow velocity of the last rotating wheel), for which with small turbines we can allow 10%, and with larger ones about 5%. The friction of the drums or wheels and that of the face area of the blades against the steam, and the friction of the bearings, all of which are difficult to estimate, we shall assume at from 10 to 7%. Finally, the loss due to leakage should be added, which will be different according to the turbine system, and might be anywhere from 10 to 5%.

We allow for these losses by adding their corresponding values finally to the theoretically required steam volume, but the velocities and cross-sections are calculated with the theoretical volume. The total loss varies between 55 and 35% for small or large units, respectively.

If the condenser pressure is chosen $p_2 = 0.1$ kg. per sq. cm. (1.42 lb. per sq. in.) or less, we then calculate the corresponding heat contents λ_2' of the adiabatic frictionless expansion from p_1 to p_2.

It gives

$$H_0 = \lambda_1 - \lambda_2' \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

as the "*theoretical drop of heat.*" From this is lost the part

$$\zeta H_0 = Z \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4a)$$

with $\zeta = 0.2$ to 0.3, and there remains as "useful drop"

$$H_w = (1 - \zeta) H_0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

which serves for producing the velocities, and from which the exhaust losses and the wheel frictions are to be deducted, and for furnishing the delivered effective power from the actual working steam volume. We can now arrange as many turbines consecutively as the separate drops h_a, h_b, h_c, . . . combined can utilize the useful drop corresponding to that velocity height in heat units at entrance to the first guide wheel

$$A \frac{c_{oa}^2}{2g} = h_1,$$

that is, until

$$h_1 + h_a + h_b + h_c \cdots = H_w \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

If we could specify *equal velocities for entire groups of single wheels,* then the turbine can be calculated in this manner without much trouble.

In general, however, we allow the velocities to *increase constantly,* and for fifty or more stages this calculation would become too inconvenient, and we find for such a graphical determination.

GRAPHICAL DETERMINATION.

We replace by this method, instead of the usual application of small differences by differentiation, an easy graphical integration. We next assume the separate turbines represented by the equal (but yet unknown) distances of the division points on the base

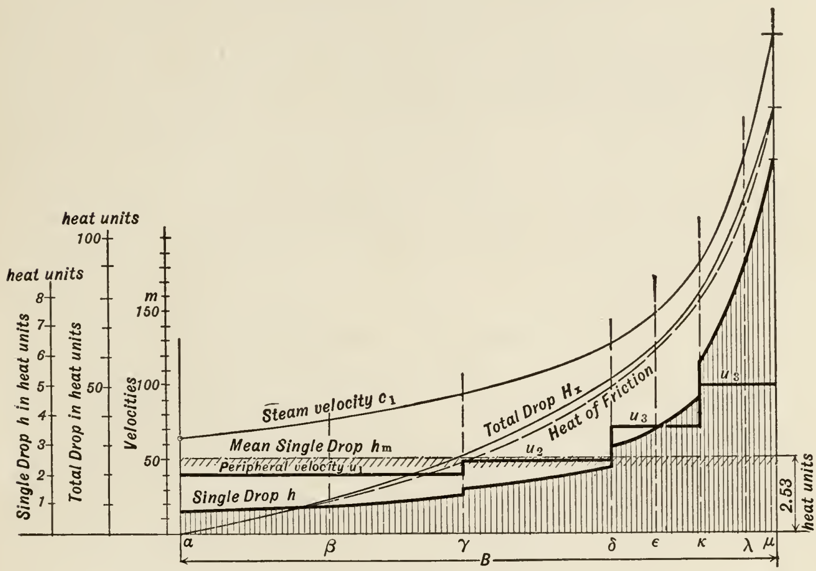


Fig. 59.

line B, (Fig. 59). In these division points, as will be explained below, we shall draw as ordinates the velocities, pressures, and "drops" of the turbine under consideration. We begin with the choice of peripheral velocity u.

CHOICE OF PERIPHERAL VELOCITY (u).

The larger this may be, the better the steam utilization; still there is a limit placed by two considerations. The entrance cross-section that can be calculated at the very beginning from the expected efficiency and power (therefore the steam volume), is at 1 000 h. p. so small that with about 1 500 revolutions and over 50 meters (164 feet) peripheral velocity, the blades with *full peripheral admission* turbines, are only a few millimeters long. As, for instance, with the *Parson's* construction the clearance x in Fig. 60,

between the blade and casings or drum respectively, is a place where leakage occurs; we would not wish to make the ratio of this clearance space to blade length less than $\frac{1}{40}$ to $\frac{1}{50}$, whereby (by similarity of the ratio of the guide and rotating wheel blades) the loss due to clearance would be from 4 to 5%. This leads us, to begin at times with velocities of 35 to 40 meters (114.8–131.2 ft.). With the long blades of the last wheel the clearance would be of small consequence; here u *would be chosen as large as the stability of the wheels and blade attachments would admit.* From the small initial value, u then increases in stages, as Fig. 59 shows, to the final value.

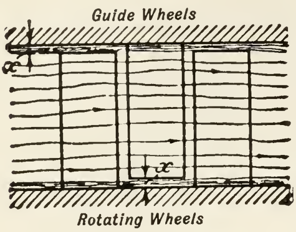


Fig. 60.

CHOICE OF ANGLES.

The smaller the exit angles of guide and rotating wheels, that is, α and α_2, the more drop we utilize with given steam and peripheral velocity, and the smaller will be the number of stages that would be favorable. Too small angles require narrow and long channels, thereby increasing the steam friction, and require, on account of the proportionately large blade thickness, larger cross-sections, and consequently produce eddy currents. As a practical mean we would have, with reaction turbines, the value 20° to 25°. With impulse turbines we find α_2 larger, mostly $= \alpha_1$.

THE CHOICE OF STEAM VELOCITIES.

The choice of steam velocities is governed by the desire to get a *turbine with the smallest possible friction losses.* As the friction increases with the square of the velocity and with the length of the friction path, that is, with the number of turbines, there will be a favorable value for c_1, but which still cannot be exactly found. If we make c_1 small, so that, as with hydraulic turbines, c_2 would have an axial direction, then we would use too small a drop in one wheel and obtain too many stages, too large a friction path, and above all *too many blade shocks,* that play a certain part in

the losses due to back pressure. If we make c_1 large, then few wheels are obtained, but the friction increases because c_1^2 increases too quickly. There appears to be a correct practical mean for reaction turbines, of $\frac{u}{c_1} = 0.5 \ldots 0.3$; for impulse turbines, still less. We allow c_1 in Fig. 59 to increase approximately according to a hyperbolic curve which rises towards the end too rapidly. The final value of c_1 is determined by taking into consideration the exhaust loss and the usually too great blade lengths of the last wheel.

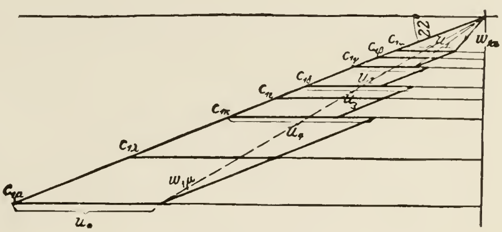


Fig. 61.

The combination for several turbines α, β, γ, δ . . . of the value c_1 with $-u$ gives in the *velocity diagram*, Fig. 61, (of which it is sufficient when $\alpha = \alpha_2$, to draw only one-half) the velocity w_1.

We are free to choose the pressure in the clearance space. The simplest would be to work with the assumption that

$$\alpha = \alpha_2, \qquad w_2 = c_1, \qquad c_2 = w_1 \quad . \quad . \quad . \quad . \quad (7)$$

that is, to assume the axial components c_a of the four velocities c_1, w_1, w_2, c_2 equally large. If we prescribe equal number of blades and blade thicknesses in guide and rotating wheels, and neglect the very small change of specific volume occasioned by flowing through a turbine system, then the blades of such a system need not be widened radially. We also can design the guide and rotating wheel blades with the same profile. With a large number of stages the exit velocity c_2 of a certain wheel *differs only slightly from the exit velocity of the preceding turbine. We may neglect this difference entirely,* and place for instance, referring to the systems a and b, the value $c_{2a} = c_{2b}$, so that in equation 1, $c_{1b}^2 - c_{2a}^2 = c_{1b}^2 - c_{2b}^2$. If we omit the index b, then the equations may be written, taking into account equation 7,

$$\left.\begin{aligned} h' &= A\frac{c_1^2 - c_2^2}{2g} = A\frac{c_1^2 - w_1^2}{2g} \\ h'' &= A\frac{w_2^2 - w_1^2}{2g} = A\frac{c_1^2 - w_1^2}{2g} \end{aligned}\right\} \quad . \quad . \quad . \quad (8)$$

then $h' = h''$, and we have *half reaction degree.* By addition we get the *single drop* for a guide and rotating wheel together as

$$h = h' + h'' = 2A\frac{c_1^2 - w_1^2}{2g} \quad . \quad . \quad . \quad . \quad . \quad (9)$$

By going from the last wheel of a group with equal u to the first wheel of the succeeding, there occurs on account of the sudden change of u a sudden change in h, which is worthy of notice. The values of h that occur at the places referred to, give the curve of single drop (h), of which it is sufficient to determine about three points for each group.*

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* We would save time if we would represent the drop h in the form as shown in Fig. 62, and write

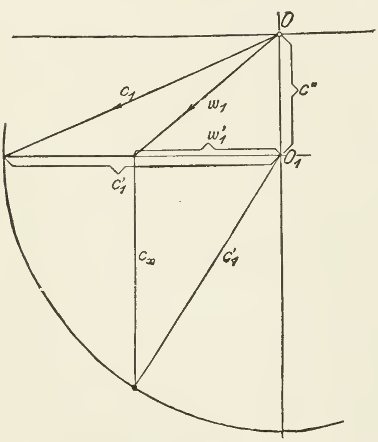

Fig. 62.

$$\begin{aligned} h &= A\frac{c_1^2 - w_1^2}{g} \\ &= A\frac{(c_1'^2 + c''^2) - (w_1'^2 + c''^2)}{g} \\ &= A\frac{c_1'^2 - w_1'^2}{g} \\ &= \frac{A}{g}(c_1' + w_1')(c_1' - w_1'), \end{aligned}$$

and observe that the geometrical mean $(c_1' + w_1')(c_1' - w_1')$ may be found graphically by drawing an arc with c_1' as a radius from O_1 as center, and drop a vertical line from w_1'. This vertical line c_x extending to the arc is the desired mean, that is in French units

$$h = 2A\frac{c_x^2}{2g} = 2\left(\frac{c_x}{91.2}\right)^2 \text{ heat units}\cdot$$

or in English units,

$$h = 2A\frac{c_x^2}{2g} = 2\left(\frac{c_x}{223.7}\right)^2 \text{ heat units.}$$

THE TOTAL NUMBER OF STAGES.

In the above method of calculation the entrance velocity c_{2a} for the first guide wheel was also assumed. To accelerate the steam from the chamber to this velocity, we need the use of a drop

$$h_0 = A\frac{c_{2a}^2}{2g} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (10)$$

and the total number of stages is now to be determined from the consideration, that the sum of the individual drops h corresponding to the velocity height h_0 gives the working drop H_w;

$$h_0 + \sum_1^{z_0} h = H_w \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (11)$$

The unknown distance of the points of the turbines represented on base line B is now

$$\Delta x = \frac{B}{z_0} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (12)$$

where z_0 stands for the number of stages.

If we bring Δx in the numerator and denominator of the second part of equation 11 as a factor, it follows

$$H_w = \frac{\Sigma\, h\, \Delta x}{\Delta x} + h_0 = \Big[h_1\, \Delta x + h_2\, \Delta x + \cdots h_z\, \Delta x \Big] \frac{z_0}{B} + h_0 . \quad (13)$$

The sum of the numerator can approximately be superseded by the integral

$$\int_0^B h\, dx,$$

that is, by the contents of the sectioned area limited by h in Fig. 59. The division by B gives the *mean heat drop* h_m; we have, therefore,

$$H_w = z_0\, h_m + h_0,$$

from which the number of stages

$$z_0 = \frac{H_w - h_0}{h_m} \text{ or simply} \cong \frac{H_w}{h_m} \quad . \quad . \quad . \quad . \quad . \quad (14)$$

with the usually allowable negligence of h_0.

We then divide B into z_0 equal parts, and the beginning of each group is moved to one of these division points.

DETERMINATION OF DISTRIBUTION OF PRESSURE AND BLADE DIMENSIONS.

The former depends upon the laws of the division of the steam friction losses among the individual wheels. The steam friction is influenced by the width and length of the blade channels, and depends upon the degree of bending, and above all upon the velocity. It is permissible to take the friction losses in a wheel as a ratio to the mean of the square of the velocities, or

$$R_1 = A\,\zeta_1\,\frac{c_m^{\,2}}{2\,g},$$

in which c_m is a mean value of the steam velocity. As all velocities of the same wheel stand in a fixed ratio to one another, we also can say

$$R_1 = A\,\zeta_1'\,\frac{c_1^{\,2}}{2\,g}$$

with an assumed empirical and unchangeable coefficient ζ_1'. If we add the friction heats from the first to a certain intermediate wheel x, we have

$$\sum_1^x R_1 = A\,\zeta_1'\,\Sigma\frac{c_1^{\,2}}{2g} = A\,\frac{\zeta_1'}{2\,g\,\Delta x}\sum_0^x c_1^{\,2}\,\Delta x = A\,\frac{\zeta_1'}{2\,g\,\Delta x}\int_0^x c_1^{\,2}\,dx. \quad (15)$$

This quantity of heat must be carried into the entropy diagram, Fig. 63 as R_x in the manner before described, in order to get the point P_x of the true condition curve at the referred-to intermediate pressure p_x. As we do not know which pressure p_x belongs to the abscissa x, we must *assume the course of the condition curve for*

trial, so that the increase of c_1 corresponds to the loss towards the end. As the result teaches, good performance is reached when the energy loss is represented (Fig. 3) in heat units

$$Q_x = \zeta\,(\lambda_1 - \lambda_x'),$$

in which ζ is the unchangeable loss coefficient defined by equation 5, and λ_x' the heat contents of adiabatic expansion at the assumed pressure p_x at point P_x'. The points P_x determine pressure, temperature, quality of steam, and steam heat of the true condition change.*

Especially is

$$\lambda_x = \lambda_x' + Q_x,$$

and at P_2, that is, the condenser pressure, $Q_x = Z$, therefore as required, equal to the total energy loss.

At expansion to the pressure p_x the available steam heat or the drop of heat is

$$H_x = \lambda_1 - \lambda_x \quad . \quad . \quad (16)$$

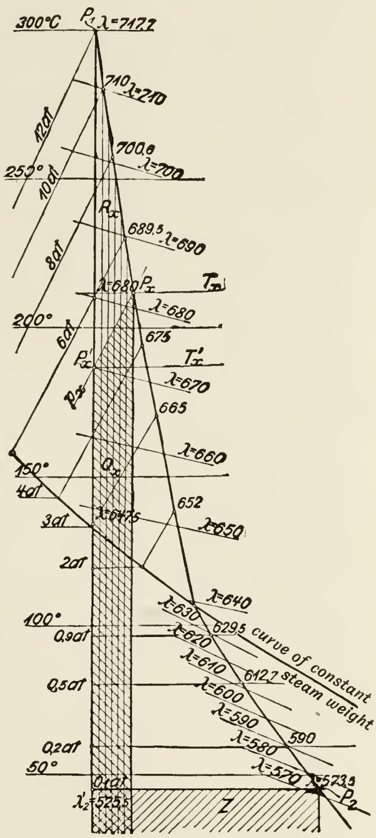

Fig. 63.

and this is represented in Fig. 64 as a function of p_x. For $p = p_1$ is obviously $H_x = 0$; for $p = p_2$, $H_x = H_w$.

---

* If P_x' is in the superheated territory, we obtain from the adiabatic temperature T_x' the true temperature T_x, from the formula

$$Q_x = c\ (T_x - T_x').$$

If P_x' lies in the saturated territory, we obtain the quality of steam x from the equation

$$Q_x = r_x\,(x - x'),$$

in which x' is the quality of steam on the adiabatic.

And now to find the *corresponding pressure for the xth turbine*, we must take the sum of the utilized drops to the xth wheel, that is,

$$H_x = h_0 + h_1 + h_2 + \cdots h_{x-1},$$

or when we again multiply and divide by Δx

$$H_x = \frac{\Sigma h \, \Delta x}{\Delta x} + h_0 = \frac{1}{\Delta x}\int_1^x h\,dx + h_0 = \frac{z_0}{B}\int_1^x h\,dx,$$

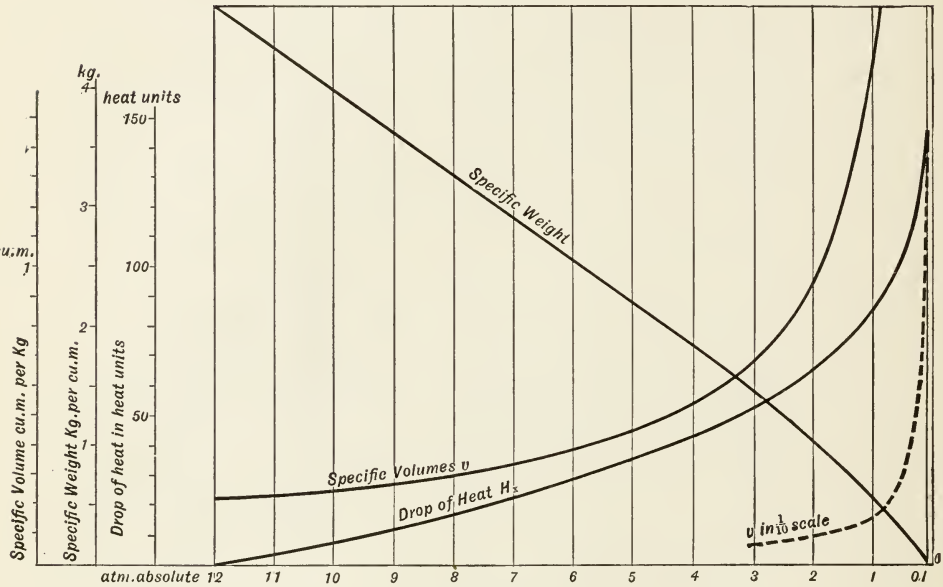

Fig. 64.

that is, we must draw the integral curve h which would give the end point corresponding to H_w, and insert it in Fig. 59. Now we have to look up in Fig. 64 the p_x corresponding to H_x and place it in Fig. 59 as ordinate to the corresponding abscissa x. To avoid drawing too many curves, this has been drawn in Fig. 65.

From the now known $p_x T_x$ of the trial condition curve we obtain finally the specific volume v_x at the corresponding place. If G_{sc} lb. (or kilograms) is to flow through the wheel in one second, we get from the law of continuity the cross-sections :

$$\left.\begin{array}{l}\text{Exit from the } x\text{th guide wheel . .} \\ \text{Exit from the } x\text{th rotating wheel .}\end{array}\right\} f_1 = \frac{G_{sc}\, v_x}{c_{1x}} \quad . \quad (18)$$

$$\left.\begin{array}{l}\text{Entrance in the } x\text{th guide wheel .} \\ \text{Entrance in the } x\text{th rotating wheel}\end{array}\right\} f' = \frac{G_{sc}\, v_x}{w_{1x}} \quad . \quad (19)$$

We may, as has been remarked, take no note of a change of v inside of a turbine; still nothing prevents us from carrying our calculations to any degree of exactness. From the assumed blade

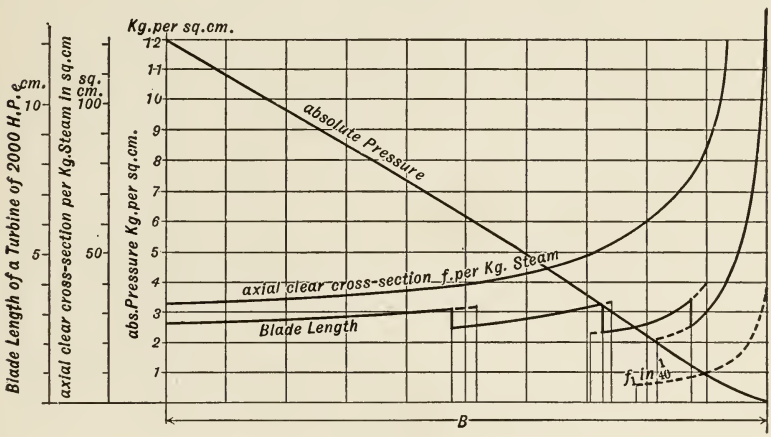

Fig. 65.

thickness, division and angles, we can get the blade lengths. If the blades were infinitely thin, we would have with a blade length a_0,

$$f_1 = \pi D a_0 \sin \alpha$$

On account of the decrease in width due to the blade thickness, and the blades of the guide wheel running past, a_0 must be increased in cross-section to about $1\frac{1}{2}$ times. The value

$$\frac{f_1}{\sin \alpha} = \pi D a_0$$

is inserted in Fig. 65 as the *axial clear cross-section.*

The example solved in Figs. 59 to 65 is referred to the initial data $p_1 = 12$ kg. per sq. cm. absolute (170.7 lb. per sq.

in. absolute), $t_1 = 300°$ C. (572° F.); the condenser pressure $p_2 = 0.1$ kg. per sq. cm. (1.42 lb. per sq. in.) and the loss of energy $\zeta = 0.25$. Finally, the friction heat was determined according to equation 15

$$R_x = A \frac{\zeta_1'}{2g\,\Delta x} \int_0^x c_1^2\,dx$$

and was drawn in Fig. 59 to such a scale that H_w and the total value R coincided. We must now compare from the entropy diagram by measuring the vertically sectioned area which was used to get the line R_x with the value that has already been obtained. If the assumption of the condition curve was correct, then the curves of R_x must coincide. To be more exact, would only be of use if we knew more about the value of the coefficient of resistances. We shall also neglect including a part of the wheel friction in R_x.

Instead of a constantly variable blade length we will choose longer or shorter divisions and make changes in the choice of c_1, so that for *larger divisions of turbine lengths constant cross-sections are obtained.*

Let the final exit velocity $= c_{2z}$; then the total losses in heat units for one pound (or kilogram) of steam are

$$H_z = Q_z + A\left(\frac{c_{2z}^2}{2g}\right). \quad . \quad . \quad . \quad . \quad . \quad . \quad (20)$$

The available energy is

$$H_0 = \lambda_1 - \lambda_2',$$

and

$$H_i = H_0 - H_z. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (21)$$

is the "indicated" steam work in heat units; therefore

$$L_i = \frac{H_i}{A} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (22)$$

is the same in ft.-lbs. or meter kilograms per lb. or kilogram of steam. From this we obtain the indicated power in h. p.

In the French units

$$N_i = \frac{G_{sc} L_i}{75} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (23)$$

in the English units

$$N_i = \frac{G_{sc} L_i}{550}.$$

The efficiency of the indicated work is

$$\eta_i = \frac{H_i}{H_0} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (24)$$

The consumption of steam per indicated h. p. hour in the French units is,

$$D_i = \frac{3\,600\, G_{sc}}{N_i} = \frac{270\,000}{L_i} = \frac{637}{H_i} \quad . \quad . \quad . \quad (25)$$

in the English units

$$D_i = \frac{3\,600\, G_{sc}}{N_i} = \frac{1\,980\,000}{L_i} = \frac{2\,544.65}{H_i}.$$

The other work of friction, such as steam friction of the drums, blade face areas, stuffing boxes, including the work required for running without load (that is, journal friction and the like), is in h. p. N_r; then follows the effective power in h. p.

$$N_e = N_i - N_r \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (26)$$

and the steam consumption per h. p.$_e$ hour

$$D_e = 3\,600 \frac{G_{sc}}{N_e} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (27)$$

30. THE MANY STAGE IMPULSE TURBINE.

The many stage impulse turbine is calculated in a similar manner, provided the exhaust velocity c_2 can be utilized. This is true above all, for full peripheral admission of the so-called limit turbine. The choice of velocities and the construction of the condition curve is obtained as before.

The drop of heat in the guide wheel has, with the same assumptions as with the reaction turbine, the value

$$h' = A \frac{c_1^2 - c_2^2}{2\,g} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

As we should choose w_2 about $= 0.8\ w_1$, there occurs no acceleration in the rotating wheel, but rather a transformation of kinetic energy into heat; that is,

$$h'' = A\frac{w_2^2 - w_1^2}{2g} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

therefore negative in value. The single drop for a turbine system is

$$h = h' + h''$$

and is therefore smaller than h'.

But if the construction does not permit of entire utilization of c_2, then we could use of the exit energy of a wheel, that is $\frac{c_2^2}{2g}$, only the value

$$(1 - \zeta)\frac{c_2^2}{2g}$$

for the succeeding guide wheel, in which the value of ζ is estimated.

For a turbine with radial entrance, for instance, the steam would have to travel such long paths to the next guide wheel, that we can place $\zeta = 1$, that is, the entire exit energy must be considered as lost. With axial turbines, closely following each other, ζ becomes smaller the more they approach full peripheral admission. The high pressure wheels have admission only for a small part of the circumference, in order to gain longer blades and have the advantage that at the beginning they can work immediately with high peripheral velocity. Here ζ could also approach unity. It is therefore important for turbines of this type to keep u large, and the angle α_2 small, in order that c_2 will be small, and can be omitted without harm.

After we have fixed upon ζ, we get for the guide wheel

$$h' = A\frac{c_1^2 - (1 - \zeta)\,c_2^2}{2g} \quad . \quad . \quad . \quad . \quad . \quad . \quad (47)$$

(in which we neglect the difference between c_2 for two successive turbines). In the rotating wheel we have as before,

$$h'' = A\frac{w_2^2 - w_1^2}{2g} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (48)$$

From the exit of the rotating wheel up to the entrance in the guide wheel there would be added (algebraically), as third drop

$$h''' = A\left[(1 - \zeta)\frac{c_2^2}{2g} - \frac{c_2^2}{2g}\right] = -\zeta A\frac{c_2^2}{2g} \quad . \quad . \quad (49)$$

Finally, there must be added, with turbines that are composed of single discs, the steam friction of the wheel in question, L_{rx} in heat units per second, divided by the steam weight per second G, which must at present be estimated ; that is

$$h_r = \frac{L_{rx}}{G} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (50)$$

The hereby necessary knowledge of the values of the steam pressures can be obtained by a temporarily approximate solution. The single drops or losses

$$h = h' + h'' + h''' + h_r = \frac{A}{2g}\left[(c_1^2 - c_2^2) - (w_1^2 - w_2^2)\right] + h_r \quad (51)$$

are independent of the value ζ, by which we do not mean to say that these are not dependent on it. The number of stages will remain the same, still the greater ζ the more the entropy increases, and the greater therefore are also the final losses.

In impulse turbines, *partial peripheral admission* is permissible and is generally used. It has the advantage that we can work with larger peripheral velocities (60 — 80 m.) (196.9 — 262.5 ft.) starting from the first wheel, whereby the number of stages is greatly decreased.

31. MANY STAGE TURBINES WITH CONSTANTLY VARIABLE PERIPHERAL AND STEAM VELOCITIES.

THE HYPERBOLIC TURBINE.

This turbine type is not recommended for construction because it would constantly lead to sudden changes of u. But an especially simple example of such a turbine type will aid to a better understanding of the conditions of many stage expansion. We shall assume that u, as well as c_1 increases according to a *hyperbolic law,*

in which we give the velocities at the distance x along the base B, measured from the beginning, the values

$$u_x = \frac{a}{x - x_1}, \qquad c_{1x} = \frac{b}{x - x_1} \quad . \quad . \quad . \quad . \quad . \quad (1)$$

in which a, b, x_1, are determined from the smallest and the largest values of the peripheral velocity, namely $u = u_1$, for $x = 0$; and $u = u_2$ for $x = B$, and from the initial value of c_{1x}, which we will call c_1.

We find

$$x_1 = B\frac{u_2}{u_2 - u_1}; \qquad a = B\frac{u_1 u_2}{u_2 - u_1}; \qquad b = a\frac{c_1}{u_1} \quad . \quad (2)$$

and the last entrance velocity c_{1z} becomes

$$c_{1z} = c_1 \frac{u_2}{u_1}.$$

The velocities c_{1x} and u_x are proportional.

We shall assume a turbine, in which the axial components of c_1, w_1, w_2, c_2 are equal, and with impulse turbines, $\alpha_1 = \alpha_2$, with reaction turbines $\alpha = \alpha_2$, and the angles for all wheels are equal. The drop of heat per single turbine is found, after easy transposition, for impulse turbines as

$$h_a = \frac{A}{g}(2\,c_{1x}\cos\alpha - 2\,u_x)\,u_x. \quad . \quad . \quad . \quad . \quad (3)$$

for reaction turbines as

$$h_r = \frac{A}{g}(2\,c_{1x}\cos\alpha - u_x)\,u_x \quad . \quad . \quad . \quad . \quad . \quad (4)$$

The number of stages is determined by neglecting h_0 from the equation

$$H_w = \Sigma\,h_x = \frac{1}{\Delta x}\Sigma h_x\,\Delta x = \frac{z_0}{B}\int_0^B h_x\,dx = z_0\,h_m \quad . \quad . \quad (5)$$

From this follows that the mean drop

$$h_m = \frac{1}{B}\int_0^B h_x\,dx \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

which is analytically determinable, and is expressed by means of the *geometrical mean of the initial and final velocities* u_1 and u_2 as well as c_1 and c_{1z}, that is, by

$$\left.\begin{aligned} u_m &= \sqrt{u_1 u_2} \\ c_{1m} &= \sqrt{c_1 c_{1z}} \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

We get with impulse turbines

$$h_{am} = \frac{A}{g}(z c_{1m} \cos \alpha - 2 u_m) u_m \quad . \quad . \quad . \quad . \quad (8)$$

with reaction turbines,

$$h_{rm} = \frac{A}{g}(2 c_{1m} \cos \alpha - u_m) u_m \quad . \quad . \quad . \quad . \quad (9)$$

From this follows the important conclusion: *In the "hyperbolic turbine" the mean wheel drop, therefore also the number of stages of the same, is as though all wheels work with the (constant) geometrical mean of the initial and final values of the peripheral and steam velocities.*

We can also prove that with equal width of blades, and so long as we can estimate values of friction so that it is proportional to the blade breadth and the square of the steam velocities, and in case the ratio $\frac{u_x}{c_x}$ remains unchanged, the *total work of steam friction, of a turbine does not depend on the absolute values of the velocities, and is equally large whether many or few stages be chosen.*

This is of great value in the construction of the many stage turbine under the assumption of a constant resistance coefficient derived from the stated hypothetical solution. Theoretically it is allowable, by increasing the number of stages, to decrease the velocities to any desired value. But we should observe that this refers to any system (therefore to such as have equal degree of reaction), and the friction of the drums or wheels is not taken into account. In various systems the work of friction would vary and must be calculated individually.

RADIAL TURBINES.

Let λ_0, λ_1, λ_2 be the heat contents of steam at entrance to the guide wheel, at entrance to the rotating wheel, and at exit from the latter, respectively; r_0, r_1, r_2, and further, u_0, u_1, u_2 the correspond-

ing radii and peripheral velocities of a radial steam admission turbine. For the flow in the guide wheel we get as before

$$\frac{c_1^2}{2g} - \frac{c_0^2}{2g} = \frac{1}{A}(\lambda_0 - \lambda_1) \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

But for the flow in the rotating wheel we must take into consideration the work of the supplementary forces of relative motion, that is, the contrifugal force, and we get *

$$\frac{w_2^2}{2\,g} - \frac{w_1^2}{2\,g} = (\lambda_1 - \lambda_2) + \frac{u_2^2 - u_1^2}{2\,g} \quad . \quad . \quad . \quad . \quad (2)$$

With a single stage turbine we can usually neglect the last part of the equation; with many stages, not without further consideration. Through addition of both equations 1 and 2 we get the single drop of one stage

$$h = \lambda_0 - \lambda_2 = \frac{A}{2\,g}\Big[(c_1^2 - c_0^2) + (w_2^2 - w_1^2) - (u_2^2 - u_1^2)\Big] \quad (3)$$

The summation of all stages leads to the "useful drop"

$$H_w = h_0 + \Sigma\, h,$$

where h_0 stands for the former defined drop for the entrance in the first guide wheel. In the sum $\Sigma\, h$ there appears also $-\Sigma\,(u_2^2 - u_1^2)$,

---

* Taking into consideration the results in Article 14, we have:

The work of contrifugal force on a mass element dm, whose distance from the turning axis increases from r_a to r_e, with an angular velocity ω, is

$$\int_{r_a}^{r_e} dm.\, r\, \omega^2\, dr = \frac{dm\ \omega^2}{2}\,(r_e^2 - r_a^2).$$

If we imagine in Fig. 22, the entire enclosed mass between the cross-sections A_1, A_2, in rotation around a fixed axis, then the work of the contrifugal force upon the same

$$= \Sigma\,\tfrac{1}{2}\,dm\ \omega^2\,(r_e^2 - r_a^2) = \Sigma\,\tfrac{1}{2}\,dm\ \omega^2\,r_e^2 - \Sigma\,\tfrac{1}{2}\,dm\ \omega^2\,r_a^2 = \Sigma\,\tfrac{1}{2}\,dm\ u_e^2 = \Sigma\tfrac{1}{2}\,dm\ u_a^2.$$

Herein the first sum is the negative "potential energy" of the mass system at the end of the occurrence; the second sum is the same at the beginning of the occurrence, taking into account the direction of the rotation. We observe a constant flow and a movement of the cross-section $A_1\,A_2$ to $B_1\,B_2$; herewith the potential energy of the mass particles contained between the planes B_1 and A_2 in the initial and final conditions balance each other, and there remains only

$$\frac{1}{2}\,\frac{dG}{g}\,dt\,u_2^2 - \frac{1}{2}\,\frac{dG}{g}\,dt\,u_1^2,$$

which when added to the work of the upper surface forces in equation 1, Article 14, will give the above equation.

which cannot be neglected without further consideration, because the single sums are small. If we assume that all stages (radial) follow one another closely, and if we say that approximately

$$u_2^2 - u_1^2 = \frac{1}{2}(u_2^2 - u_0^2)$$

we get

$$\Sigma (u_2^2 - u_1^2) = \frac{1}{2} \Sigma (u_2^2 - u_0^2),$$

in which, if we add through all the stages, the intermediate parts of the equation eliminate one another we will have left only

$$\frac{1}{2}(u_e^2 - u_a^2) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

in which u_e is the velocity of the last wheel and u_a the velocity of the first wheel, and this part of the equation can be under certain circumstances of importance.

The construction of a new turbine does not offer any serious difficulties when, with the use of Fig. 59, etc., we follow the method formerly explained.

Lately *Brady* suggested a radial turbine, in which the guide and the rotating wheels rotate with equal, but opposite angular velocity. Here Formula 2 is applicable for the movement between the blades of the guide as well as the rotating wheels, and the drop of heat for one stage becomes

$$h = \frac{A}{2g}\Big[(c_1^2 - c_0^2) + (w_2^2 - w_1^2) - (u_2^2 - u_0^2)\Big] \quad . \quad . \quad (5)$$

The summation through all stages gives, for the influence of the peripheral velocity, the factor

$$\Sigma u_2^2 - u_0^2 = u_e^2 - u_a^2 \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

By drawing the triangle of velocities for a turbine of *Brady's* system we must also observe that we must combine for a trial c_1 with the negative u_1 (referred to the rotating wheel) in order to obtain the absolute exit velocity from the guide wheel. The (geometric) use of $-u_1$ gives first w_1, etc., not taking into account the small part of the work of centrifugal force (Equation 6); the rotation of the guide wheel acts just as a doubling of the peripheral velocity; or we can, with a given velocity, decrease the revolutions one-half.

32. MOLLIER'S DIAGRAM FOR SATURATED STEAM.*

If we fix upon a certain condition of the steam (0° C. and 1 kg. per sq. cm. pressure, as is usual) as an initial point, then with any other condition, the steam contents or total heat of the steam, as well as the entropy, will have one and only one certain value.

Mollier uses the heat contents of a condition of steam determined by p and v as the ordinate, and the entropy as the abscissa of a right angled coördinate system, in which any condition of the steam can be expressed by a point in this plane. The points of equal pressure are connected and there results a series of curves in which $p =$ constant. Similarly the curves $T =$ constant and $x =$ constant can be found, from which results an especially useful diagram for steam turbine calculations.

A vertical line in the diagram expresses the equation $S =$ constant; that is, it expresses, as in the ordinary entropy diagram, the reversible adiabatic, and also the flow without losses in a nozzle. The expansion from condition A_1 with the pressure p_1 to the pressure p_2 (See Fig. 66) leads, by drawing the vertical from A_1 to the point A_2'; and the decrease of heat contents is the distance $A_1 A_2'$ which can be read along the margin of the table directly in heat units. We have, therefore, $A_1 A_2' = H_0 = \lambda_1 - \lambda_2'$ heat units. If the initial flow velocity were equal to nothing, then

$$\left(\frac{w}{91.2}\right)^2 = H_0; \qquad w = 91.2\sqrt{H_0}.$$

Mollier has also added along the left margin of the table a scale of velocities so that w can be directly ascertained.

The horizontal lines play an especially important part. For these $\lambda =$ constant, that is, the heat contents or total heat of steam in the initial condition is equal to that in the final condition. As the decrease of heat contents of a flow without addition of heat and without performing work, is equal to the increase of kinetic energy, there follows, that the latter, in our case, through friction and eddy currents, is again entirely retransposed into heat. Such a change of steam condition from high to low pressure is called *throttling*, and we may therefore call these horizontal lines "*thrott-*

* With permission of Professor Mollier.

ling lines." The throttling of steam pressure p_1, to the same final pressure p_2 as before, gives the steam condition at point A_3, Fig. 66; and we can easily determine by means of the table the increase of the quality of steam or the temperature.

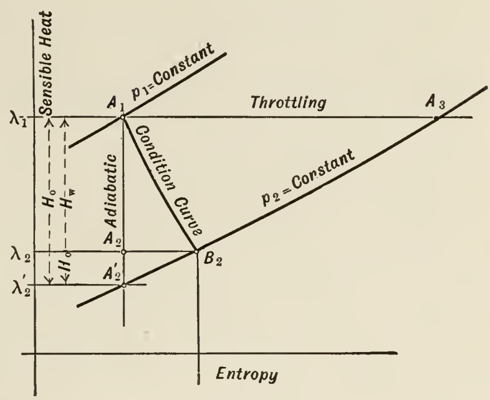

Fig. 66.

A change of condition of any certain kind is represented by a curve which combines the points representing the successive changes of condition. The flow in a nozzle, taking into consideration the resistances, can, for instance, be represented by the curve $A_1 B_2$, Fig. 66, and we can get by projecting B_2 towards A_2 *the distance* $A_1 A_2$, which is the *actual* decrease of heat contents or total heat, therefore, in the scale of the velocities, the actual final velocity. The loss of *kinetic energy* compared to flow without resistance, is

$$H_z = \lambda_2 - \lambda_2' = \zeta H_0 = \text{distance } A_2' A_2 .$$

In the reversed order, from the known assumed coefficient of losses ζ, the distance $A_2' A_2 = \zeta A_1 A_2'$, and by projecting point B^2 over along curve $p_2 =$ constant, the final condition of expansion is determined.

DIAGRAM FOR SATURATED STEAM IN ENGLISH UNITS.

A diagram similar to Mollier's diagram, but in English units, has been prepared by the translator. To correspond with the diagram in French units, 32° F. and 14.22 lb. per square inch have been taken as the initial point. Along the left margin of the table is a scale of velocities from which w may be directly ascertained in feet, provided the initial flow velocity $= 0$.

In Fig. 66,

$$A_1 A_2' = H_0 = \lambda_1 = \lambda_2' \quad \text{B. t. u.}$$

$$H_0 = \left(\frac{w}{223.7}\right)^2; \qquad w = 223.7\sqrt{H_0}.$$

FINAL CONDITION OF THE STEAM FOR ANY TURBINE SYSTEM.

Let the heat contents of the final condition of the steam at the prescribed final point with adiabatic expansion be λ_2'.

The estimated loss of heat $H_z = \zeta H_0$ gives the heat contents $\lambda_2 = \lambda_2' + \zeta H_0$, therefore also the distance $\lambda_1 - \lambda_2 = A_1 A_2$, Fig. 66. Carrying A_2 along the horizontal towards B_2 on the curve $p_2 =$ constant, will determine the final condition. The condition curve must, for the time being, be estimated and drawn between A_1 and B_2.

APPLICATION TO THE DESIGN OF STEAM TURBINES.

A. SINGLE STAGE IMPULSE TURBINE.

The condition curve $A_1 B_2$, Fig. 66, as mentioned above, permits the calculation of the steam velocity and the specific volume for any intermediate point, from which may be found the shape of the nozzle, the velocity diagram, and everything else, as in Article 27.

B. FEW STAGE IMPULSE TURBINE.

We shall confine ourselves to the case where the exit velocity from each rotating wheel is entirely consumed by eddy currents. The determination of the final point B_k of condition curve, Fig. 67, follows as above and gives $A_1 A_k = H_w$, as available drop of heat. We divide this into as many equal divisions (or divisions proportional to the peripheral velocity), as there are stages, and design a simple turbine for each division drop.

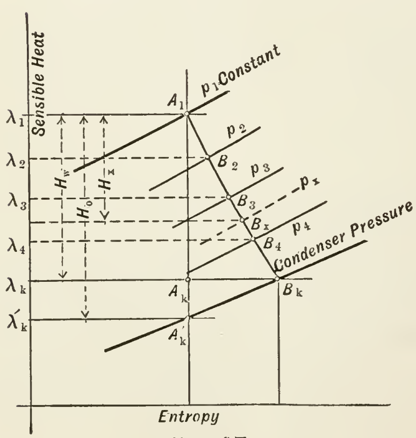

Fig. 67.

C. MANY STAGE TURBINE.

The approximate method of procedure given in Article 29 is again used, the table serving only as a means for drawing the condition curve, and for determining the division drop H_x.

The ordinary entropy diagram gives the best representation of the change of heat. The diagram of *Mollier*, on the other hand, has the advantage, that the quantity of heat can be determined as distances along straight lines.

33. THE STEAM FRICTION OF ROTATING DISCS.

The resistances that a turbine wheel experiences while rotating in steam can be divided into two parts. The first, that which is in general caused by the smooth discs, and second, that which is caused by the blades. The latter can be determined more simply because the resistance is principally caused by the churning or fan work of the blades. In observing the flow of air against an unenclosed rotating wheel, (which for instance could very easily be done by means of a small tassel that is fastened to a wire by a short thread) we find that the disc causes only small velocities in almost radial direction. Even to about two-thirds of the blade length the velocity remains very small, with of course a somewhat stronger leaning towards the direction of the circumference. Only in the last third of the distance does the air flow in nearly tangential direction and with greater velocity. A part of the air thrown aside returns in regular stream paths back to the wheel.

It is evident that an unenclosed wheel absorbs greater work running empty than a wheel that is enclosed in a closely fitted cover, because in the latter case the air is prevented from circulating freely.

If we should try to calculate the work of fan resistance, we would soon see the difficulties of such a trial. At one time the angle of the blade surfaces is unfavorable (fortunately) to the entrance of the air, and causes an eddy current; and then there would be no certain path for the flow of air (or steam) and we could not give the values of its cross-section. With the enclosed wheel the air in the clearance space between the wheel and the cover receives a considerable velocity that can be utilized at entrance; still we are not able to estimate its value accurately. If

the angles at entrance to and exit from a wheel are unequal, then there occurs, as observations have shown, an effect similar to the axial turbine pump; that is, there occurs, besides the ordinary churning effect, an air stream flowing through the wheel which increases the work necessary to drive the wheel without load. It is still more difficult to calculate the friction of the smooth disc. There are, of course, experiments at hand made by physicists concerning the "friction of gases." Still these were made under conditions such that they cannot be applied to the turbine without further investigation.

The author performed a series of experiments to investigate the above questions, and the results are given in Table 1 and

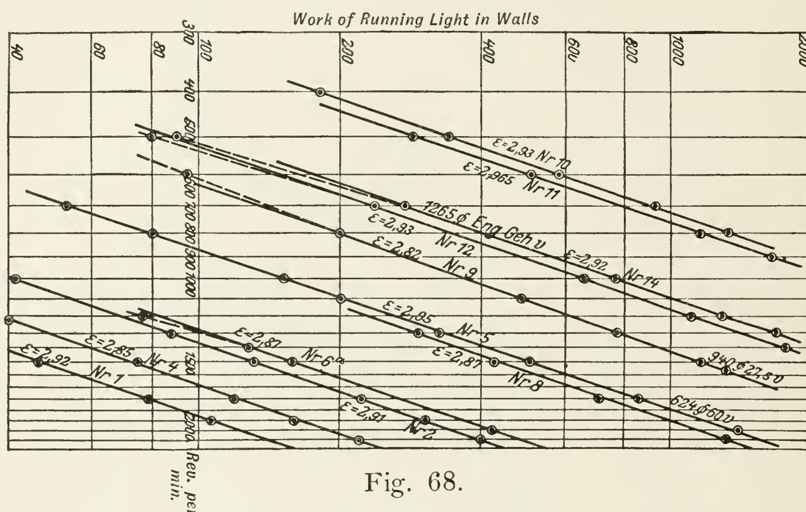

Fig. 68.

shown in Figs. 68 and 69. There was used a smooth unmachined disc made of boiler plate of 537 mm. (21.142 in.) diameter and five turbine wheels of 545, 624, 722, 940, 1 265 mm. (21.46, 24.57, 28.43, 37.01, 49.8 in.) outer diameter. The wheels were at times overhung on the shaft end of a direct current motor, at other times fastened to an extension of the motor shaft which was supported between bearings, and were either driven in the open air or enclosed in a cover. From the total power consumed there was subtracted the power required to drive the mechanism without load, subtracting also the heat consumed in the field coils. The temperature of the field coils was taken from time to time and

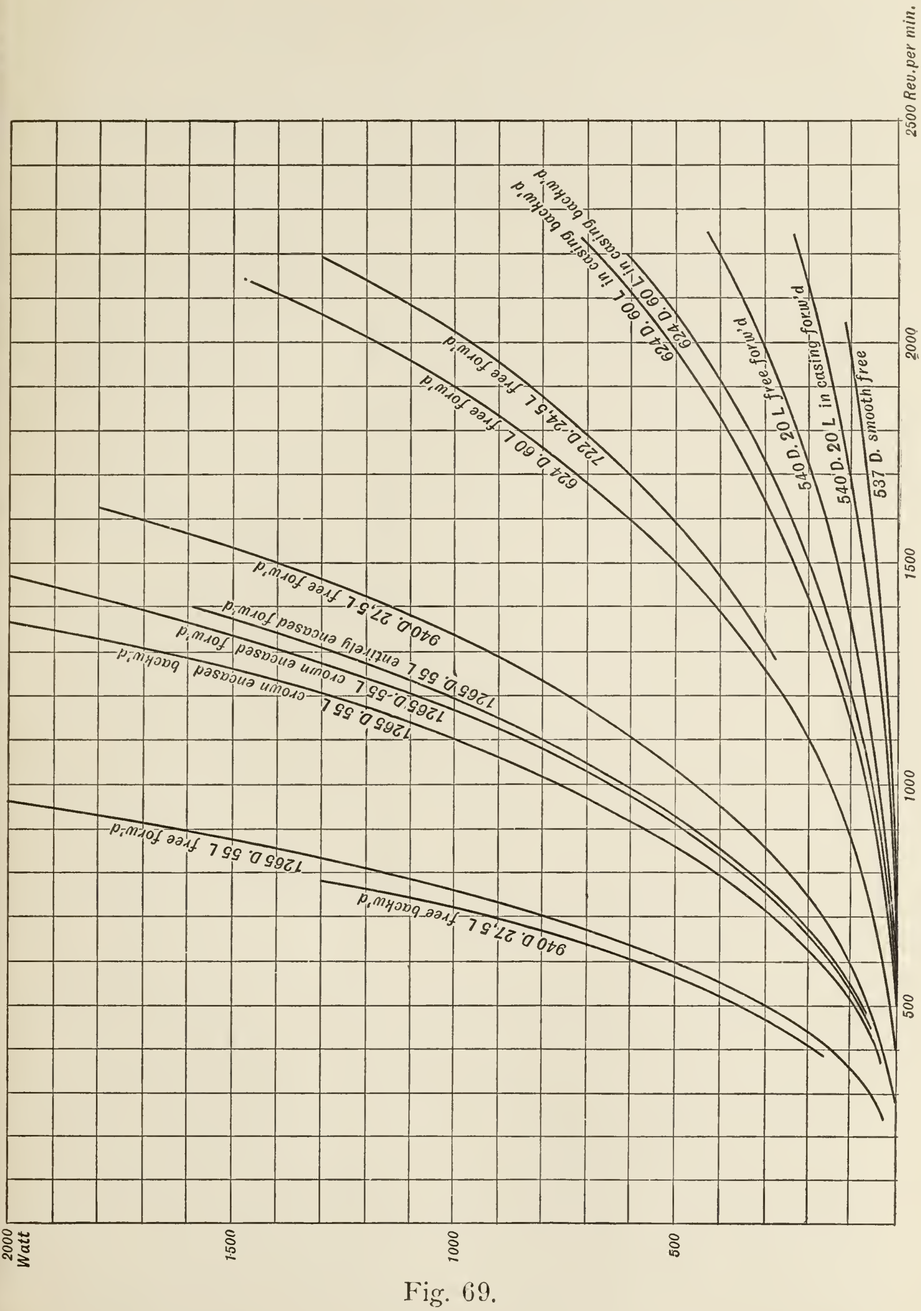
2500 Rev. per min.
2000
1500
1000
500
2000 Watt
1500
1000
500
624 D. 60 L in casing backw'd
624 D. 60 L in casing backw'd
540 D. 20 L free forw'd
540 D. 20 L in casing forw'd
537 D. smooth free
722 D. 24,5 L free forw'd
624 D. 60 L free forw'd
940 D. 27,5 L free forw'd
1265 D. 55 L entirely encased forw'd
1265 D. 55 L crown encased forw'd
1265 D. 55 L crown encased backw'd
1265 D. 55 L free forw'd
940 D. 27,5 L free backw'd

Fig. 69.

their resistances were corrected from these values. The current through the field coils was kept constant. As the work of running the mechanism without load was determined by the power consumed, this included therefore the bearing friction caused by the weight of the wheel. Still the coefficient of friction, as is well known, varies almost exactly in the inverse ratio of the pressure, and this increase of friction is therefore of small consequence.

Fig. 68 is drawn with the logarithms of the consumed power W in watts as ordinates, and the logarithms of the revolutions per minute, n, as abscissas. The points determined for each experiment lie practically in a straight line over quite a considerable distance, and can be represented by the formula

$$\log W = \log W_0 + \epsilon \log n \quad . \quad . \quad . \quad . \quad . \quad (1)$$

The value ϵ is the trigonometrical tangent of the angle of slope with the abscissa. From equation 1 it follows that

$$W = W_0 n^\epsilon \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

The values of ϵ are shown in the figure, their mean value being 2.90. We can therefore approximately let

$$\epsilon = 3,$$

which will simplify the calculations exceedingly. We can express these results by the following law :

THE WORK OF ROTATING WHEELS AND DISCS WITHOUT LOAD IN THE OPEN AIR OR IN AN ENCLOSED SPACE INCREASES VERY APPROXIMATELY AS THE THIRD POWER OF THE NUMBER OF REVOLUTIONS.

It is sufficient, therefore, to give a single point for each series of experiments performed with varying number of revolutions but otherwise similar conditions. Table 1 gives the maximum values of each experiment, the term "forward running" meaning the ordinary direction of rotation, that is, rotation with the convex side of the blade in advance. The resistances of backward running were determined for several wheels, because this is important in marine turbines, as they must be capable of rotation in either direction. The casings were made of sheet metal for the smaller discs, and of wood for the larger ones. The clearance given was the space between the blades and the casing.

TABLE 1.

Number.	Conditions of Experiment.	Forward or Backward Running.	External Diameter in mm.	*External Diameter in in.*	Blades.						Maximum Revolutions per Minute.	Corresponding Peripheral Velocity in Meters per Second.	*Corresponding Peripheral Velocity in Feet per Second.*	Power Consumed.			a
					Length (Radial) in mm.	*Length (Radial) in in.*	Breadth (Axial) in mm.	*Breadth (Axial) in in.*	Spaces between Blades in mm.	*Spaces between Blades in in.*				Watts.	Horse Power in French Units.	*Horse Power in English Units.*	
1	Smooth disc, 4 mm. (0.157 in.) thick, free in air	Forward	537	21.142	. .	. . .	. .	. . .	. .	. . .	2 000	56.3	184.7	110	.149	0 147	. . .
2	Rotating wheel A free in air	Forward	545	21.467	20	0.787	20	0.787	12.3	0.484	2 200	62.8	206.0	400	0.544	0.536	6.38
3	" " " " " "	Backward	545	21.467	20	0.787	20	0.787	12.3	0.484	2 100	59.9	196.5	1 880	2.554	2.518	34.42
4	" " " encased with 4 mm. (0.157 in.) side play	Forward	545	21.467	20	0.787	20	0.787	12.3	0.484	2 200	62.8	206.0	218	0.296	0.292	3.48
5	Rotating wheel B free in air	Forward	624	24.567	60	2.362	20	0.787	13.7	0.539	2 100	68.6	226.1	1 380	1.875	1.849	12.86
6	" " " encased with 4 mm. (0.157 in.) side play	Forward	624	24.567	60	2.362	20	0.787	13.7	0.539	2 100	68.6	226.1	525	0.713	0.702	4.89
7	Rotating wheel B encased with 4 mm. (0.157 in.) side play	Backward	624	24.567	60	2.362	20	0.787	13.7	0.539	2 200	71.9	235.9	680	0.924	0.911	5.51
8	Rotating wheel C free in air	Forward	722	28.425	24.5	0.965	20	0.787	12.6	0.496	2 200	83.2	272.5	1 315	1.787	1.762	5.13
9	Rotating wheel D free in air	Forward	940	37.008	27.5	1.073	25	0.984	16.3	0.642	1 600	78.7	258.2	1 720	2.336	2.303	4.67
10	" " " " " "	Backward	940	37.008	27.5	1.073	25	0.984	16.3	0.642	750	36.9	121.1	1 120	1.522	1.501	9.34
11	Rotating wheel E free in air	Forward	1 265	49.803	55	2.165	25	0.984	12	0.472	980	64.9	213.0	2 160	2.935	2.894	5.77
12	" " " crown encased to depth of 160 mm. (6.3 in.), about 6.5 mm. (6.257 in.) side play	Forward	1 265	49.803	55	2.165	25	0.984	12	0.472	1 650	109.3	358.6	2 870	3.901	3.856	1.61
13	Rotating wheel E crown encased to depth of 160 mm. (6.3 in.), about 6.5 mm. (0.256 in.) side play	Backward	1 265	49.803	55	2.165	25	0.984	12	0.472	1 400	92.7	304.1	2 290	3.110	3.066	2.10
14	Rotating wheel E entirely encased, 6.5 mm. (0.256 in.) side play	Forward	1 265	49.803	55	2.165	25	0.984	12	0.472	1 400	92.7	304.1	1 590	2.16	2.13	1.48

The important influence of the blade lengths is clearly seen from these results. The work of fan resistance depends on the freedom of air circulation, and is shown by comparing the values for wheels in open air and in an enclosed space. Because of the better entrance of the air into the blades the work of running the uncovered wheel backwards is 5 to 6 times as large as in running it forward. If the wheel is enclosed, this ratio decreases to about 1.2. Experiments 12 and 14 further show the interesting fact that enclosing only the crown of a wheel does away with the main part of the resistances, and very little is gained in addition by enclosing the entire wheel.

THE FRICTION OF SMOOTH WHEELS without blades has also been investigated by *Odell,** who used four discs made of cardboard and drawing-paper with diameters of about 381, 559, 686, 1 194 mm. (15, 22, 27 and 47 in.).

Odell found the consumption of power with the first three to be proportional to the 3.5th power of the number of revolutions; with the fourth the exponent was 3.1, therefore in close agreement with the values found by us. The diameter entered as a factor with a certain exponent; this exponent with the small discs was between 6 and 7, and with the large discs between 5 and 6. Experiments conducted by the author with cardboard discs were unsuccessful because the cardboard would bend under the strain of centrifugal force. As the force required by smooth discs was in itself small, and as my experiments, as well as those of *Odell,* with large discs gave for the number of revolutions an exponent nearly equal to 3, it is found advisable, since no experiments more nearly exact are known, to express the power in h.p. required to rotate a smooth disc unloaded, by the formula:

In French units

$$N_0' = a_1 D_1^{2.5} \left(\frac{u_1}{100}\right)^3 \gamma \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

in which

D is the diameter of the disc in meters.

u_1 is the peripheral velocity of the disc in meters per second.

γ is the specific weight of the surrounding medium in kilograms per cu. meter.

---

* Engineering, Jan., 1904, p. 33.

In the English units

$$N' = 0.02295\, \alpha_1 D_1^{2.5} \left(\frac{u_1}{100}\right)^3 \gamma,$$

in which

D is the diameter of the disc in feet.

u_1' is the peripheral velocity of the disc in feet per second.

γ is the specific weight of the surrounding medium in pounds per cu. ft.

Table 2 gives the data of a series of experiments from which was calculated the value α_1, in which for the experiments of Odell γ was taken equal to 1.2, while for my experiments $\gamma = 1.12$.

TABLE 2.

EXPERIMENTS OF ODELL (NOS. 1–5) AND BY THE AUTHOR (NO. 6).

Number of Experiment.	1	2	3	4	5	6
Diameter of disc in mm . . .	**381**	**559**	**686**	**1 194**	**1 194**	**537**
Diameter of disc in inches . .	15	22	27	47	47	21.135
Maximum revolutions per min.	**2 000**	**850**	**525**	**530**	**740**	**2 000**
Corresponding peripheral velocity in meters per second .	**39.9**	**24.9**	**18.8**	**33.1**	**46.2**	**56.2**
Corresponding peripheral velocity in feet per second . .	130.9	81.7	61.7	108.6	159.8	184.4
Consumption of power in Watts	**17.7**	**8.14**	**5.56**	**101.3**	**229.1**	**110**
Consumption of power in French horse power	**0.0240**	**0.0111**	**0.00755**	**0.138**	**0.309**	**0.149**
Consumption of power in English horse power	0.0237	0.0109	0.00744	0.136	0.305	0.147
Constant α_1 in formula 2 . . .	**3.52**	**2.58**	**2.41**	**2.02**	**1.68**	**3.43**

For the further interpretation of our experiments (Table 1) we may apply the approximate mean value

$$\alpha_1 = 3.14.$$

Experiment 6 shows a higher consumption of power, because the disc had two holes placed there for balancing which caused considerable more work of fan resistance.

THE DEPENDENCE OF THE TOTAL WORK OF FRICTION OF A WHEEL ON THE DENSITY OF THE SURROUNDING STEAM ATMOSPHERE is shown graphically in Fig. 70, which was derived from experiments the author

made with a few stage impulse turbine. The turbine shaft and all the rotating wheels were placed in stagnant steam and were driven by a direct current motor. The consumption of work increased lineally with the specific weight of the steam. That the consumption did not become nothing at zero density, is explained by the existence of bearing friction which could not be neglected on account of the great weight of shaft and rotating wheels. As the steam was saturated, the consumption of work approximately increased directly with the absolute pressure.

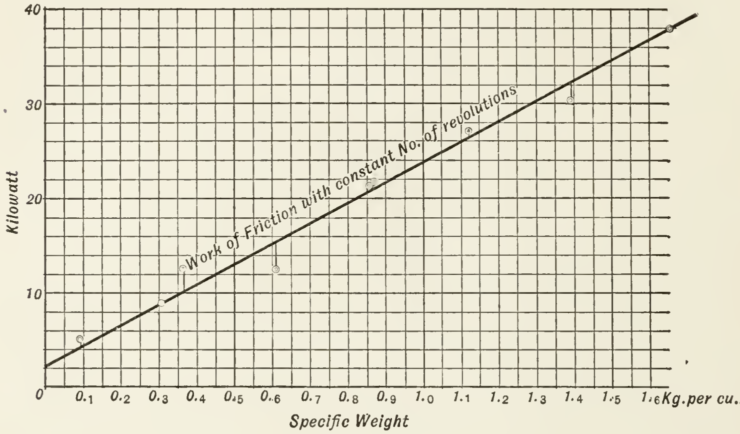

Fig. 70.

With the same turbine, experiments were also conducted with increasing number of revolutions, which also satisfied the law that the work of friction approximately increases with the third power of the number of revolutions.

THE DEPENDENCE OF THE WORK OF FRICTION ON THE DEGREE OF SUPERHEAT OF THE STEAM is shown by *Lewecki's* * valuable experiments. The rotating wheel of the Laval turbine he investigated had a diameter of 220 mm. (8.66 in.) and 20 mm. (0.787 in.) blade length, 10 mm. (0.394 in.) width of blade, about 6 mm. (0.236 in.) clearance space, and rotated successively in air, and in saturated and superheated steam. The pressure varied from 1 kg. per sq. cm. absolute (14.2 lb. per sq. in. absolute) to 0.36 kg. per sq. cm. absolute, (5.12 lb. per sq. in. absolute). Table 3

* Zeitschr. d. V. deutsch. Ing, 1903.

shows the results of these experiments for the constant speed of 20 000 revolutions per minute.

TABLE 3.

The Wheel Ran with 20 000 Revolutions per Minute in	Temperature.		Total Horse Power Used to Run Turbine without Load at Atmospheric Pressure.		Wheel Resistance Alone.					
					At Atmospheric Pressure.			At 0.36 Atmospheres (5.3 Pounds per Square Inch) Absolute.		
					Horse Power.			Horse Power.		
	C.°	*F.°*	In French Units.	*In English Units.*	French Units.	*English Units.*	a	French Units.	*English Units.*	a
Air . . .	30	86.0	6.8	6.7	4.6	4.5	6.44	. . .	. . .	. . .
Saturated Steam .	. .	. . .	5.5	5.42	3.3	3.25	8.83	1.5	1.47	10.48
Superheated Steam .	123	253.4	5.10	5.03	2.85	2.81	8.12	0.95	0.938	7.60
	184	363.2	4.55	4.49	2.25	2.22	7.39	. . .	. . .	. . .
	244	471.2	4.30	4.24	2.05	2.02	7.62	. . .	. . .	. . .
	300	572.0	4.15	4.09	1.88	1.85	7.06	0.60	0.5916	6.94

The work of friction, under otherwise constant conditions, decreases greatly with increasing superheat.

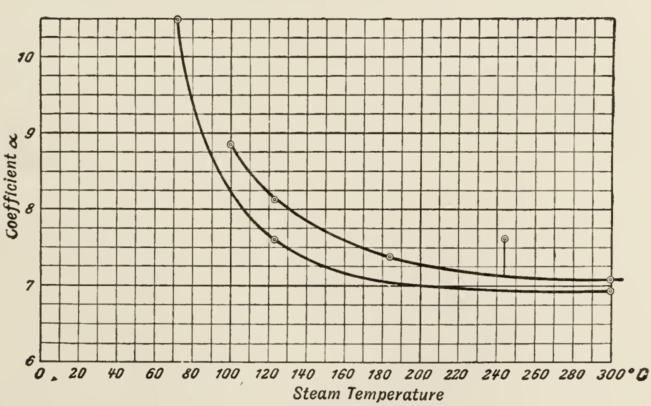

Fig. 71.

The values of the constant a (see below) are given in Fig. 71.

From *Lewecki's* experiments it was also found that the work of rotation without load increased as the third power of the number

of revolutions. The wheel resistance in saturated steam at atmospheric pressure was

$$N_0 = \quad 1.34 \quad 1.40 \quad 2.25 \quad 3.26 \quad \text{h. p. in French units,}$$

with

$$n = \quad 14\,850 \quad 15\,330 \quad 17\,660 \quad 20\,000 \quad \text{revolutions per minute,}$$

and therefore

$$10^{12}\frac{N_0}{n^3} = \quad 0.41 \quad 0.39 \quad 0.41 \quad 0.41 \quad \text{that is, practically constant.}$$

Another series of experiments with constant number of revolutions, decreasing pressure, and with saturated steam gave, after subtracting 0.23 h.p., the estimated value of bearing and stuffing box friction, the following:

TABLE 4.

In French Units.

Total h. p., running without load N_0 =	1.51	2.08	3.26
Deduct for bearing and stuffing boxes	0.23	0.23	0.23
Actual steam friction N_0' =	1.28	1.85	3.03
Absolute steam pressure kg. per sq. cm. =	0.40	0.60	1.00
Specific weight γ =	0.248	0.363	0.587
Ratio $\frac{N_0'}{\gamma}$ =	5.16	5.10	5.16
a =	8.10	8.03	8.10

In English Units.

Total h.p., running without load N_0 =	1.49	2.05	3.21
Deduct for bearing and stuffing boxes	0.23	0.23	0.23
Actual steam friction N_0' =	1.26	1.82	2.98
Absolute steam pressure lb. per sq. in. =	5.69	8.53	14.2
Specific weight γ =	0.0155	0.0227	0.0364
Ratio $\frac{N_0'}{\gamma}$ =	81.3	80.2	81.3
a =	8.10	8.03	8.10

The actual steam friction is therefore again proportional to the specific weight of the steam.

FORMULA FOR THE VALUE OF THE TOTAL WORK OF ROTATION WITHOUT LOAD.

A formula which would represent the wheel friction in free air or with an encased wheel, for forward and backward running, would be very difficult to derive. Our experiments at least permit

us to express the work of rotation in open air for forward running. The friction of smooth discs of which wheel bodies are built can be expressed, as was given above, by the equation,
In French units,

$$N_0' = \alpha_1 D^{2.5}\left(\frac{u}{100}\right)^3 \gamma;$$

In English units,

$$N_0' = 0.02295\ \alpha_1 D^{2.5}\left(\frac{u}{100}\right)^3 \gamma.$$

If we subtract this part from the total power, we have left the power to overcome fan resistance, which is empirically expressed as,
In French units,

$$N_0'' = \alpha_2 L^{1.25}\left(\frac{u}{100}\right)^3 \gamma,$$

In English units,

$$N_0'' = 1.4346\ \alpha_2\ L^{1.25}\left(\frac{u}{100}\right)^3 \gamma.$$

In all, the horse power for rotating without load would be,
In French units,

$$N_0 = N_0' + N_0'' = \left[\alpha_1 D^{2.5} + \alpha_2 L^{1.25}\right]\left(\frac{u}{100}\right)^3 \gamma;$$

In English units,

$$N_0 = N_0' + N_0'' = \left[0.02295\ \alpha_1 D^{2.5} + 1.4346\ \alpha_2 L^{1.25}\right]\left(\frac{u}{100}\right)^3 \gamma,$$

with the following values:

$$\alpha_1 = 3.14,$$

$$\alpha_2 = 0.42.$$

In which the outside diameter D is expressed in meters or feet, the blade length L in centimeters or inches, the outside peripheral velocity u in meters or feet per second, the specific weight γ in kilograms per cu. cm. or in lb. per cu. ft.

The formula gives for the wheels A to E that were used, an error of 5.9%, 1.6%, 0.6%, 1.2%, 0.2%, respectively; therefore, for practical use, it is sufficiently accurate. Of course, should

more experiments be made, there may be a change in the form of this formula.

In dealing with an encased wheel there is considerable decrease of friction power, for which a simplified formula has been derived; In French units,

$$N_0 = \alpha D^2 \left(\frac{u}{100}\right)^3 \gamma.$$

In English units,

$$N_0 = 0.041566\, \alpha D^2 \left(\frac{u}{100}\right)^3 \gamma.$$

The values of the total coefficient α are given in Tables 1 and 3. With 545 mm. (21.467 in.) diameter the consumption of power for an enclosed wheel is about one-half that in free air; with 1 265 mm. (49.803 in.) diameter, only about one-quarter.

If we wish to calculate the work of friction of a wheel in steam, we must make a recalculation of the values as found by *Lewicki.* In order to carry out the comparison correctly, we must subtract, in Table 3, the bearing and stuffing box friction load; that is, 0.23 h. p. from the total power. Then a consideration of the constant α will permit us to summarize as follows: *The work of friction of a wheel in saturated steam with equal specific weight, equal size of wheel and equal velocity, is 1.3 times that in air.*

The work of friction of a wheel in superheated steam at atmospheric pressure and 300° C. (572° F.) is the same as that in air.

At 300° C. (572° F.) temperature in a vacuum of 0.36 atm. abs. (5.3 lb. per sq. in.), the work of friction in steam, according to *Lewicki's* experiments, is, surprisingly, even smaller than in air at atmospheric temperature.

The reason for the decrease of the work of rotation without load when the wheel works with partial or full peripheral admission is yet undetermined. The surrounding steam is thrown into eddy currents, then enters the wheel and disturbs still more the outflowing steam stream, and thus causes losses. Though we may look upon the work of rotation without load as small in itself, the new loss must nevertheless be considered.

After all, our experiments with actual turbine wheels show that the work of fan resistance is not so great as was formerly supposed.

III.

CONSTRUCTION OF THE MOST IMPORTANT TURBINE PARTS.

34. FORM OF BLADE.

The form of the blade should be such that the steam stream expands to the desired final pressure with the smallest possible friction and eddy current losses, and is turned in the desired direction. For the rotating wheel channels it is sufficient to keep in mind the relative motion, especially as the difficulties of certain hydraulic radial turbines, in which it may happen that along a part of the water path the blade can transmit work to the water instead of receiving work from it, disappear with steam turbines, on account of higher velocities and sharper blade curvature. As to turning the stream in the desired direction, the direction of exit from the guide and rotating wheel is of greatest importance. In order to get the desired angle, we should try to keep the last part of the blade straight, at least to the foot of the perpendicular dropped

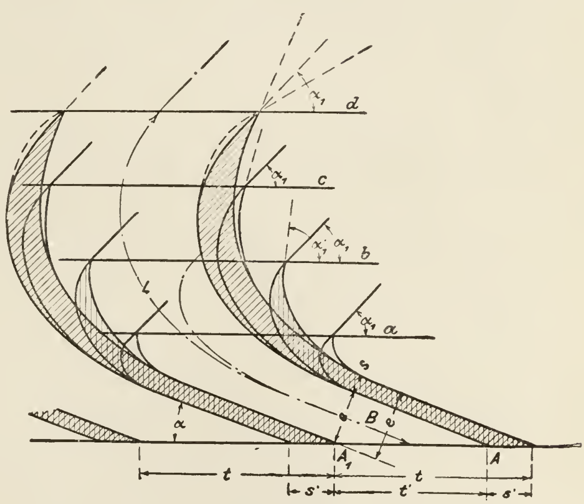

Fig. 72.

from A_1, or the length $A\,B$ in Fig. 72. From there on, the channel should lead in easy curvature to the angle α_1. The construction according to a in Fig. 72 would obviously be too sharp, and would cause the steam stream to separate from the wall. The construction according to b would suffice, and the wheel radius would depend, above all, upon how far we wish to diminish the shock at entrance. For the profile b the angle α_1 is taken as the slope of the blade back, from which we obtain for the guiding blade surface the somewhat large angle α_1'. This would be more favorable with c, and d, but the latter would obviously give a needlessly long steam path. Besides, a pointing of the blade such that α_1 is half of α_1', as is shown dotted at d, could be considered just as correct as the first mentioned. By drawing the absolute steam path and finding the decrease of peripheral speed, we get useful results concerning the regularity of delivering work.

The proper length of the channel, that is, the steam path, can be determined only by practical experience, and with given curvature the ratio of length to breadth can be considered fairly constant. The question arises whether we should use wide and long or short and narrow channels.

35. WIDE AND LONG, OR SHORT AND NARROW CHANNELS.

Following our analogy with hydraulics, we may give an expression similar to that representing the "loss of head" in hydraulic resistance, as follows:

$$h_z = \zeta \frac{U}{F}\, l\, \frac{w^2}{2\,g} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

In this, U is the circumference, F the area of cross-section, l the curved length of the channel, and w the velocity. More exactly, the mean value of $\frac{U}{F} w^2$ should be inserted, which can be found graphically, if necessary, from the expression for mean value

$$\int_0^l \zeta \frac{U}{F} \frac{w^2}{2\,g} dl = \frac{\zeta}{2\,g} \left(\frac{U}{F} w^2\right)_m l.$$

If a is the radial length, and e the least width of the channel, we get

$$h_z = \zeta \frac{2(a+e)}{a\,e} l \frac{w^2}{2g} = 2\zeta\left(1+\frac{e}{a}\right)\frac{l}{e}\frac{w^2}{2g} \quad . \quad . \quad . \quad (2)$$

$\frac{l}{e}$ becomes, as was said at the close of the above article, nearly constant; and equation 2 shows that it is advisable, within certain limits, to choose the spacing (therefore also e) small. A limit to the reduction of e is set by the influence of the blade thickness, due to which there is an expansion of the stream in the clearance space accompanied by eddy currents.

This spacing also depends on the length of the blades, hence necessitates a minimum axial breadth. As practical limits we could consider with lengths of 20 to 30 mm. (0.79 to 1.18 in.) an axial breadth of about 8 to 10 mm. (0.315 to 0.394 in.), and a spacing of 5 to 6 mm. (0.2 to 0.24 in.); with very long blades 200 to 300 mm. (7.87 to 11.81 in.), about 25 mm. (0.985 in.) breadth and 14 to 16 mm. (0.55 to 0.63 in.) spacing.

36. CONSTRUCTION OF BLADE AND METHOD OF FASTENING IT.

1. HIGH PERIPHERAL VELOCITY.

A. BLADES CONSTRUCTED SEPARATELY.

For wheels with high velocity, that is, somewhat over 150 m. (492.12 ft.) *de Laval* has designed the excellent construction shown in Fig. 73. The blades are pressed from ingot steel and finished to caliper. The principle of the interchangeability of parts is strongly adhered to, and the cost of renewing a blade rim is small. At their outer ends the blades have flanges which touch one another and form a closed boundary ring, the joints being lightly caulked. The centers of the blades are greatly thickened throughout their length in order to get approximately steady flow through the blades, (see section *A B*, Fig. 73). They could be made thinner at the outer end to decrease centrifugal force, but there would not

be as good a steam channel, especially with small wheels and long blades, because of the noticeably greater spacing at the outer end

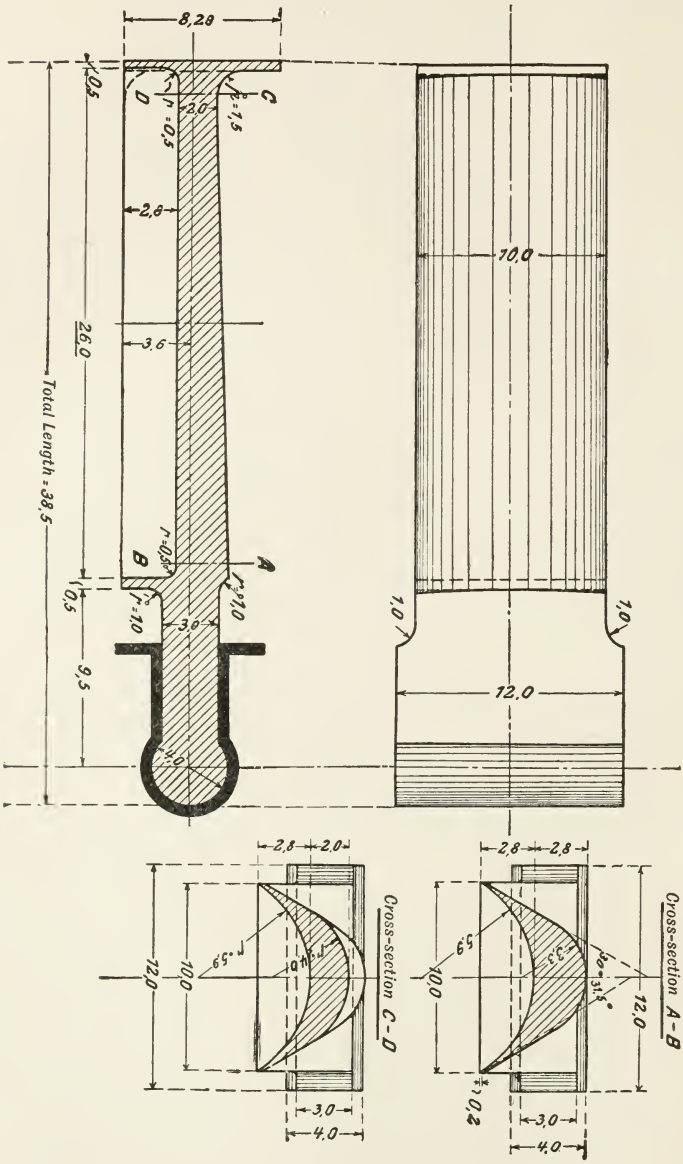

Fig. 73.

(see section *CD*, Fig. 73). The construction is suitable for the greatest velocities that have so far been attained, about 430 m.

(1 410.76 ft.), but it is limited to those single wheels which are accessible from the side.

In the earlier types *Laval* used the construction shown in Fig. 74, with the wheel body made in two parts. It was not suitable for large wheels, and was more costly than that shown in Fig. 73.

Seger patented the idea shown in Fig. 75, in the year 1894 by an English patent, No. 4 611. The blade *a* is cut to length from a

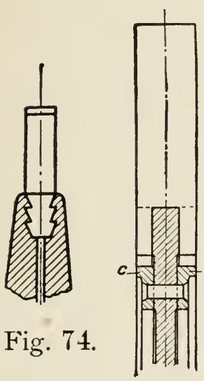

Fig. 74.

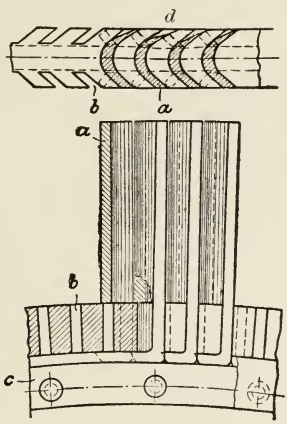

Fig. 75.

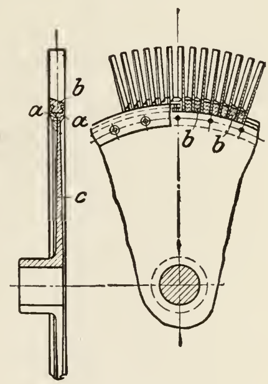

Fig. 76.

drawn section, the bottom end is machined in the form of a fork and set in the machined groove *b* of the wheel rim. The forked ends are now bent over and kept from straightening out again by

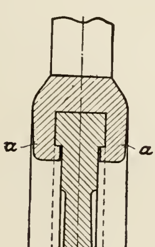

Fig. 77.

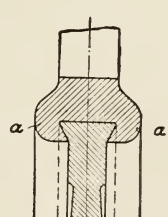

Fig. 78.

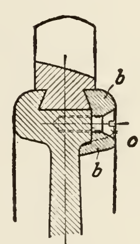

Fig. 79.

the rings *c* which are riveted together. Since the spaces *b* are most easily machined straight, the blade section has to consist of two straight lines and one curved line; at the entrance *d* the otherwise uniformly thick blade is sharpened. A decrease of the blade thickness towards the outer end is beneficial, but requires special

machine work or rolled sheet metal strips of unequal thickness, from which the blades must be cut and bent.

Zölly used for his impulse wheels drawn wrought iron blades which were machined narrower towards the outer ends and finished to a polish. To fasten the blades, they were provided on both sides with right angled lugs *a* as shown in Fig. 76; these lugs fitting into corresponding slots *c* on the wheel. A covering ring was then fitted into place, and permanently riveted after inserting the blades. The blades were kept at the correct distance from each other by

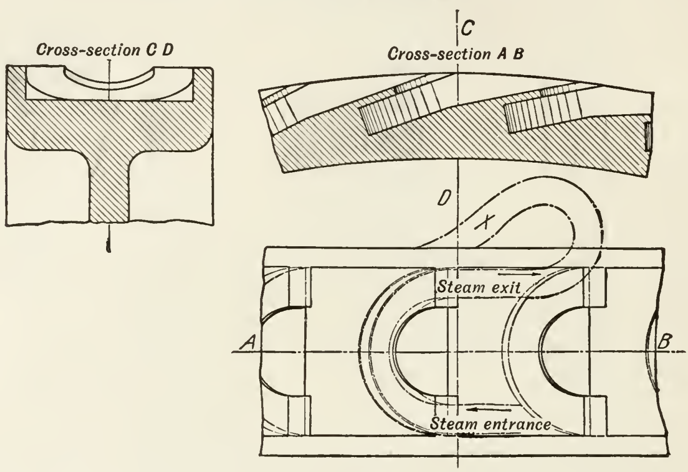

Fig. 80.

the distance pieces *b*, which were machined all over, and also provided with lugs similar to those on the blades. The blades were widened radially and their outer ends remained free.

The reverse of this construction, as shown in Fig. 77, is also possible. The solid dovetail shown in Fig. 78 requires the parts *a–a* to be stronger, because the pressure on the fitted surfaces is greater and its lever arm is greater. Here, as well as in Fig. 79, where the rim itself receives the dovetail, the dead weight is appreciably larger with the same width of blade. The radial stress of the disc increases in proportion to its weight, as is shown in Article 39.

The weight is also increased because we must leave the blades full at the bottom for some distance so that the guide apparatus can come close to the blade channel. At a few symmetrically arranged

places a fitted blade must be fastened by means of bolts; or as is shown in Fig. 79, an opening for the introduction of the blades closed by means of a fitted plate *b*, which is held in place by machine screws *c*.

B. BLADES AND RIM IN ONE PIECE.

In the Riedler-Stumpf turbine we have Pelton-like buckets machined directly into the rim of the turbine wheel. Fig. 80 shows the bucket form described in *Stumpf's* French patent 310 020 of the year 1901.

The overhung milling cutter machines the semicircular channel and the half round opening in the division wall between the buckets at the same time. Fig. 81 shows a portion of a wheel with two buckets side by side, the edge between them splitting the jet of steam into two equal parts.

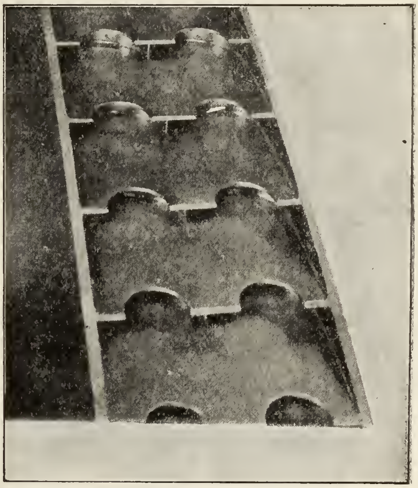

Fig. 81.

The General Electric Company of Schenectady planes the buckets of their axial impulse turbines with special machines, whose cutter, properly shaped to the curvature of the blade, is moved to and fro in a curved path. In Figs. 82 and 83 are shown segments with blades, with small and large spacing. Over the ends of the blades a band is placed and riveted to the projection, which may be seen in the illustration.

2. **MODERATE PERIPHERAL VELOCITIES.** ($u < 150$ m., 492 ft.)

The many stage turbine works, as we have said, with peripheral velocities not greatly exceeding 100 m. (394 ft). The blades have only to withstand small centrifugal forces, hence their construction is much simpler.

The Parson's Turbine, according to the catalogue of their

licensed firms, uses blades made from bars of drawn bronze or other metals, placed in dovetailed grooves and clamped by small pieces of

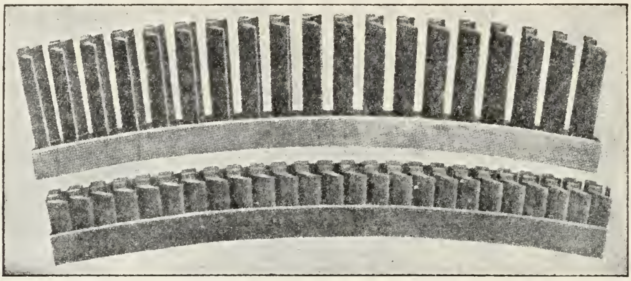

Fig. 82.

the same material. These small pieces are caulked in order to make everything tight. There is no need for an enclosure on the outer circumference as one is formed by the turbine casing itself, enclosing the wheel with a clearance space so slight that the losses due to it are kept sufficiently small.

The guide blades are fastened in a similar manner in the walls of the casings. Guide and rotating wheels follow closely one another.

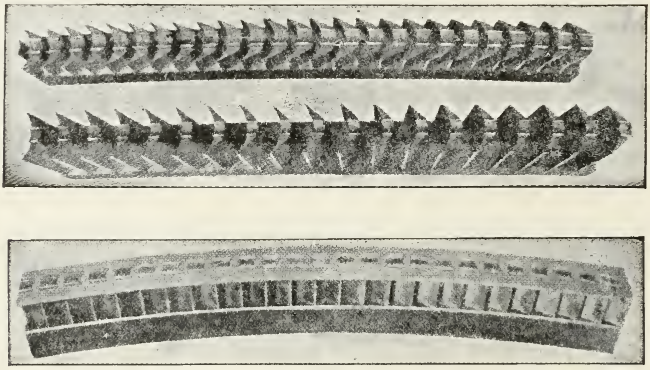

Fig. 83.

In the direction of the axis a play of a few millimeters (less than 0.1 inch) is allowable.

The latest efforts, as may be seen from the patents* granted to

* H. F. Fullagar, Swiss Patent No. 24 039, Class 93; Parson's Foreign Patents Co. and A. G. for Brown, Boveri-Parsons. German Patent No. 144 528, Class 14c. The latter again, Swiss, Patent No. 26 718, Class 93, etc.

firms manufacturing these turbines, aim to get a solid means of machining the blade and to avoid losses due to leakage through

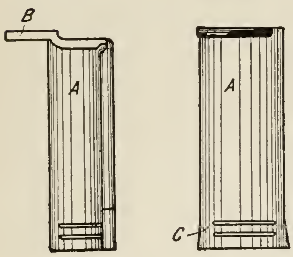

Fig. 84.

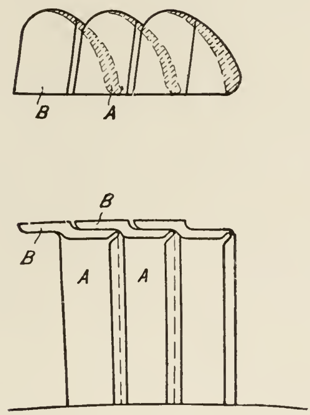

Fig. 85.

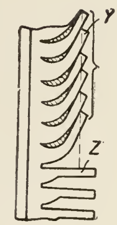

Fig. 86.

suitable partition rings. Thus, according to Fig. 84, the ends of the blades are bent, overlapped, and finally soldered so that the pro-

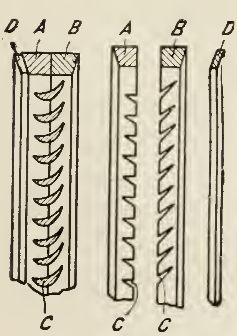

Fig. 87.

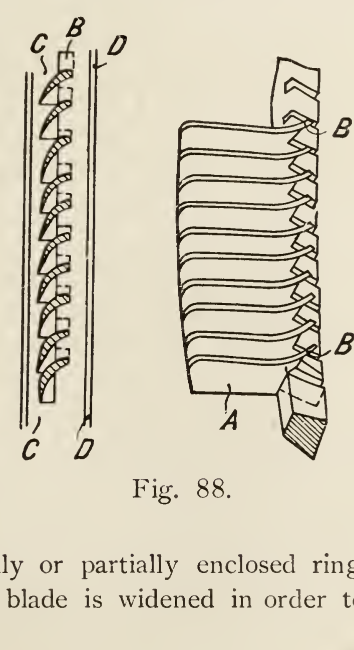

Fig. 88.

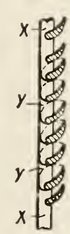

Fig. 89.

jection *B* forms a wholly or partially enclosed ring. (Fig. 85.) The inner end *C* of the blade is widened in order to fill out the dovetail groove.

The rings should be made wider than the blades, so that with caulking, the blade will not be damaged. Fig. 86 shows a method of fastening the blades by bending the teeth *z* of a ring, this ring being inserted in a groove and held there by means of a special caulking ring. How the exposed corners *y* are to be turned off is not stated. In Fig. 87 the fastening ring is divided into halves *A* and *B*, and is provided with grooves *C* in which the blades fit. *D* is the caulking ring.

The blade *A* is slit as Fig. 88 shows, at the lower end *B*, and the two parts spread apart. The grooves in the two halves of the fastening rings *C* and *D* are shaped so as to fit these spread ends.

The outer ends of the blades are held together, Fig. 89, by a ring *x* in which they are partly inserted, soldered wires assisting in tying them.

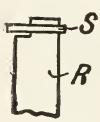

Fig. 90.

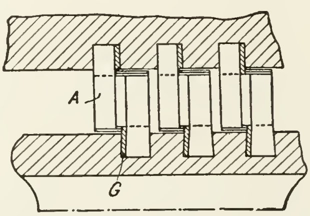

Fig. 91.

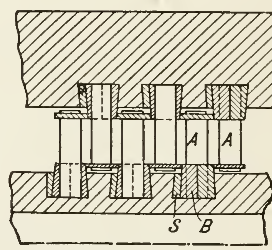

Fig. 92.

The same result is accomplished by means of the perforated ring *S* (Fig. 90) which is suitably attached to the blade element *R*.

These methods allow of the production of a much better steam tight turbine, as Fig. 91 shows. Here the partition ring shown in Fig. 90 may just clear the caulking ring *G* with the smallest possible play. The wheel may have any desired radial play, but must, on the other hand, be placed very exactly axially. In Fig. 92 we get a perfectly steady flow of steam, but the rotating drum must be prevented from moving axially in either direction.

Ideas of this kind obviously require similar materials for the rotating drum and the casing, and similar cooling conditions, so that the expansions in length of both parts are equal.

Fig. 93 represents one of Parson's constructions, with pressed

blades inserted in dovetailed grooves. The partition ring was formed similarly to *Laval's*.

In the *Rateau* Turbine the blades were pressed out of sheet iron and were riveted to a drum, also of wrought iron as is shown in Fig. 94.

An outer partition ring is always used. According to the German patent No. 143 960, it is desirable to keep the blades thin on account of their weight, and the strength is increased by filling the hollow part at the place of bending with lead. The strain in the

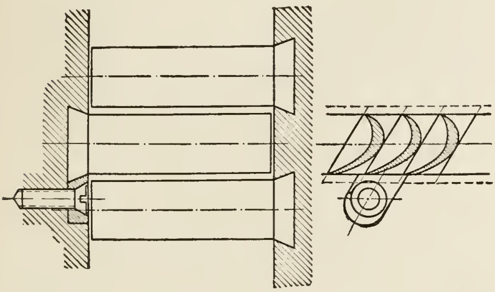

Fig. 93.

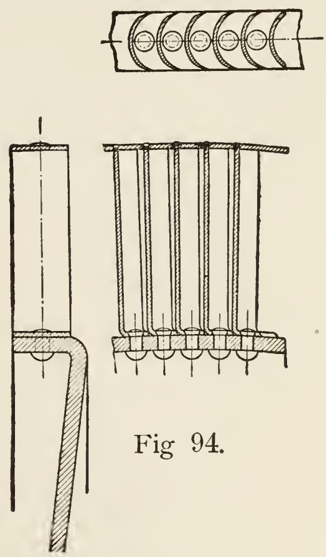

Fig 94.

rivet is not larger nor of a different nature from that, for instance, which occurs in the rivets of a steam dome of a boiler. If the blades are of large dimensions, it is recommended that the flange for riveting be split into two parts, so that the blade may be pressed into a proper form.

37. CONSTRUCTION OF THE GUIDE APPARATUS.

Fig. 95 shows the construction of the entrance to a nozzle, with its operating spindle made steam tight by a stuffing-box. The nozzle is made steam tight metallically in the slightly conical bored hole. To remove the nozzle, we use an apparatus with a pressure screw.

With the round nozzle a section taken in the plane of the wheel forms a very flat ellipse, and the first and last blades are not fully filled. They draw steam from about them with loss of work. This

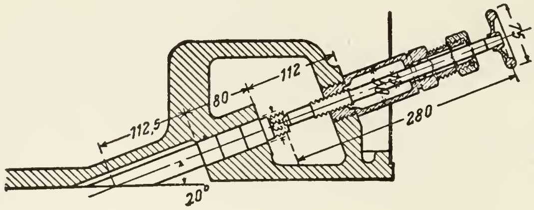

Fig. 95.

disadvantage is avoided by the *Stumpf* nozzle, which is originally turned round but is pressed into a right angled section (Fig. 96). These nozzles are placed so near together that the steam jet enters the rotating wheel as if from one large nozzle. Partial peripheral admission is easily obtained.

*Th. Reuter** machines the guide channels screw fashion in the circumference of a ring *a* (Fig. 97), and makes this steam tight by covering it with ring *c*. The section of the nozzle is also right angled and the division walls run out pointed, in order to bring

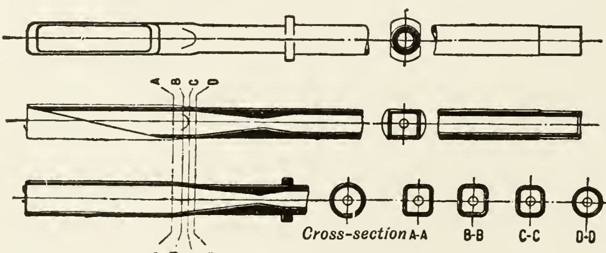

Fig. 96.

about a union of the individual jets. In the figure are also drawn the individual stop valves used for regulating, and which will be discussed later.

Where an expansion is permissible, that is, with many stage turbines, we work obviously with the simple blade form, and every-

* Swiss Patent No. 25 441, Class 93, February, 1902.

thing depends on its construction. Fig. 98 shows the entrance of a *Zölly* turbine whose blades are bent of steel plate, and are cut out

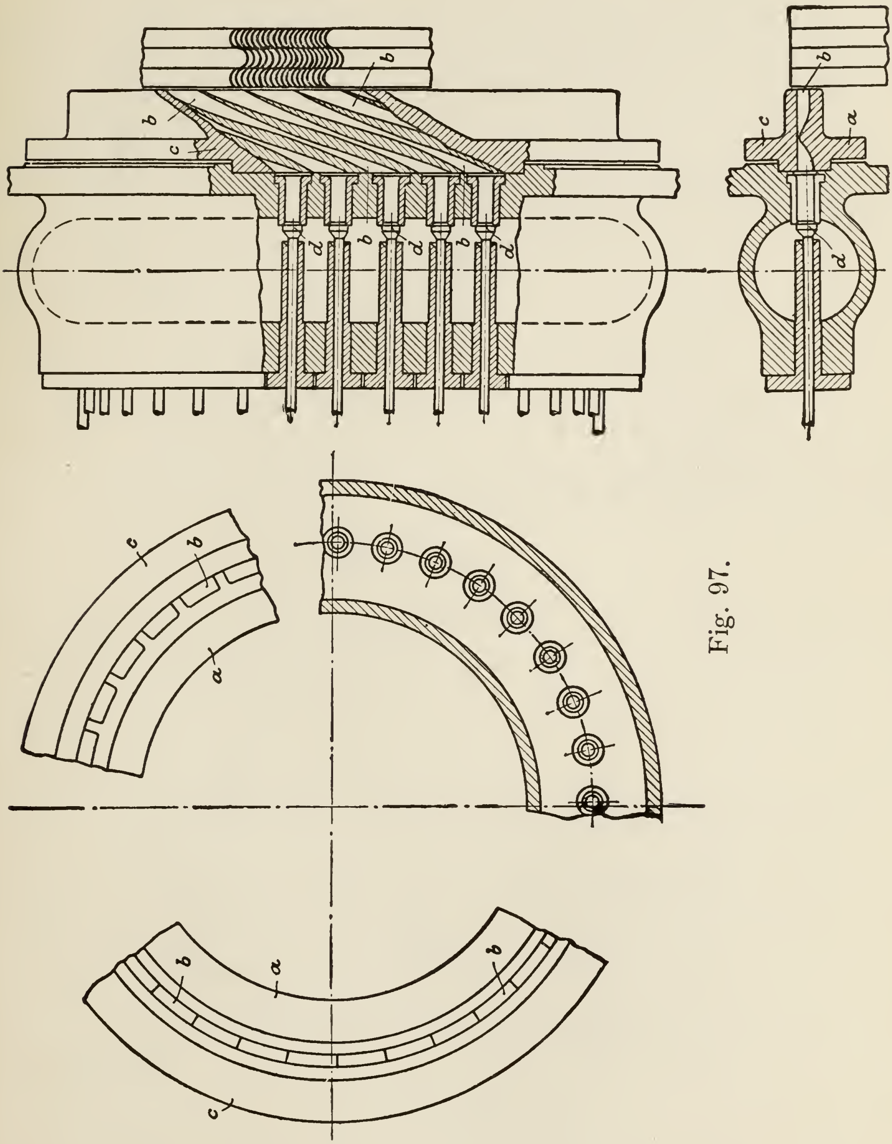

Fig. 97.

according to sketch a. To receive the projections a, grooves b are provided, in which the blades are held fast by the rings c and d.

Rateau places the guide blade groups in a helical form, as shown in Fig. 99, so that the steam may follow the natural flow

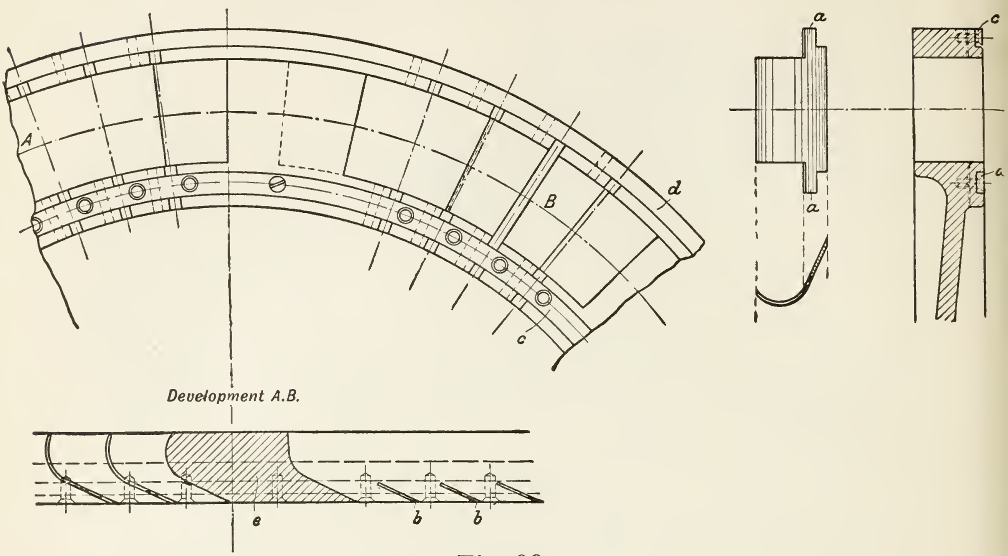

Fig. 98.

due to its velocity. The absolute steam path is drawn in the figure, according to the Swiss patent No. 24 473 (dotted) with the assumption of normal exit.

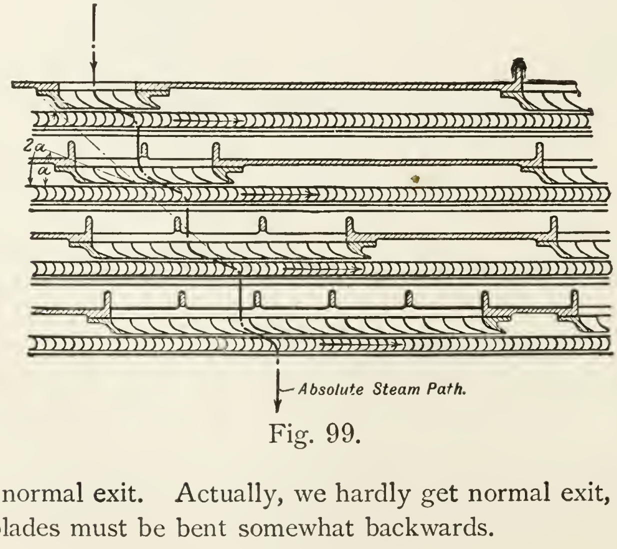

Fig. 99.

tion of normal exit. Actually, we hardly get normal exit, and the guide blades must be bent somewhat backwards.

38. WHEEL DRUMS.

The rotating wheels of many stage turbines are designed either as *individual wheels*, or in groups of successive blade-rings forming *drums* of greater or less length. In the latter case, the inside of the drum should be bored out to obtain complete mass-balance. (See Art. 43.) These rings may be secured either by inserted spoked rings, or by means of flanges on both ends of the drum and forged on the shaft, or by means of end-discs forced on the shaft.

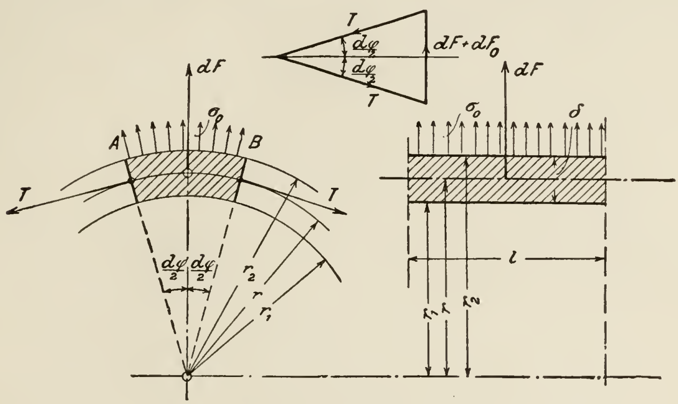

Fig. 100.

A drum fastened in this manner is to be calculated, at least so far as the middle part is concerned, as a free rotating ring, because the influence of the end attachments does not reach far.

Consider in Fig. 100 the elementary mass $A\ B$ of the cylindrical drum of length 1, bounded by the axial planes through A and B, which diverge on either side of the vertical axial plane by the angle $\frac{d\phi}{2}$. Take first the action of centrifugal force on this elementary mass. It is given by the equation

$$dF = (r\, d\phi\, \delta\, l\, \mu)\, r\, \omega^2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

in which

$$\mu = \frac{\gamma}{g} = \text{the specific mass.}$$

$$\omega = \text{the angular velocity.}$$

The remainder of the notation is apparent from the figure.

Further we may assume, if the thickness δ be not too great, that the stress σ in the direction of the circumference is of uniform intensity, and that its effective resultant on the area $l\delta$ amounts to

$$T = l\,\delta\,\sigma \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

Finally, we must take into consideration the centrifugal force of the blades and of the parts by which they are held together. This force may be imagined to be equally distributed. If σ_0 represents this force per unit area (sq. cm. or sq. in.) on the *mean* cylinder area of radius r, the ensuing resultant on the element will be

$$dF_0 = r\,d\phi\,l\,\sigma_0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

These forces are in equilibrium if

$$dF + dF_0 = 2\,T \sin\frac{d\phi}{2} = T\,d\phi.$$

Substituting equation 1 in equation 3, and taking $r\omega$ equal to the peripheral velocity w, we have

$$\sigma = \frac{r\,\sigma_0}{\delta} + \mu\,w^2.$$

From the above the important fact is determined that the term

$$\sigma' = \mu\,w^2,$$

that is, the stresses due to centrifugal force *depend only on the peripheral velocity*, no matter how large the radius, and we get the following values for wrought iron :

In the French units,

$w =$ 25	50	75	100	150	200	400	meters per second.
$\sigma' =$ 50	200	450	800	1 800	3 200	12 800	kg. per sq. cm.

Or in the English units,

$w =$ 82.02	164.04	246.06	328.08	492.13	656.17	1 312.33	ft. per sec.
$\sigma' =$ 711.16	2 844.6	6 400.4	11 378 5	25 602	45 514	182 060	lb. per sq. in.

Beyond velocities of about 100 to 120 meters (328 to 393.7 ft.) the stresses due to centrifugal force in a free drum are too high for ordinary materials. We must therefore strengthen the drum by spokes, or better still, by solid discs. Still, these spokes or discs must be placed close together in order to be fully effective,

thus preventing the boring out of the drum. It is therefore best to divide the drum into short sections, and to construct and calculate these sections in the same manner as the disc-wheels discussed in the next article.

39. DISC-WHEEL OF VARYING THICKNESS.

The stresses in a disc-wheel produced by its own centrifugal forces can be determined by the following investigation:

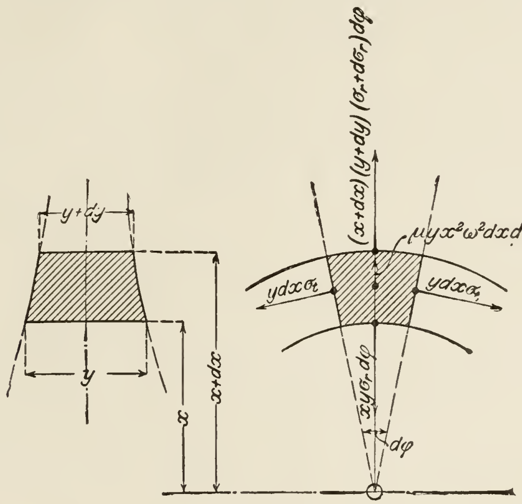

Fig. 101.

In Fig. 101 let

$x =$ the radial distance of a point from the axis,
$y =$ the thickness of the disc at the distance x,
$\sigma_r =$ the radial stress per unit area,
$\sigma_t =$ the tangential stress per unit area,
$\mu =$ the specific mass of the material of the disc,
$\omega =$ the angular velocity of rotation,
$m = \frac{1}{\nu}$ the ratio of longitudinal expansion to sectional contraction.

We shall assume the thickness to vary so slightly that we can neglect the angle formed by the direction of the radial stresses to the plane of symmetry of the wheel, and furthermore, that these stresses are distributed uniformly over the cross-section. This is true in most cases.

The disc-element represented in Fig. 101 has the volume

$$dV = y\,x\,d\phi\,dx \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

and the mass

$$dm = \mu\,dV \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

and is acted upon by the following forces: its own centrifugal force

$$dF = dm\,x\,\omega^2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

(with infinitely small thicknesses, we may substitute x for the radius of the center of gravity).

The side forces,

$$dT = y\,dx\,\sigma_t \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

the radial force on the inner face surface,

$$dR = y\,x\,d\phi\,\sigma_r \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

and the similar force on the outer face surface,

$$dR' = (y + dy)\,(x + dx)\,d\phi\,(\sigma_r + d\sigma_r) \quad . \quad . \quad . \quad (6)$$

The condition of equilibrium demands that the radial components of these forces neutralize each other; that is,

$$dR' - dR - T\,d\phi + dF = 0 \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

Substituting equation 1 in equation 6,

$$\frac{d\,(x\,y\,\sigma_r)}{dx} - y\,\sigma_t + \mu\,\omega^2\,x^2\,y = 0 \quad . \quad . \quad . \quad . \quad (8)$$

Further, let

ξ = the radial deflection of the end point of radius x,
ϵ_r = the specific expansion radially,
ϵ_t = the specific expansion tangentially.

The fundamental law of elastic deformation* teaches, that when an elastic body is subjected to a tensile stress which causes a specific *elongation* ϵ in the direction of the stress (that is, per unit length), a contraction occurs in all directions at right angles thereto, whose value (also referred to unit length) is equal to $\nu\epsilon$. The constant ν has a mean value for wrought iron, of 0.3. An element of the disc under consideration undergoes through the radial stress

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* *Grashof*, Theorie der Elastizität and Festigkeit, 1878, p. 32.

σ_r the radial expansion $\frac{\sigma_r}{E}$. The tangential stress acting simultaneously, produces the cross-section contraction whose algebraic sum is equal to $\frac{\nu \sigma_t}{E}$, and the resultant elongation radially is, therefore,

Similarly is found

$$\left.\begin{aligned} \epsilon_r &= \frac{1}{E}[\sigma_r - \nu \sigma_t] \\ \epsilon_t &= \frac{1}{E}[\sigma_t - \nu \sigma_r] \end{aligned}\right\} \quad \ldots \ldots \quad (9)$$

The elongations can be expressed by the deflection ξ. An infinitely thin ring of radius x has, before expansion, a circumference $2\pi x$; after expansion, $2\pi(x+\xi)$. Therefore the specific expansion at the circumference is

$$\epsilon_t = \frac{2\pi(x+\xi) - 2\pi x}{2\pi x} = \frac{\xi}{x} \quad \ldots \ldots \quad (10)$$

As the deflection of the point A at a distance x is given by ξ, the deflection of the point B, which was originally at a distance $x + dx$, will be

$$\xi' = \xi + \frac{d\xi}{dx} dx.$$

The original length of the distance $A B$ is dx; after expansion it becomes

$$dx' = (x + dx + \xi') - (x + \xi) = \xi' - \xi + dx = \frac{d\xi}{dx} dx + dx.$$

The specific elongation, therefore, is

$$\epsilon_r = \frac{dx' - dx}{dx} = \frac{d\xi}{dx} \quad \ldots \ldots \quad (11)$$

Substituting the values of ϵ_t and ϵ_r in equation 9, we have

$$\left.\begin{aligned} \sigma_r &= \frac{E}{1-\nu^2}\left[\nu \frac{\xi}{x} + \frac{d\xi}{dx}\right] \\ \sigma_t &= \frac{E}{1-\nu^2}\left[\frac{\xi}{x} + \nu \frac{d\xi}{dx}\right] \end{aligned}\right\} \quad \ldots \ldots \quad (12)$$

Introducing the above into equation 8, we have

$$\frac{d^2\xi}{dx^2}+\left[\frac{d\,(\log y)}{dx}+\frac{1}{x}\right]\frac{d\xi}{dx}+\left[\frac{\nu}{x}\frac{d\,(\log y)}{dx}-\frac{1}{x_2}\right]\xi+A\,x=0, \quad (13)$$

with the simplified notation

$$A=\frac{(1-\nu^2)\,\mu\,\omega^2}{E}.$$

Equation 13 becomes integrable, if we take, for instance,

$$y=c\,x^{\alpha} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (14)$$

and assumes the form

$$\frac{d^2\xi}{dx^2}+\frac{\alpha+1}{x}\frac{d\xi}{dx}+\frac{\alpha\,\nu-1}{x^2}\xi+A\,x=0 \quad . \quad . \quad . \quad (15)$$

To dispense with the term containing x, place

$$\xi=z+a\,x^3 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (16)$$

and we obtain, after substitution,

$$\frac{d^2z}{dx^2}+\frac{\alpha+1}{x}\frac{dz}{dx}+\frac{\alpha\,\nu-1}{x^2}z=0. \quad . \quad . \quad . \quad . \quad . \quad (17)$$

provided we have chosen

$$a=\frac{-(1-\nu^2)\,\mu\,\omega^2}{E\,[8-(3+\nu)\,\alpha]} \quad . \quad . \quad . \quad . \quad . \quad . \quad (18)$$

The solution of equation 17 follows by taking $z=b\,x^{\psi}$, which leads to the calculation of ψ in the equation

$$\psi^2-\alpha\,\psi-(1+\alpha\,\nu)=0 \quad . \quad . \quad . \quad . \quad . \quad . \quad (19)$$

We obtain two values for ψ:

$$\left.\begin{aligned}\psi_1&=\frac{\alpha}{2}+\sqrt{\frac{\alpha^2}{4}+\alpha\,\nu+1}\\ \psi_2&=\frac{\alpha}{2}-\sqrt{\frac{\alpha^2}{2}+\alpha\,\nu+1}\end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad (20)$$

from which, (with α positive), ψ_1 is always positive, and ψ_2 is always negative. The solutions then give with equation 16 the complete integral,

$$\xi = a x^3 + b_1 x^{\psi_1} + b_2 x^{\psi_2} \quad . \quad . \quad . \quad . \quad . \quad . \quad (21)$$

in which $b_1 b_2$ are constants determined by the rim conditions (see below). We now find $\frac{\xi}{x}$ and $\frac{d\xi}{dx}$ which when placed in equation 12 will give the values of the stresses

$$\left.\begin{aligned}\sigma_r &= \frac{E}{1-\nu^2}\left[(3+\nu)\,a x^2 + (\psi_1+\nu)\,b_1 x^{\psi_1-1} + (\psi_2+\nu)\,b_2 x^{\psi_2-1}\right]\\ \sigma_t &= \frac{E}{1-\nu^2}\left[(1+3\nu)\,a x^2 + (1+\psi_1\nu)\,b_1 x^{\psi_1-1} + (1+\psi_2\nu)\,b_2 x^{\psi_2-1}\right]\end{aligned}\right\} (22)$$

RIM CONDITIONS.

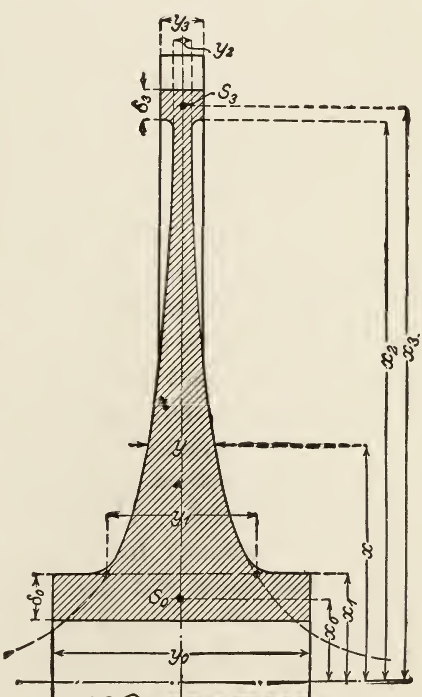

Fig. 102.

With positive values of α, the disc takes the form of the disc-section used by *de Laval* for small wheels. Fig. 102 shows such a section, consisting of a disc proper, a hub, and a strengthened outer ring to which the blades are attached.

If we suppose the blades made in one piece with the wheel, they exercise a centrifugal force per sq. in. (sq. cm.) of the cylindrical surface of radius x_3 and width y_3, which force may be expressed as σ_3. This ring suffers an elongation ξ_2' due to its own centrifugal force, to the radial stress σ_{r2} of the disc through the width y_2, and to the load σ_3, according to the formula

$$\xi_2' = \frac{x_3^2}{E\,\delta_3 y_3}\left[\sigma_3 y_3 + \mu\,\omega^2\,\delta_3 y_3 x_3 - \sigma_{r2}\frac{x_2 y_2}{x_3}\right] \quad . \quad . \quad (23)$$

in which for σ_{r2} the expression in equation 22 may be placed, letting $x = x_2$.

Similarly, if we neglect the irregularity of the distribution of the stresses between the disc and the hub, as well as the radial stresses in the latter, we have for the hub

$$\xi_1' = \frac{x_0^2}{E\delta_0 y_0}\left[\sigma_0 y_0 + \mu\,\omega^2\,\delta_0 y_0 x_0 + \sigma_{r1}\frac{x_1 y_1}{x_0}\right] \quad . \quad . \quad (24)$$

in which σ_0 represents the pressure per sq. in. (sq. cm.) of area of internal surface of the hub, $2\pi x_0\,y_0$, due to forcing the hub on the shaft. On the other hand, the radial expansion of the disc due to its own stress-condition is, at x_1 and x_2, respectively:

$$\left.\begin{aligned}\xi_1 &= a\,x_1^3 + b_1\,x_1^{\psi_1} + b_2\,x_1^{\psi_2}\\ \xi_2 &= a\,x_2^3 + b_1\,x_2^{\psi_1} + b_2\,x_2^{\psi_2}\end{aligned}\right\} \quad . \quad . \quad . \quad . \quad (25)$$

As the disc is in permanent connection with the rim and the hub, so must

$$\left.\begin{aligned}\xi_1 &= \xi_1'\\ \xi_2 &= \xi_2'\end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (26)$$

From these equations the values of the constants b_1 and b_2 are determined, after the stresses in equations 23 and 24 have been expressed by the aid of x_1 and x_2 of equation 22. The complicated form of the equations demands a trial assumption of all dimensions and a checking of the resulting stresses.

The cross-section of the rim is governed by the size of the blades and the method of attaching them, which demands a fairly arbitrary value of δ_3. The thickness of the disc, y_2, can be determined approximately from the assumption, that the centrifugal force of the blades and rim is carried directly to the wheel disc, causing the stress σ_{r2}. If we substitute for this stress the allowable value σ_{r2}', then y_2 can be determined from the equation,

$$\sigma_{r2}' y_2 = \sigma_3 y_3 + \mu\,y_3\,\delta_3\,\omega^2 x_3 \quad . \quad . \quad . \quad . \quad . \quad (27)$$

Now choose a trial value for y_1, and the equations

$$y_1 = c\, x_1^{\alpha}$$

$$y_2 = c\, x_2^{\alpha}$$

determine the exponents,

$$\alpha = \frac{\log\left(\frac{y_1}{y_2}\right)}{\log\left(\frac{x_2}{x_1}\right)} \quad \ldots \ldots \ldots \quad (27a)$$

The dimensions of the hub must also be somewhat freely chosen, in conformity with the size of the disc and the diameter of the shaft. Not until now will the calculation give exact values for the stresses σ_r and σ_t, of which σ_t for $x = x_2$, is the most vital.

If it is desired to carry on this discussion to results of greater accuracy, we must take into account the "cross-sectional contraction" of the rim and hub. For the former there is a tangential stress

$$\sigma_t' = \frac{x_3}{\delta_3\, y_3}\left[\sigma_3\, y_3 + \mu\, \omega^2\, \delta_3\, y_3\, x_3 - \sigma_{2r}\, y_2 \frac{x_2}{x_3}\right].$$

The radial stress can be estimated as follows: at the outer circumference the approximate stress σ_3 exists. At the inner circumference, we have $\sigma_{2r}\, y_2$ per unit length, which we can consider uniformly distributed over the width y_3 through a suitable fillet, giving the stress $\frac{\sigma_{r2}\, y_2}{y_3}$. As a mean value for the entire rim or ring we have approximately

$$\sigma_r' = \frac{1}{2}\left(\sigma_3 + \frac{y_2}{y_3}\,\sigma_{r2}\right).$$

Further, the tangential expansion of the ring per unit length is

$$\epsilon' = \frac{1}{E}\left(\sigma_t' - \nu\, \sigma_r'\right) \quad \ldots \ldots \quad (28)$$

and the elongation of the radius is

$$\xi_2' = x_3\, \epsilon' \quad \ldots \ldots \ldots \quad (29)$$

which value can be used in place of equation 23. In like manner we can consider the hub, and apply the formulæ developed below for "discs of uniform thickness." In general σ_r' is small in comparison to σ_t', and in order to simplify the calculation, already sufficiently complicated, equations 23 and 24 may be used. For similar reasons it is recommended that we do not attempt to solve for the constants b_1 b_2 in the equations expressed in algebraic symbols, but rather substitute directly the given and assumed numerical values in equation 23.

SIMPLIFIED RIM CONDITIONS.

For thin discs, not heavily called upon for strength, the following approximate method may be used: first assume the hub to be so strongly designed that in formula 21, for $x = 0$, also $\xi = 0$; that is, that the disc behaves in such a manner that it can be considered as solid through to the center of the shaft. As ψ is negative,

$$b_2 = 0;$$

so that only the constant b_1 remains unknown in

$$\xi = a\,x^3 + b_1\,x^{\psi_1} \quad . \quad . \quad . \quad . \quad . \quad (30)$$

In order to determine this value, we make for the disc the further unfavorable supposition that the rigidity of the ring $y_3\ \delta_3$ be neglected, so that the value represented by σ'_{r2} in equation 27 becomes the actual radial stress. We have, therefore, according to equation 22,

$$\sigma_{r\,2} = \frac{E}{1-\nu^2}\left[(3+\nu)\,a\,x_2^2 + (\psi_1+\nu)\,b_1\,x_2^{\psi_1-1}\right] . \quad (31)$$

an (assumed) fixed value, with the help of which we calculate the dimension y_2 according to Formula 27,

$$y_2 = y_3\left[\frac{\sigma_3}{\sigma_{2\,r}} + \frac{\mu\,\delta_3\,\omega^2\,x_3}{\sigma_{2\,r}}\right] \quad . \quad . \quad . \quad . \quad . \quad (32)$$

The constant b_1 is calculated by solving Formula 31. If we use the abbreviated form

$$\sigma_a = \frac{(3+\nu)\,\mu\,\omega^2\,x_2^2}{8-(3+\nu)\,a}, \text{ that is, } a = \frac{-(1-\nu^2)\,\sigma_a}{E\,(3+\nu)x_2^2} \quad . \quad (33)$$

then,

$$b_1 = \frac{1 - \nu^2}{E\,(\psi_1 + \nu)} \cdot \frac{\sigma_{2r} + \sigma_a}{x_2^{\psi_1 - 1}} \quad . \quad . \quad . \quad . \quad . \quad (34)$$

and the stresses are

$$\sigma_r = -\,\sigma_a \left(\frac{x}{x_2}\right)^2 + \left(\sigma_{r2} + \sigma_a\right)\left(\frac{x}{x_2}\right)^{\psi_1 - 1} \quad . \quad . \quad . \quad . \quad (35)$$

$$\sigma_t = -\,\frac{1 + 3\,\nu}{3 + \nu}\,\sigma_a \left(\frac{x}{x_0}\right)^2 + \frac{1 + \psi_1\,\nu}{\psi_1 + \nu}\left(\sigma_{r2} + \sigma_a\right)\left(\frac{x}{x_2}\right)^{\psi_1 - 1} \quad . \quad . \quad (36)$$

Now we still have the hub cross-section $y_0\,\delta_0$ to determine, in order to satisfy the conditions in equation 26. Here the centrifugal force of the hub is neglected, the stress σ_0 due to forcing the hub on the shaft being taken into consideration by multiplying the stress σ_{r1} by a factor

$$\beta > 1,$$

so that with the assumed value of δ_0 (and consequently x_0), we have the following :

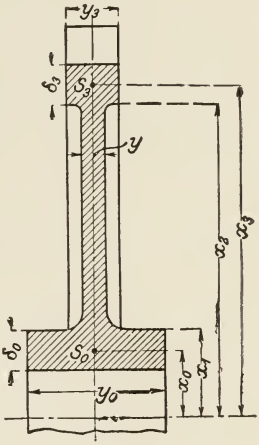

Fig. 103.

$$y_0 = \beta\,\frac{y_1\,x_0}{\delta_0}\left(\frac{\psi_1 + \nu}{1 - \nu^2}\right)\frac{\left(\sigma_{r2} + \sigma_a\right)\left(\frac{x_1}{x_2}\right)^{\psi_1 - 1} - \sigma_0\left(\frac{x_1}{x_2}\right)^2}{\left(\sigma_{r2} + \sigma_a\right)\left(\frac{x_1}{x_2}\right)^{\psi_1 - 1} - \frac{\psi_1 + \nu}{3 + \nu}\left(\frac{x_1}{x_2}\right)^2} \quad . \quad (37)$$

This value of y_0 is the minimum. If y_0 is increased beyond this value, the stresses on the disc become smaller. With strong discs, y_0 is impractically large according to this calculation; in such cases the diameter of the hub may be freely assumed, and the stresses calculated according to Formulæ 23 and 24.

For the *disc of uniform thickness*, Fig. 103, y = constant, the solution is greatly simplified, and we have

$$\xi = a\,x^3 + b_1 x + \frac{b_2}{x}$$

with

$$a = -\,\frac{\mu\,\omega^2\,(1 - \nu^2)}{8\,E} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (38)$$

The limiting conditions give as before for the expansion of the hub and of the outer ring, the values of ξ_1' and ξ_2' according to equations 23 and 24, while for the disc we have

$$\left.\begin{aligned}\xi_1 &= ax_1^3 + b_1x_1 + \frac{b_2}{x_1}\\ \xi_2 &= ax_2^3 + b_1x_2 + \frac{b_2}{x_2}\end{aligned}\right\} \quad . \; . \; . \; . \; . \; . \quad (39)$$

After determining the corresponding values of σ_{r1} and σ_{r2} from equation 22, we obtain the values of b_1 and b_2 from the relations

$$\xi_1 = \xi_1' \text{ and } \xi_2 = \xi_2'.$$

Using b_1 and b_2 we find ξ, and finally from equation 22, the stresses.

40. THE DISC OF UNIFORM STRENGTH.

The disc of uniform strength should fulfill the conditions, that the radial and tangential stresses throughout its mass are of constant value. Accordingly, in equation 8 we substitute

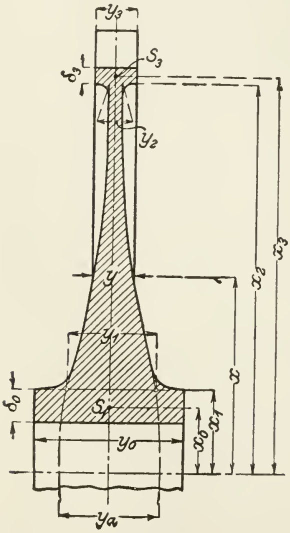

Fig. 104.

$$\sigma_r = \sigma_t = \sigma = \text{constant};$$

and get

$$\frac{dy}{dx} + \frac{\mu\omega^2}{\sigma}xy = 0 \quad . \quad (40)$$

and by integration,

$$y = y_a e^{\frac{-\mu\omega^2}{2\sigma}x^2} = y_a e^{\frac{-\mu w^2}{2\sigma}} \quad . \quad (41)$$

in which y_a is the thickness of the disc carried to the shaft center, and w the peripheral velocity at a distance x. These formulæ have been used for quite a time in the design of the Laval turbine.

The specific elongation is found to be equally large in all directions, and the linear expansion is

$$\xi = \frac{1-\nu}{E}\sigma x \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (42)$$

It is here to be noted, that a considerable error would be made by neglecting the contraction of cross-section, that is, by assuming $\nu = 0$, as may be seen from the limit conditions.

We can construct the disc either solid across or with a hub, (see Fig. 104). In the latter case, Formulæ 23 to 26 can be applied if $\sigma_{r1} = \sigma_{r2} = \sigma$. The conditions $\xi_1 = \xi_1'$ and $\xi_2 = \xi_2'$ are in full:

$$\frac{1-\nu}{E}\sigma x_1 = \frac{x_0^2}{E\delta_0 y_0}\left[\sigma_0 y_0 + \mu\omega^2\delta_0 y_0 x_0 + \sigma\frac{x_1 y_1}{x_0}\right] \quad . \quad (43)$$

$$\frac{1-\nu}{E}\sigma x_2 = \frac{x_3^2}{E\delta_3 y_3}\left[\sigma_3 y_3 + \mu\omega^2\delta_3 y_3 x_3 - \sigma\frac{x_2 y_2}{x_3}\right] \quad . \quad (44)$$

The strength of the rim $y_3\ \delta_3$ is again determined by constructive reasons, and equation 44 serves for the calculation of y_2. In order that this may not be negative, σ must not be assumed too large. We then find for the simple case, that $\sigma_3 = 0$ and approximately $x_3 = x_2$, the condition:

$$\sigma < \frac{\sigma_u}{1-\nu} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (45)$$

in which

$$\sigma_u = \mu\omega^2 x_2^2$$

representing that stress that a freely rotating ring of radius x_2 would develop. This condition will no doubt always be fulfilled.

Equation 41 serves for the calculation of y_1 from y_2 according to the formula

$$y_1 = y_2\, e^{+\frac{\mu\omega^2}{2\sigma}(x_2^2 - x_1^2)} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (46)$$

and finally, equation 43 serves for the calculation of y_0 from the values x_0 and δ_0, which are freely assumed.

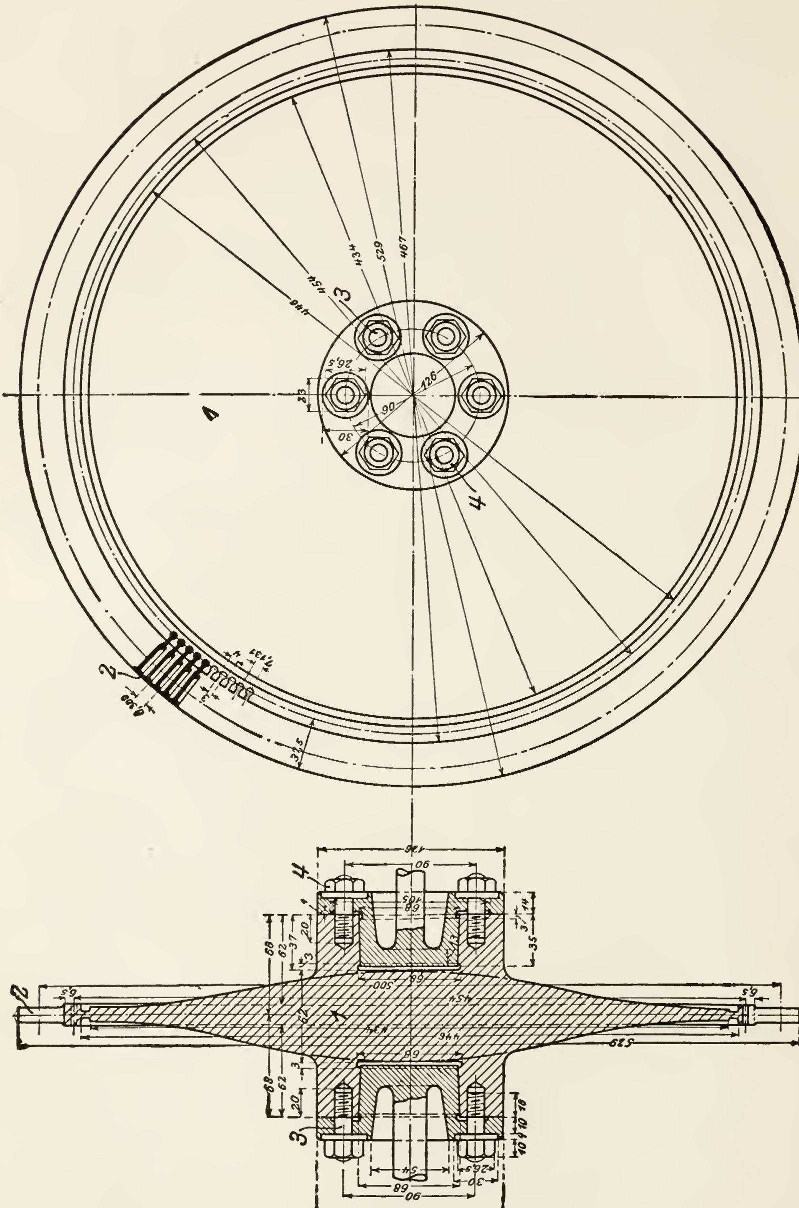

Fig. 105.
Dimensions are given in mm.

The construction of the solid disc, that is, without hub, is, according to *de Laval,* recommended wherever possible, as the taking into account of the hub influences in the calculation forms only

a rough approximation. In this case, only equations 44 and 41 need be taken into account to find the unknown quantities y_2 and y_a. For attaching the shaft, *de Laval* uses two strengthening-rings to decrease the stresses at this place, and to get rid of the weakness of the disc at this circumference. (See Fig. 105.) For this can be also used a device fitting closely under the blade-rim, patented by the Humbolt Machine Works.

Grübler has shown in the discussion of his formulæ* for discs of uniform thickness, that by even a very small hole at the center of the disc, the strains in the disc are double those in a solid disc. We must therefore be prepared to meet similar influences in discs of unequal thickness or with eccentrically-bored holes, and in this direction the utmost caution is recommended.†

41. GEOMETRICALLY SIMILAR DISC-WHEELS.

It is to be noted that in the above formulæ, all stresses depend only on the square of the peripheral velocity, and not on the absolute value of the radius. This is not alone characteristic of the formulæ already discussed, but on the other hand, is general, as can be proved from the following discussion.

Let us compare two geometrically similar disc-wheels of any given form (including blades, etc.), of which the second has k times as large linear dimensions as the first, and whose distortions at similar points on each wheel, due to the influence of centrifugal force, we may imagine to be proportional. From this assumption, the stresses in the same directions at similar points are equal.—The angular velocities of the rotating discs are ω and ω', respectively. We cut from these disc-bodies two homologously situated and geometrically similar elements. The corresponding element of the second disc has k^3 times as great a volume, therefore k^3 times as great a mass, as the element of the first disc. The distance of this element of the second disc from its axis is k times as great as, and therefore the total centrifugal force $k^4 \frac{\omega'^2}{\omega^2}$ times greater than, that of the first element. The surface forces give

* Zeitschr. d. Ver. deutsch. Ing., 1897, p. 860.

† See also the exceedingly interesting discussion by *Kirsch*, Z., 1897, p. 798.

only k^2 times as great resultants; and in order to preserve equilibrium of forces, it is necessary and sufficient that $\omega^2 = k^2 \omega'^2$; that is, that $\omega' = \frac{\omega}{k}$. But then, the velocity of the outmost circumference of the discs is equally great, and we may say: *The strains in geometrically similar discs of any given form are, with equal peripheral velocity, equally great at similarly situated points.*

Imagine a symmetrical disc divided into two equal parts by a plane of symmetry at right angles to its axis. The centrifugal forces of each half are obviously in equilibrium. Now remodel each half into a disc also symmetrical with respect to a plane at right angles to the axis. Then we can see that the axial dimensions of the wheel (and of course also the blades, etc.) can be proportionally increased or decreased at will, without any changes in the stresses, velocity remaining constant.

Let us now geometrically enlarge a wheel, so that the linear dimensions are doubled, and retain the original peripheral velocity. In similarly situated points there exist the same stresses. Divide the wheel by a plane of symmetry as before, and the same is then true for each half. But we can imagine these halves to have been so formed that all radial dimensions are doubled. These two latter discussions may be united in the following rule:

With constant peripheral velocity, both axial and radial dimensions of a wheel may be increased or decreased either separately or in any independent ratio, without in any way changing the specific stresses at similarly situated points.

42. STRESSES AND MATERIALS OF CONSTRUCTION.

If it is desired to ascertain the highest velocity which can be attained, the question must first be decided: what material can be used, and what stress may be permitted? According to Formula 41, it is sufficient to use ordinary wrought iron or wrought steel for velocities of 200 meters (656.17 ft.). For 300 meters (984.25 ft.) crucible cast steel can be used. When 400 meters (1 312.33 ft.) or over is reached, then new construction materials are necessary. In fact, at 1 500 kg. per sq. cm. (21 300 lb. per sq. in.) according to Formula 41, with $w = 400$ meters (1 312.33 ft.), the ratio $\frac{y_a}{y_2}$ is about

70; that is, if y_2 is assumed as only 5 mm. (0.2 in.) then $y_a = 350$ mm. (13.78 in.). But if we could take 2 500 kg. per sq. cm. (35 500 lb. per sq. in.), then is $\frac{y_a}{y_2}$ about 13, and hence practicable. Here we can take advantage of the properties of *nickel-steel.* The Krupp Co. of Essen recommends as suitable material for turbine-discs, a nickel-steel of about 90 kg. per sq. mm. (127 800 lb. per sq. in.) tensile strength, and 12% elongation, with 65 kg. per sq. mm. (92 300 lb. per sq. in.) as the elatics limit. Furthermore, Krupp states that there is also a nickel-steel of still higher strength, but correspondingly less elongation, and that with small forged pieces a tensile strength has been reached of over 200 kg. per sq. mm. (284 000 lb. per sq. in.) and an elastic limit of over 160 kg. per sq. mm. (227 200 lb. per sq. in.). The following figures were taken from Krupp's tables :

IN FRENCH UNITS.

Ultimate Tensile Strength. Kilogram per Square Millimeters.	Elongation, Per Cent.	Elastic Limit. Kilogram per Square Millimeters.	
180	7.0	96	Measured on rod, 12 millimeters diameter, and between points, 100 millimeters apart.
178	5.5	108	
177	6.0	148	
182	4.1	160	
149	6.8	132	
219	(?)*	150	

IN ENGLISH UNITS.

Ultimate Tensile Strength. Pounds per Square Inch.	Elongation, Per Cent.	Elastic Limit, Pounds per Square Inch.	
255 600	7.0	136 320	Measured on rod, 0.472 inch diameter, and between points, 3.937 in. apart.
252 760	5.5	153 360	
251 340	6.0	210 160	
258 440	4.1	227 200	
211 580	6.8	187 440	
310 980	(?)*	213 000	

* Elongation was not measured, because rod broke on center-punch mark near end.

Whether the application of such a hard nickel-steel is practicable for turbine discs, can only be determined by experiment and actual practice. Up to the present time discs have been furnished that show under test about 95 kg. per sq. mm. (134 900 lb. per sq. in.) ultimate tensile strength, and 14% elongation with an elastic limit of 73 kg. (103 630 lb. per sq. in.).

The value of the allowable working stress must evidently be left to the judgment of the designer. According to the opinion of Krupp, the stresses in one and the same direction may be permitted to run up to about one-third of the elastic limit, and eventually even higher.

Disc-wheels furnished by Krupp up to the present time fail to show any indications of internal stresses, nor have any flaws been found in the finished machine parts ; and it may be assumed that internal stresses do not exist.

The price of turned and finished disc-wheels of 1 000 to 3 000 mm. (39.37 in. to 118.11 in.) diameter is given by Krupp as, roughly, 350 to 270 marks per 1 000 kg. ($40 to $30 per 1 000 pounds). As, however, wheels of 2 000 mm. (78.75 in.) diameter and 1 000 kg. (2 200 pounds) are entirely practicable, the cost of this item in the construction of the turbine will prove no hindrance at all.

43. THE MASS-BALANCING OF ROTATING RIGID BODIES.

Next in importance to sufficient strength in the design and construction of turbine drums and wheels is the avoidance or vibrations. The importance of the danger of neglecting this fact may be seen by considering a Laval wheel of 760 mm. (30 in.) diameter and 420 meters (1 378 feet) peripheral velocity. An unbalanced weight of 0.1 kg. (0.22 lb.) at the rim of this wheel causes a centrifugal force in the disc of nearly 5 000 kg. (11 000 lb.). We must, therefore, by adding weights, try to bring about such a distribution of masses around the rotating axis that the centrifugal forces all balance each other. The determination of these additional masses we call *Mass-Balancing*.

If the axis of rotation is assumed as fixed, then it is necessary and sufficient to suppose the centers of gravity of all masses to lie in this axis, and the so-called centrifugal moment to disappear. It

is especially emphasized, because of the still frequent misunderstandings, that the first condition alone does not suffice, as is immediately clear from a reference to Fig. 106. The center of gravity of the two masses m, equally large and equally distant, falls in the axis; but their centrifugal forces, instead of balancing, constitute a moment and generate opposite forces in the bearings.

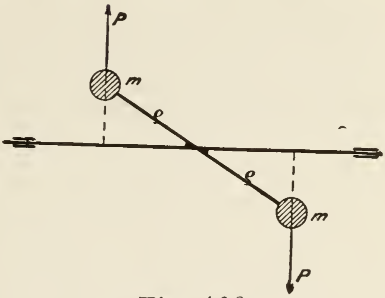

Fig. 106.

If the exact position of the forces creating unbalanced moments in rotating bodies is known, as for instance m_1 and m_2 in Fig. 107, we can arrive at complete balancing by bringing the additional masses into the planes E' and E''. The action of the mass m_1 at radius r_1 is neutralized by the additional masses m_1' and m_1'' at radii r_1', r_1'', provided r_1, r_1', r_1''

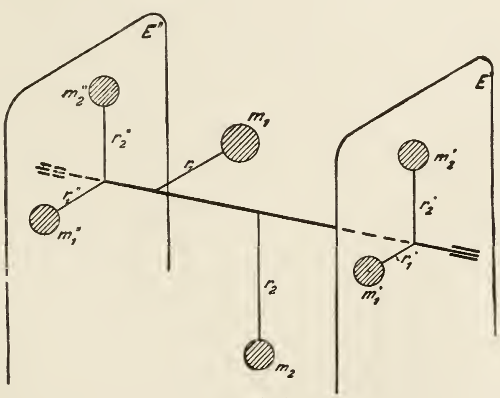

Fig. 107.

lie in the same plane and the centrifugal forces of m_1, m_1', m_1'' are in equilibrium. We must therefore have

$$\left.\begin{aligned} m_1 r_1 \omega^2 &= m_1' r_1' \omega^2 + m_1'' r_1'' \omega^2 \\ m_1' r_1' \omega^2 a_1' &= m_1'' r_1'' \omega^2 a_1'' \quad . \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad (1)$$

in which a_1', a_1'' are the respective distances of the planes E', E'' from m_1. These equations are especially well satisfied if we take

$$m_1 = m_1' + m_1'' \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

and determine the radii from

$$\left.\begin{array}{l} r_1 = r_1' + r_1'' \\ r_1' a_1' = r_1'' a_1'' \end{array}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

which expresses nothing else, than *that the additional masses, equal to* m_1 *at radii* r_1', r_1'' *have moments which balance the moments of* m_1 *at* r_1. In the same manner m_2 and any other masses creating further unbalanced moments should be treated. The individual masses m_1', m_2' . . . m_k' in E', and m_1'', m_2'' . . . in E'' are compensated for by an added single mass equal to the sum of the individual masses, located at their common center of gravity. This determination, according to the above method, is not possible without further investigation, because the location of the masses creating unbalanced moments is unknown; but this method deserves the consideration of the designer. It may then be stated: *Complete mass-balance can be obtained in the case of rotation about a fixed axis by the addition of two suitable masses in the two otherwise arbitrary planes at right angles to this axis.*

Theoretical mechanics aid us in the determination of the location and magnitude of the masses creating unbalanced moments, as for instance by pendulum experiments the moments of inertia may be found; and from these, by calculation, the so-called centrifugal moments. However, we may be fairly certain that because of the heavy machine parts that enter into this question, this method cannot be depended upon for sufficiently accurate results. We often try with drums, after their center of gravity has been brought into the axis of rotation in the ordinary way, by adding masses to determine the position of the unbalanced moments. This is accomplished by rotating the drums in bearings by means of vertically-led belts, the bearings being horizontally adjustable on rollers. The drum then vibrates about a vertical axis, and by chalking the places of greatest deviation the location of the unbalanced masses is found.

This method can also be used for a mathematical determination of the unbalanced moments.* The sensitiveness of this method was

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* Let the "balance-mass" m found in the ordinary way be brought to the circumference at radius r_0, so that its center of gravity lies in the shaft's axis.

During the rotation the vibrations about a vertical axis Z (Fig. 108) passing through the center of gravity S, follow the law that the loss of the moment of momentum (the "impulse moment") at any instant is equal to the moment of external forces. A certain point P has the velocity $r\omega$ due to rotation about axis o, and the velocity $\rho\epsilon$ due to rotation about axis Z, according to Fig. 108. The impulse moment for Z is, if $r\omega$ is resolved into two components, $y\omega$ and $-z\omega$

$$\Theta = \Sigma\,\delta m\,\rho^2\epsilon - \Sigma\,\delta m\,\omega\,x\,z \quad . \quad (4)$$

The so-called "centrifugal moment" $\Sigma\,\delta m\,x\,z$ is a maximum for a certain position in the coördinate plane $X\,O\,Y$, and disappears for a position at right angles thereto. To prove this, take a second position in the coördinate system $X'\,Y'\,Z'$, in which the X and X axes coincide. We have according to the notation in the figure,

$$z = y'\sin\phi + z'\cos\phi \quad . \quad . \quad (5)$$

and

$$K = \Sigma\,\delta m\,x\,z$$
$$= \sin\phi\,\Sigma\,\delta m\,x\,y' + \cos\phi\,\Sigma\,\delta m\,x\,z',$$

or simplified,

$$K = A\sin\phi + B\cos\phi \quad . \quad . \quad (6)$$

If $\quad A = R\cos\alpha;\; B = R\sin\alpha,$

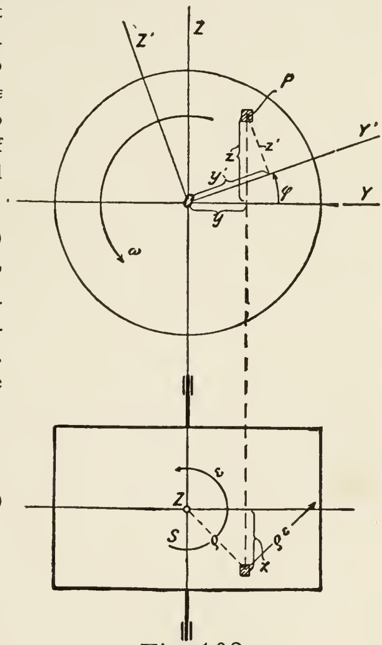

Fig. 108.

then is

$$K = R\sin(\phi + \alpha)\,.\;.\;.\;.\;.\;.\;.\;.\;.\;.\;. \quad (7)$$

If we consider $Y'\,Z'$ as fixed and $Y\,Z$ as movable, then $K = R$, and is a maximum when

$$\phi = \phi_0 = \frac{\pi}{2} - \alpha$$

If $\phi = \frac{\pi}{2} + \phi_0$, then $K = 0$. We take ϕ_0 as the angle of the $Y'\,X'$ plane, and $\frac{\pi}{2} + \phi_0$ as the angle of the $Z'\,X'$ plane, so that

$$\Sigma\,\delta m\,x\,y' = 0, \qquad \Sigma\,\delta m\,x\,z' = R\;.\;.\;.\;.\;.\;.\;.\;. \quad (8)$$

Now imagine the system $X'\,Y'\,Z'$ rotating with the bodies, and we have for the stationary system $X\,Y\,Z$

$$\Sigma\,\delta m\,x\,z = R\cos\phi\;.\;.\;.\;.\;.\;.\;.\;.\;.\;. \quad (9)$$

in which R is unknown. If

$$\Sigma\,\delta m\,\rho^2 = J \text{ and } \phi = \omega t\;.\;.\;.\;.\;.\;.\;.\;.\;. \quad (10)$$

equation 4 becomes

$$\Theta = J\epsilon - R\,\omega\cos\omega t\;.\;.\;.\;.\;.\;.\;.\;.\;.\;.\;. \quad (11)$$

The moment of the external forces is 0, hence the derivative $\frac{d\theta}{dt}$ disappears. As

increased by placing springs horizontally, so that they acted on the bearings. If the number of revolutions attained the value for the vibrations of the system, consisting of drum and springs, then "resonance" occurred; that is the vibrations were increased to a certain degree by the friction of the atmosphere, etc., and very

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the unbalanced mass-moments are small, ω can, as a first approximation, be considered constant, and we have

$$J\frac{d\epsilon}{dt} = -R\,\omega^2 \sin \omega t \quad . \quad . \quad . \quad . \quad (12)$$

From this

$$J\epsilon = R\,\omega \cos \omega t \quad . \quad . \quad . \quad . \quad (13)$$

and with $\frac{d\psi}{dt} = \epsilon$ we get for the purely periodical angle ψ for the horizontal vibrations of the body

$$J\psi = R \sin \omega t \quad . \quad . \quad . \quad . \quad (14)$$

When $\omega t = \frac{\pi}{2}$, then ψ reaches the positive maximum value ψ_0, while the plane R has turned through the angle π. R and ψ_0 are opposite to each other if we regard them as vectors.

If the amplitude ψ_0 is determined experimentally, then can also be found the unknown

$$R = J\psi_0 \quad . \quad . \quad . \quad . \quad (15)$$

For J may be placed, under the assumption of homogeneous mass distribution, the calculated (or experimentally determined) polar moment of inertia of the masses.

The moment R should be brought equal to 0, by the added masses m_1 and m_2, for which purpose it is necessary that their center of gravity should coincide with S; that is, with the notation in Fig. 109

$$\left.\begin{aligned} m_1\rho_1 &= m_2\rho_2 \\ m_1x_1z_1' + m_2x_2z_2' &= -R \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad (16)$$

from which with assumed ρ_1, ρ_2 and α, we can determine the masses $m_1 m_2$. The planes in which $m_1 m_2$ are to lie are determined by chalking while rotating, and $m_1 m_2$ so placed that they act against each other in the sense of R. According to equation 14, ψ reaches its maximum when the "heavy" place of the drum lies diametrically opposite on the other side; that is, the chalk-mark appears on the "light side." *Hodgkinson* says he observed that the chalk mark plane deviates 90° from the plane of R. With sufficiently small friction, this is not possible from the above discussion.

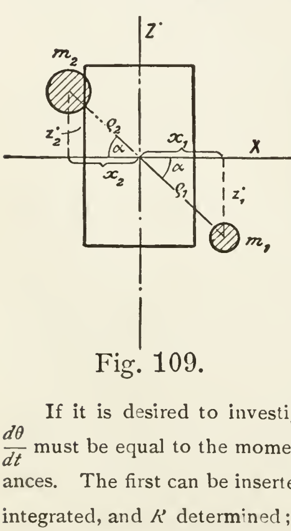


Fig. 109.

The original balance-mass m can be resolved in the normal planes into m_1 and m_2 and the values m' and m'' can be united with m_1 and m_2 respectively, according to the law locating their center of gravity (see above).

If it is desired to investigate the action of the springs above mentioned, then $\frac{d\theta}{dt}$ must be equal to the moments of the springs, plus the moments of the air-resistances. The first can be inserted in the calculation as $k\,\psi$, the second as $k'\left(\frac{d\psi}{dt}\right)$, then integrated, and R determined; k' can be determined by pendulum experiments, provided the entire process does not fail on account of its impracticability.

small errors of masses were sufficient to create visible deflections. The influence of these springs can be easily taken into account in the calculation given below. After all, practice requires that we follow *the path pointed out by experiments.*

It might be useful to bear in mind in this experimental procedure that the action of an unbalanced centrifugal force must be compensated for by a moment. We may therefore imagine the unbalanced masses to be situated as shown in Fig. 110, in which are given the unbalanced centrifugal force m_0 and the moments of the equal masses m. The plane of m can also coincide with that of m_0. First we determine in the ordinary way the counterweight of m_0, and replace this, for instance, with the masses m_0', m_0'' in the end planes of the drum. We must now experiment in order

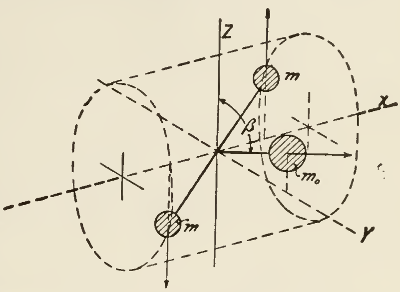


Fig. 110.

to neutralize the moment of m, with two other equally large masses m', which are also placed in the end planes of the drum, concentrically and symmetrically with the center of gravity, in which we vary the magnitude and position of m'. If it is proved experimentally by allowing the drum to rotate, that the balancing was correctly done, then we can combine into one mass m_0' and m' on one side, and m_0'' and m'' on the other side, each pair according to their common center of gravity. It is recommended not to fasten these masses permanently, but to so attach them that each is adjustable concentrically, so that they can be placed in a new position if necessary on account of the change in the center of gravity frequently caused by subsequent vibrations, as for instance, those due to the fields of a dynamo.

44. THE FLEXIBLE SHAFT OF LAVAL.

If the balancing of masses as explained above could be accurately determined mathematically, then we still would not get true running of the shaft. As the shaft is not rigid, it is bent by the opposing centrifugal forces of the unbalanced mass moments and balancing masses, and can therefore run considerably out of true. Actually, the balancing is never complete, and there

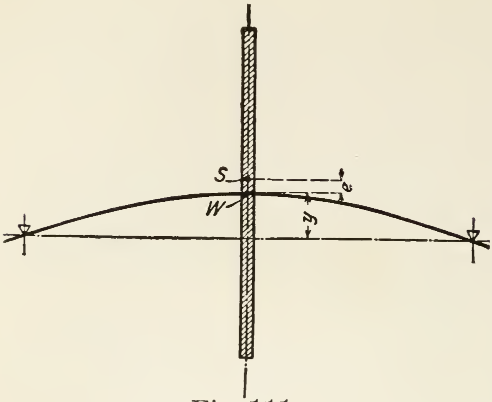


Fig. 111.

remains an unbalanced centrifugal force, whose action *de Laval* tried to render harmless, by placing the turbine shaft in well separated bearings, thus permitting a large degree of flexibility. In this way the wheel can rotate at high velocities about its center of gravity on a "free" rotating axis, and there occurs the phenomenon of *critical velocity* already recognized by *de Laval*, but which was first scientifically explained by *Rankine*, *Reynolds** and *Föppl*.†

Let us imagine an otherwise symmetrical disc with center of gravity lying eccentrically at e, Fig. 111, set into comparatively slow rotation. The shaft will be sprung by centrifugal force the distance y (measured from the position occupied by the shaft when bent only by its own weight) which is calculated for the case of relative equilibrium from the condition that the centrifugal force $m(y+e)\omega^2$, in which m is the mass of the disc (weight of shaft being neglected), and must be equal to the produced elastic opposing force of the shaft. We may place this opposing force proportional to the deflection. If, then, α is a constant depending upon a relation of shaft-lengths, styles of bearings, etc., the elastic resisting (opposing) force is

$$P = \alpha y \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

and the conditions of equilibrium demand

$$m(y+e)\omega^2 = P = \alpha y \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

from which we get the deflection

$$y = \frac{m\omega^2 e}{\alpha - m\omega^2}$$

If we increase the angular velocity, y increases and becomes infinitely large when $\alpha - m\omega^2 = 0$, or

* See Philos. Trans. of the Royal Soc., London, Vol. 185, year 1895, page 281.
† Civil-Ingenieur, 1895, p. 333.

$$\omega = \omega_k = \sqrt{\frac{a}{m}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

that is, the centrifugal force would bend the shaft until it breaks (or to the limits of the hub's strength). This value of ω_k we shall call the "*critical*" *angular velocity*, and shall refer also to the *critical number of revolutions.* If we calculate in French units we shall use the *cm. kg. sec.* system; in English units, the *ft. lb. sec.* system. Then α, according to equation 1, stands for the force in lb. (kg.) which would cause the shaft to bend 1 ft. (1 cm.). Let

$G = mg$ be the weight of the wheel,

the number of revolutions $n = \frac{30\,\omega}{\pi}$. Taking $g = 32.16$ ft. (981 cm.), then according to the formula of *Föppl,*

in the French units, $$n = 300\sqrt{\frac{a}{G}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3a)$$

or in the English units, $$n = 54.14\sqrt{\frac{a}{G}}.$$

For instance, in the case of a shaft supported in two bearings distant $2l$ apart, and loaded in the middle with a disc

$$y = \frac{1}{6}\,\frac{Pl^3}{JE} \text{ and } \alpha = \frac{6\,JE}{l^3};$$

for the shaft fixed at both ends, under similar conditions,

$$y = \frac{1}{24}\,\frac{Pl^3}{JE}; \qquad \alpha = \frac{24\,JE}{l^3}$$

We can only increase the number of revolutions beyond the "critical" value when bearings are provided that prevent the excessive bending of the shaft while it is passing through the critical speed.*

Theory and practice agree that now a new condition of stable equi-

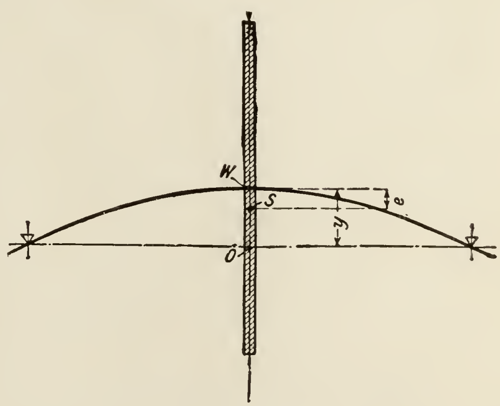


Fig. 112.

* Or when the velocity increases so rapidly that the disc "has no time" to deviate from the axis.

librium occurs, at which the loaded point W of the shaft exchanges its position with the center of gravity S, as is shown in Fig. 112. The deflection is found from the formula

$$m\,(y - e)\,\omega^2 = a\,y,$$

and we get

$$y = \frac{m\,\omega^2\,e}{-a + m\,\omega^2} = \frac{e}{1 - \frac{a}{m\,\omega^2}}.$$

The more we increase ω the smaller y becomes, and with infinitely high rotation, $y = e$. If the critical velocity ω_k is introduced, then

$$y = \frac{e}{1 - \frac{\omega_k^2}{\omega^2}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

The amount of the still-existing centrifugal force which is transmitted to the bearings, is

$$P = a\,y = \frac{m\,e\,\omega^2}{\frac{\omega^2}{\omega_k^2} - 1} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

By a suitable choice of $\frac{\omega}{\omega_k}$, that is, with given ω and by decreasing ω_k it is possible to decrease P at will without taking into account the eccentricity e, which in practice must obviously be made as small as possible. *De Laval* thus gives his turbine shafts a flexibility so that ω reaches a value of seven times ω_k, and no doubt the good running of the *Laval* turbines is due to this excellent idea of the inventor.

Föppl, in his theoretical investigations under simplified assumptions, proved that conditions represented in Fig. 112 give not only a *probable* but a *stable* equilibrium. The general proof follows below.

45. DEVIATION OF ROTATING DISC, DUE TO SHAFT BENDING.

The deviation of a rotating disc due to the bending of the shaft has a noticeable effect on the critical velocity. As an example, take an overhanging disc at the end of a shaft of length l and moment

of inertia J (see Fig. 113). The centrifugal forces give with an angle of deviation τ of the disc an effective moment $\Theta\,\omega^2\,\tau$ in a clock-wise direction, in which Θ is the mass moment of inertia of the disc referred to an axis projected through S at right angles to the elastic curve. Under the influences of this moment and the centrifugal force $m\,(y+e)\,\omega^2$, we get the deflection

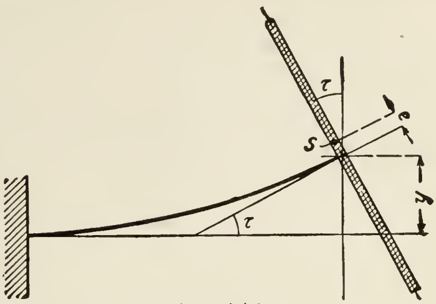


Fig. 113.

$$\left.\begin{aligned} y &= \frac{m\,e\,\omega^2 l^3}{3JE}\,\frac{Z}{N} \\ \text{with}\qquad Z &= 1 - \frac{3}{4\left(1 + \dfrac{JE}{\Theta\,\omega^2 l}\right)} \\ N &= 1 - \frac{m\,\omega^2 l^3}{3JE}\,Z \end{aligned}\right\} \quad . \; . \; . \; . \; . \; . \quad (6)$$

This deflection increases beyond all bounds when $N = 0$, that is, when ω reaches the value calculated from the equation

$$m\,\omega_k^{\,2} = \frac{3JE}{l^3 Z} \quad . \; . \; . \; . \; . \; . \; . \; . \quad (7)$$

or, completely,

$$\frac{1}{12}\,\frac{m\,\Theta\,l^4}{J^2E^2}\,\omega_k^{\,4} + \frac{l}{JE}\left(\frac{m\,l^2}{3} - \Theta\right)\omega_k^{\,2} - 1 = 0 \quad . \; . \quad (8)$$

Abbreviating this equation, we have

$$A\,\omega_k^{\,4} + 2\,B\,\omega_k^{\,2} - 1 = 0 \quad . \; . \; . \; . \; . \; . \quad (9)$$

from which for our problem, the useful solution gives a constant finite positive value,

$$\omega_k^{\,2} = \frac{-B + \sqrt{B^2 + A}}{A} \quad . \; . \; . \; . \; . \; . \quad (10)$$

*Dunkerley** had already derived equation 8, but did not interpret it quite correctly.

* Philos. Transact. of Royal Soc., Vol. 185, year 1895, p. 305.

If $\Theta = 0$, we get $$\omega_k^2 = \frac{3JE}{ml^3} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (11)$$

If $\Theta = \infty$, we have $$\omega_k'^2 = \frac{12JE}{ml^3} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (12)$$

For a fair value of Θ we can approximately place ω_k^2 of equation 11 in the denominator of equation 7, and get

$$\omega_k'^2 = \frac{\omega_k^2}{1 - \dfrac{3}{4\left(1 + \dfrac{ml^2}{3\Theta}\right)}} \quad . \quad . \quad . \quad . \quad . \quad . \qquad (13)$$

The deviation of the disc for this particular case therefore causes an *increase* of the critical velocity. If the disc has been mounted out of true at the beginning, it would not change Formula 7, as can easily be seen by recalculation.

46. CRITICAL VELOCITY, AND THE PERIOD OF OSCILLATION DUE TO ELASTICITY.

Suppose we give an impulse to the disc at right angles to its position of rest. (See Fig. 111.) This will set it vibrating according to the formula

$$m\frac{d^2y}{dt^2} = -P = -ay$$

(counting y from the position of equilibrium, which is equivalent to the deflection due to its own weight). From this equation is readily deduced the time required for a complete (to and fro) vibration.

$$T = 2\pi\sqrt{\frac{m}{a}};$$

and the *number of vibrations per second*

$$n = \frac{1}{T} = \frac{1}{2\pi}\sqrt{\frac{a}{m}};$$

but the *critical number of revolutions of the shaft per second* is

$$n' = \frac{\omega_k}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{a}{m}},$$

that is, n' and n *are identical.*

Dunkerley first pointed out this relation. This is not true, however, when the moment of inertia Θ of the disc enters into the question, due to the latter being obliquely mounted.

With the modern many-stage turbine we must above all consider the case of a continuous shaft carrying a number of wheels, whose centers of gravity will in general be all shifted out of the center line of the shaft and through their unbalanced centrifugal forces create phenomena analogous to the case of the single disc.

THE CRITICAL ANGULAR VELOCITY OF A MULTIPLE-LOADED SHAFT.

47. TWO SINGLE WHEELS.

Fig. 96 shows the condition of equilibrium corresponding to the angular velocity ω. In the coördinate system XYZ let O_1, O_2 be the points of application of the bending forces on the geometric axis of rotation through the two bearings; x_1, y_1, the coördinates of the middle point of the one disc; and x_2, y_2, those of the other.

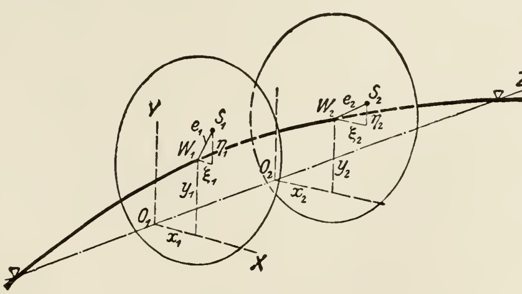


Fig. 114.

Let the coördinates of the centers of gravity S_1 and S_2 referred to the axes ξ and η parallel to the original XY axes, with their origins at $x_1 y_1$ and $x_2 y_2$ be ξ_1, η_1 and ξ_2, η_2; therefore e_1, e_2 are their "eccentricities." The torsional deformation compared to the bending is always so small that a change of the angles formed by e_1 and e_2 may be neglected. The centrifugal forces derived from the disc-masses m_1 and m_2 may be resolved into the components

$$\left.\begin{aligned} X_1 &= (x_1 + \xi_1) m_1 \omega^2, \qquad Y_1 = (y_1 + \eta_1) m_1 \omega^2 \\ X_2 &= (x_2 + \xi_2) m_2 \omega^2, \qquad Y_2 = (y_2 + \eta_2) m_2 \omega^2 \end{aligned}\right\} \quad . \quad . \quad (1)$$

Under their influences the shaft is deflected, and we may calculate constants a from the shaft dimensions and determine the type of support from

$$\left.\begin{aligned} x_1 &= a_{11} X_1 + a_{12} X_2, \qquad & y_1 &= a_{11} Y_1 + a_{12} Y_2 \\ x_2 &= a_{21} X_1 + a_{22} X_2, \qquad & y_2 &= a_{21} Y_1 + a_{22} Y_2 \end{aligned}\right\} \quad . \quad . \quad (2)$$

in which $a_{12} = a_{21}$. If we insert the expressions for the force-components, we have the equations:

$$(a_{11} m_1 \omega^2 - 1) x_1 + a_{12} m_2 \omega^2 x_2 + a_{11} \xi_1 m_1 \omega^2 + a_{12} \xi_2 m_2 \omega^2 = 0$$
$$a_{21} m_1 \omega^2 x_1 + (a_{22} m_2 \omega^2 - 1) x_2 + a_{21} \xi_1 m_1 \omega^2 + a_{22} \xi_2 m_2 \omega^2 = 0$$
$$(a_{11} m_1 \omega^2 - 1) y_1 + a_{12} m_2 \omega^2 y_2 + a_{11} \eta_1 m_1 \omega^2 + a_{12} \eta_2 m_2 \omega^2 = 0$$
$$a_{21} m_1 \omega^2 y_1 + (a_{22} m_2 \omega^2 - 1) y_2 + a_{21} \eta_1 m_1 \omega^2 + a_{22} \eta_2 m_2 \omega^2 = 0$$

The values $x_1\, x_2\, y_1\, y_2$ determined from the above increase to infinity when the following determinants vanish:

$$D = \begin{vmatrix} (a_{11} m_1 \omega^2 - 1), & a_{12} m_2 \omega^2 \\ a_{21} m_1 \omega^2, & (a_{22} m_2 \omega^2 - 1) \end{vmatrix}$$

The critical velocity ω_k can therefore be calculated from the equation,

$$D = (a_{11} m_1 \omega_k^2 - 1)(a_{22} m_2 \omega_k^2 - 1) - a_{12}^2 m_1 m_2 \omega_k^4 = 0$$

In the case of equal masses $m_1 = m_2 = m$ arranged symmetrically (also equal strength of shaft and bearing), we have $a_{11} = a_{22} = \alpha$, $a_{12} = \beta$; and

$$\alpha m \omega_k^2 - 1 = \pm \beta m \omega_k^2,$$

from which two values of the critical velocities result

$$\left.\begin{aligned} m \omega_{k_1}^2 &= \frac{1}{\alpha - \beta} \\ m \omega_{k_2}^2 &= \frac{1}{\alpha + \beta} \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

These correspond, for instance, to a position of the center of gravity on one or on several sides of the geometric axis that originally lay in one plane.

The investigation of the results of the arrangement of three masses becomes greatly complicated.

48. GRAPHIC DETERMINATION WITH ANY DIVISION OF MASSES AND WITH ANY VARIABLE STRENGTH OF SHAFT.

The graphic solution of this general problem can only be approximately accomplished. Such a method was used by *Vianello* to solve problems investigating bending stresses, and *Delaporte* describes a similar process, in "Reveu de mécanique," 1903, Vol. 12, p. 517. We shall assume the fundamental law of *Mohr* for determining the elastic curve of deflected beams, and suggest a somewhat unusual method of solution.

A shaft with any arrangement of supports is loaded with the forces P_1P_2 . . . at right angles to the beam-axis; which cause the deflections y_1, y_2 . . . at their points of application. If all the forces P were brought to k-times their value, then the deflections also increase k-times. The shaft carries a number of masses whose centers of gravity each fall in the center line of the shaft; let the forces P be the centrifugal forces produced by the masses when the shaft rotates, and cause the deflections y_1, y_2 . . . So long as the angular velocity ω is small, the centrifugal forces are not enough to deflect the shaft; and not until the critical number of revolutions has been reached does equilibrium exist between the centrifugal forces and the forces of elasticity. If this is the case for one group of $y_1, y_2,$ . . . deflections, then it will also occur for that of k-times as much, since with the increase of y, P also increases in equal ratio. In other words, at the critical number of revolutions, the shaft for each deflection would be in neutral equilibrium.

We now construct a trial elastic curve depending upon the dimensions of the given shaft, and calculate the centrifugal forces $P_1, P_2,$ . . . from the deflections $y_1, y_2,$ . . . with a likewise arbitrary angular velocity ω. From these forces, the bending moment area may be found, and according to *Mohr* the "*first*" *true* elastic curve corresponding to the forces P and their ordinates, which we shall call $y_1', y_2',$ . . . can be obtained. The deflection at, say, the middle of the shaft y_m' can be distinguished from the initially assumed value y_m; for instance, it may be smaller; but can be made equal to this value if we, instead of ω, use the higher velocity

$$\omega' = \omega\sqrt{\frac{y_m}{y_m'}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

because thereby all forms are increased in the ratio $\frac{y_m}{y_m'}$. If all ordinates are increased in the ratio $\frac{\omega'^2}{\omega^2}$, then the derived "corrected" elastic curve must coincide with the assumed one, if we had guessed correctly the first time. In this case ω' would be the critical velocity. Actually, the curves would not agree, and we must repeat this method by allowing the "corrected" elastic curve to be the *second assumed* one. Its coördinates we shall call y_1^*, y_2^* . . . The construction of the second true elastic curve gives, according to *Mohr*, as deflection at the same point as before, y_m''; which differs from y_m^*, and we must choose again a new $\omega = \omega''$ according to the equation

$$\omega'' = \omega' \sqrt{\frac{y_m^*}{y_m''}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

and the ordinates are increased in the ratio $\frac{\omega''^2}{\omega'^2}$. If the so-corrected elastic curve agrees with the "second assumption," then ω'' is the critical velocity; in any other case the method must be again repeated.

49. SHAFTS OF UNIFORM DIAMETER UNDER CONSTANT AND UNIFORM LOAD, MATHEMATICALLY CONSIDERED.

Let the shaft be uniformly loaded over its entire length by closely-placed disc-wheels (Fig. 115), which shall not affect the flexibility of the shaft. The mass of the disc per unit length is m_1; the constant moment of inertia of the cross-section of the shaft is J. In order to perform the calculation in its simplest form we shall assume that the centers of gravity of all the discs lie in one and the same axial plane, and are deflected in the same direction through the constant value e away from the shaft center. The weight of the shaft is included in the weights of the discs.

When equilibrium occurs at the velocity ω, then a shaft-element of length dx, *if we at first neglect the obliquity of the disc*, is subject to the action of the centrifugal force $m_1 (y + e) dx \omega^2$ (as the supplementary forces of the relative motion), and to the bending moments M' and M as well as to the forces due to gravity S' and S.

If we call the centrifugal force pdx where p is the "load" per unit length, we get by neutralizing the vertical force components,

$$S' - S + pdx = 0, \text{ or } \frac{dS}{dx} = -p \quad . \quad . \quad . \quad . \quad (1)$$

and when the moment of the center of gravity disappears,

$$M' - M - S'\frac{dx}{2} - S\frac{dx}{2} = 0, \text{ or } \frac{dM}{dx} = S \quad . \quad . \quad . \quad (2)$$

We shall apply to these equations the well known fundamental formulæ of deflection, which for the inserted direction of the coör-

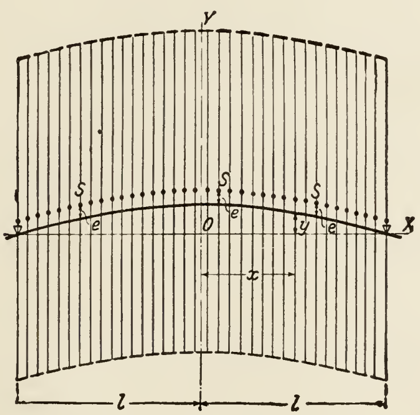


Fig. 115.

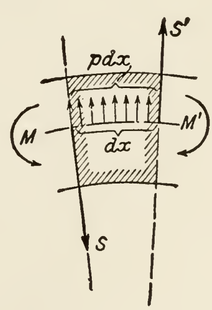


Fig. 116.

dinates in Fig. 115, M' being assumed in a positive direction, gives the following:

$$JE\frac{d^2 y}{dx^2} = -M \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

From this we have

$$JE\frac{d^4 y}{dx^4} = p = m_1\,\omega^2\,(y + e) \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

The general integration of this equation gives

$$y = a e_0^{kx} + a' e_0^{-kx} + b \cos kx + b' \sin kx - e \quad . \quad . \quad (5)$$

in which

$$k = \pm \sqrt[4]{\frac{m_1 \omega^2}{JE}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

e_0 being the base of the natural system of logarithms (in distinction from e), and the constants a, a', b, b' must conform to the conditions of the problem which might come up in the following special cases.

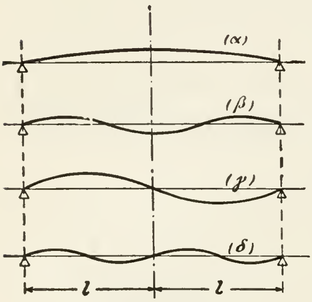


Fig. 117.

a. — A SHAFT SUPPORTED AT BOTH ENDS will either deflect as shown in α or β (Fig. 117), the elastic curve remaining symmetrical with respect to a vertical center line; or so that, as shown in γ, δ, the elastic curve is symmetrical with respect to the center of the line joining the two bearing points, in which e is assumed as negligibly small.

In the first case, y in Formula 5, if we take the abscissa as being measured from the shaft's center line, must be an *even* function, and in the second case an *uneven* function of x. In both cases there enters as a further condition the stipulation, that for $x = l$, $y = 0$, the deflection is equal to 0; that is, $\frac{d^2y}{dx^2} = 0$. Consequently, for the even function $a' = a$, $b' = 0$, and

$$a = \frac{e}{2\,(e_0^{kl} + e^{-kl})}, \qquad b = \frac{e}{2 \cos kl},$$

therefore the deflection is infinite when $\cos kl = 0$, or

$$kl = \frac{\pi}{2} \ldots \frac{3\pi}{2} \ldots \frac{5\pi}{2} \ldots$$

For the uneven function we get

$$a' = -a, \qquad b = 0, \qquad a = \frac{e}{2\,(e_0^{kl} - e^{-kl})}, \qquad b' = \frac{e}{2 \sin kl}$$

therefore, again, a critical number of revolutions when

$$k\,l = \frac{2\pi}{2},\ \frac{4\pi}{2},\ \frac{6\pi}{2},\ \ldots$$

There are, therefore, an infinite number of critical values $k\,l$, which bear the ratio 1 : 2 : 3 : 4 . . . to each other. As now ω is proportional to k^2 according to Equation 6, it follows that the critical velocities have the ratio

$$1 : 2^2 : 3^2 : 4^2 \ldots$$

In particular we find for the lowest value of the critical velocity, when

$$k\,l = \frac{\pi}{2}$$

$$\omega_k = \sqrt{\frac{\pi^4}{16} \cdot \frac{JE}{m_1 l^4}} = 3.489 \sqrt{\frac{JE}{M l^3}} \quad . \quad . \quad . \quad . \quad (7)$$

in which M is the sum of the masses of all the discs and the shaft.* Then we have for the shaft-radius corresponding to the critical velocity ω_k

$$r = 0.5686 \sqrt[4]{\frac{M\,l^3\,\omega_k^{\,2}}{E}} \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

OBLIQUITY OF DISCS.

Taking into consideration the obliquity of discs we have for the conditions of equilibrium of the forces acting upon a shaft-element, Fig. 118 :

$$M' - M + \theta_1\,dx\,\omega^2 \frac{dy}{dx} - S\,dx = 0, \text{ or } \frac{dM}{dx} = S - \theta_1\,\omega^2 \frac{dy}{dx} \quad . \quad (9)$$

θ_1 is the moment of inertia of the combined masses of the discs *per unit length of shaft* referred to an axis through S.

---

* *Mr. Wissler*, chief engineer with Sautter, Harlé & Cie., in Paris, informs me that he also derived similar formulæ for the use of his department. It was not until after the appearance of the first edition of this work that it was known that *Dunkerley* had already brought out the above solution in 1895. On the other hand, *Dunkerley* did not consider a special problem as given in Article 76.

With $\frac{dS}{dx} = -p$, the differential equation of the shaft deflection takes the form

$$J E \frac{d^4 y}{dx^4} - \Theta_1 \omega^2 \frac{d^2 y}{dx^2} = m_1 \omega^2 y \quad . \quad . \quad . \quad . \quad (10)$$

if we have here assumed $e = 0$, and determined the critical number of revolutions in the simplest manner from the conditions that the *centrifugal and elastic forces are in neutral equilibrium.*

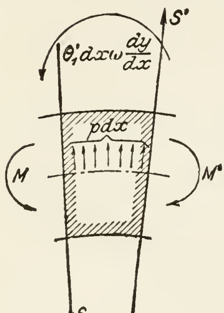


Fig. 118.

For the shaft supported at both ends, of length $2l$, we get for the calculation of k, by means of $y = a \cos kx$, the equation

$$J E k^4 + \Theta_1 \omega^2 k^2 - m_1 \omega^2 = 0 \; ; \quad . \quad (11)$$

on the other hand, in the case of (α), we have on account of rim conditions, for the deflection

$$k l = \frac{\pi}{2}$$

which value is substituted in equation 11, and gives for the critical velocity,

$$\omega_k^2 = \frac{J E \pi^4}{16 m_1 l^4} \frac{1}{\left(1 - \frac{\pi^2 \Theta_1}{4 m_1 l^2}\right)} \quad . \quad . \quad . \quad . \quad . \quad (12)$$

The critical value, therefore, in this case, is considerably increased by the obliquity of the discs.

β. — FOR A SHAFT FIXED AT BOTH ENDS *of length* $2l$ we have a possibility of deflection which is symmetrical with respect to both the vertical center-line, and to the center of the line joining the two bearing points. The further limiting conditions are $y = 0$ and $\frac{dy}{dx} = 0$ for $x = l$. We get for y as an even function the occurrence of a critical velocity, when

$$\tan (k l) = - \tan h (k l) \quad . \quad . \quad . \quad . \quad . \quad (13)$$

in which *tan h* is the so-called hyperbolic tangent, for whose values the "Hütte" (the Engineer's Pocketbook) gives complete tables. The solution gives as roots,

$$k\,l = \frac{3}{4}\pi, \quad \frac{7}{4}\pi, \quad \frac{11}{4}\pi, \cdots$$

If y is an uneven function, it follows that

$$\tan\,(k\,l) = +\tan h\,(kl) \quad . \quad . \quad . \quad . \quad . \quad . \qquad (14)$$

with the roots

$$k\,l = \frac{5}{4}\pi, \quad \frac{9}{4}\pi, \quad \frac{13}{4}\pi, \cdots$$

The critical numbers of revolutions are to each other as

$$3^2 : 5^2 : 7^2 : 9^2 : \ldots = 1 : 2.8 : 5.4 : 9 : \ldots$$

and the least value of the critical number of revolutions is

$$\omega_k \sqrt{\left(\frac{3\,\pi}{4}\right)^4 \frac{J\,E}{m_1 l^4}} = 7.851\,\sqrt{\frac{J\,E}{M\,l^3}} \quad . \quad . \quad . \quad . \qquad (15)$$

from which the shaft's radius is

$$r = 0.3791\,\sqrt[4]{\frac{M\,l^3\,\omega_k^{\,2}}{E}} \quad . \quad . \quad . \quad . \quad . \quad . \qquad (16)$$

If we assume that the shaft in the bearing always forms a small angle with the geometric axis, that is, would have to *describe a cone* during rotation, the calculation will give, surprisingly, the same velocities as for shafts horizontally mounted. The same takes place when the shaft has been mounted from the beginning in an oblique (fixed) bearing.

γ. — AN OVERHANGING SHAFT FIXED HORIZONTALLY AT ONE END gives according to the coördinate system shown in Fig. 119, the conditions $y = 0$ and $\frac{dy}{dx} = 0$, $x = 0$; further, for $x = l$ the bending moment and shearing stress $= 0$; that is, $\frac{d^2y}{dx^2} = 0$, and $\frac{d^3y}{dx^3} = 0$;

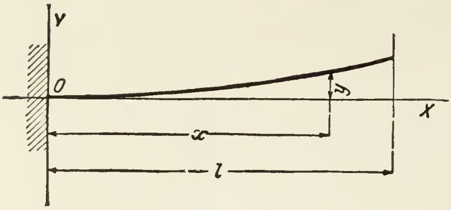


Fig. 119.

therefore, we have four equations for the determination a, a', b, b' in Formula 5.

If the determinants of the coefficients in the condition equations disappear, we again get infinitely large values for the deflection. The calculation leads to the expression

$$\cos kl\,[e^{kl} + e^{-kl}] + 2 = 0 \quad . \; . \; . \; . \; . \quad (17)$$

and the smallest root kl of this equation is $kl = 1.875$, or about $1.19\,\frac{\pi}{2}$ as compared to $\frac{\pi}{2}$ in the former case; hence, finally, with Equation 6, the critical velocity is

$$\omega_k = 3.494\sqrt{\frac{J\,E}{M\,l^3}} \quad . \; . \; . \; . \; . \; . \quad (18)$$

or the radius of the shaft is

$$r = 0.5683\sqrt[4]{\frac{M\,l^3\,\omega_k^{\,2}}{E}} \quad . \; . \; . \; . \; . \; . \quad (19)$$

Actually, the stiffness of the shaft is increased by the hubs of the disc-wheels. It must be left to actual practice to determine how large this influence is; that is, how much of the moment of inertia of the hub may be considered as assisting the shaft's moment of inertia.

50. THE UNLOADED SHAFT UNDER THE INFLUENCES OF ITS OWN MASS.

HIGH SPEED TRANSMISSION.

If an otherwise unloaded (*e.g.*, vertically conceived) shaft is bent initially, it will be still further distorted by centrifugal force, and the elastic opposing force hereby exerted is proportional to the difference between the true and initial deflections. The behavior

of the shaft would then be the same as if there were present an ideal straight line shaft which furnishes the elastic forces while it is being loaded by the eccentrically placed (otherwise considered free) masses of the actual shaft. The already derived formulæ can therefore be applied without any further conditions.

For *the shaft supported at both ends* of length $2l$, we substitute in Formula 7

$$M = \mu \pi r^2 2l$$

and get

$$\omega_k = 1.234 \frac{r}{l^2} \sqrt{\frac{E}{\mu}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

or

$$r = 0.811\, \omega_k l^2 \sqrt{\frac{\mu}{E}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

For *the shaft fixed at both ends,* we have

$$\omega_k = 2.776 \frac{r}{l^2} \sqrt{\frac{E}{\mu}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

$$r = 0.360\, \omega_k l^2 \sqrt{\frac{\mu}{E}} \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

For *the shaft fixed at one end* of length l is

$$M = \mu \pi r^2 l$$

and

$$\omega_k = 1.747 \frac{r}{l^2} \sqrt{\frac{E}{\mu}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

$$r = 0.5724\, l^2 \omega_k \sqrt{\frac{\mu}{E}} \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

Finally we have for wrought iron with $\mu = 0.0078 \div 981$ and $E = 2\,150\,000$, and introducing the revolutions n per minute, for the three cases

$$r = \frac{1.633}{10^7} l^2 n, \quad \frac{0.725}{10^7} l^2 n, \quad \frac{1.147}{10^7} l^2 n, \text{ respectively} \quad . \quad (7)$$

r and l are in centimeters.

If r and l are in inches, then

$$r = \frac{4.148}{10^7} l^2 n, \quad \frac{1.842}{10^7} l^2 n, \quad \frac{2.913}{10^7} l^2 n \text{ respectively.}$$

As an example, for a shaft supported at both ends, with $n = 1\,500$, and $l = 100$ cm., $r = 2.45$ cm. or if $l = 39.37$ in., $r = 0.965$ in. at the same speed.

The above derived forms deserve consideration in the *design of high speed transmissions*, for we demand of these that they should be kept *well within their critical number of revolutions.*

51. THE FORMULA OF DUNKERLEY.

From the preceding we see that the critical velocity can be determined mathematically only in the simplest cases, and that the graphical method is very unhandy. It is therefore fortunate that *Dunkerley* succeeded by a long series of theoretical and experimental investigations in deriving a simple empirical formula that is suitable for complicated conditions.

Imagine a shaft with any style of bearings, whose critical velocity when rotating unloaded is ω_1. On this shaft a wheel, T_1, is attached in a certain position. Neglecting the mass of the shaft, the critical velocity ω_2 of the system may be determined mathematically.

The actual critical velocity of the combined shaft and disc is, according to *Dunkerley*,

$$\omega_0 = \frac{\omega_1 \omega_2}{\sqrt{\omega_1^2 + \omega_2^2}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

After removing wheel T_1, a second wheel T_2 is mounted at another place; and then the theoretical velocity will be ω_3 if we neglect the weight of the shaft. If both T_1 and T_2 are mounted, experiment will give as actual critical velocity,

$$\omega_0 = \frac{\omega_1 \omega_2 \omega_3}{\sqrt{\omega_2^2 \omega_3^2 + \omega_3^2 \omega_1^2 + \omega_1^2 \omega_2^2}} \quad . \quad . \quad . \quad . \quad (2)$$

or, according to *Dunkerley*, the actual deviation from these figures is not larger than a few per cent.

These formulæ also hold good for the case in which the discs T_1 and T_2 are mounted between different bearings on a continuous shaft. Formula 2 is also derived when ω_0 in equation 1 is combined with ω_3 according to Formula 1. From these remarks it can be seen that the resulting ω_0 is *smaller* than any one of its components, ω_1, ω_2, ω_3

52. EXPERIMENTS ON THE CRITICAL VELOCITY OF UNLOADED AND LOADED SHAFTS.

Dunkerley has made the most extensive series of experiments, using a shaft 6.3 mm. (0.25 in.) diameter, 950 mm. (37.4 in.) long. The shaft, according to circumstances, was carried in 2, 3, or 4 bearings, and loaded with discs of 76 mm. (3 in.) and 89 mm. (3.5 in.) diameter and of about 55 g. (0.125 lb.) and 125 g. (0.271 lb.) weight, respectively. The agreement with theory in the simplest cases was nearly complete. The empirical formulæ gave, as was observed above, results to within a few per cent.

Föppl's formula was experimentally proved at the same time by *Klein*,* and also showed itself in complete harmony.

Without knowing anything of *Dunkerley's* work, the author also conducted experiments with unloaded and steady-loaded shafts, which, while undertaken with primitive apparatus, are still worthy of presentation, because they were conducted at a higher number of revolutions than those of *Dunkerley*, and because the critical number of revolutions observed were of higher order, and were omitted by *Dunkerley*. Shafts, 8.5 mm. (0.335 in.) and 3.5 mm. (0.138 in.) diameter, of calibrated round steel were coupled direct to the rotating wheel shaft of a Laval turbine in the mechanical laboratory of the Polytechnikum at Zürich, in which it was possible to obtain 25 000 revolutions per minute. By means of a brake on the gear wheel shaft the velocity was easily regulated. The bearings were 56 mm. (2.2 in.) long, giving the

* Z. 1895, p. 1192.

conditions of a "fixed" shaft. The foundation consisted of a wooden post mounted on a wooden block. To prevent fracture of the shaft, its greatest vibration was limited to about 10 mm. (0.394 in.) radius by rings about the shaft. The very first experiment proved the *existence* of higher critical velocities. The shaft, vibrating originally at about 1 mm. (0.04 in.), showed with an increase of velocity, an unstable running; at about the critical value it bent and began to rub hard against the rings. Hardly had the critical value been reached, before the shaft straightened out, and no initial vibrations could be observed. If the velocity is increased, a second critical number of revolutions is reached with the occurrence of similar phenomena, with one node at the center between bearings; and beyond that, at a third critical number of revolutions, with two nodes, etc.

These calibrated steel shafts were so well made and so homogeneous, that the vibrations at the second and the higher critical numbers of revolutions were less than 10 mm. (0.4 in. radius). The eccentricity e in our formulæ must hence be assumed very small.

In the following table the "*critical*" *numbers of revolutions* are given, at which the *pressure* against the rings or the vibrations of the entire apparatus, *appeared* to be the maximum.

1. *Unloaded Shaft, 8* mm. (0.315 in.) diameter, $l = 640$ mm. (25.2 in.) fixed at one end.

Critical Number of Revolutions per min. Theoretically, about	850	5 400	15 000	29 500
Critical Number of Revolutions per min. Observed, about	800	5 000	14 000	23 000
Ratio, Theoretical	1 :	6.3 :	17.6 :	43.6
Ratio, Observed	1 :	6.2 :	17.4 :	29.

2. *Unloaded Shaft,* 8 mm. (0.315 in.) diameter, $l = 450$ mm. (17.72 in.) fixed at one end.

Critical Number of Revolutions per min. Theoretical	1 730	11 000
Critical Number of Revolutions per min. Observed	1 600	10 300
Ratio, Theoretical	1 :	6.3
Ratio, Observed	1 :	6.4

3. *Unloaded Shaft*, 8 mm. (0.316 in.) diameter, $2\,l = 860$ mm. (33.86 in.), fixed at both ends.

Critical Number of Revolutions per min. Theoretical	2 980		8 300		16 200
Critical Number of Revolutions per min. Observed	2 700		4 800		12 000
Ratio, Theoretical	1	:	2.8	:	5.4
Ratio, Observed	1	:	1.8	:	4.4

4. *Unloaded Shaft*, 3.5 mm. (0.138 in.) diameter, $2l = 536$ mm. (21.11 in.), fixed at both ends.

Critical Number of Revolutions per min. Theoretical .	3 690		9 400		18 400
Critical Number of Revolutions per min. Observed . .	3 200	(5 200) (unstable)	8 200	(9 500) (unstable)	17 000
Ratio, Theoretical . . .	1	:	2.8	:	5.4
Ratio, Observed	1	: (1.6) :	2.55	: (2.95) :	5.3

This thin shaft shows slight vibrations ("unstable") also at those theoretical numbers of revolutions when the shaft ought to steady, which, from experiments made later, pointed to an apparent unsteadiness of driving.

5. *Shaft* of 8 mm. (0.315 in.) diameter, loaded with 20 *wrought iron* discs, each 180 mm. (7.09 in.) diameter, 2 mm. (0.079 in.) thick; total weight 8.93 kg. (19.65 lb.), $2l = 860$ mm. (33.86 in.), fixed at both ends.

Critical Number of Revolutions per min. Theoretical	5 80	1 620	3 160	5 250
Critical Number of Revolutions per min. Observed	500	1 300	2 800	7 000 (?)
Ratio, Theoretical	1 :	2.8 :	5.4 :	9
Ratio, Observed	1 :	2.6 :	5.6 :	16 (?)

In looking over these figures, the observed critical velocities are throughout smaller than the theoretical, while the ratio of the numbers of revolutions of the different orders corresponds well with the theoretical values. The reason of the first deviation may be in the sympathetic vibrations of the very light and incomplete foundation used in my experiments. As a matter of fact, the deviation is

greatest with heaviest shaft (5). The full explanation of this difference must be left to future experiments. One thing may be said, based on the observations already made, that the running of the shaft, especially for the model (5) representing a many-stage turbine, is smoother after passing the critical number of revolutions than before.

53. BEARINGS FOR STEAM TURBINES.

In the construction of bearings for steam turbines, we must take into account first the extremely high surface or sliding velocity, and second the never completely vibrationless motion of the shaft. A result of the high velocity is the exceptionally large work of friction, which is transformed into heat and increases the temperature of both bearing and shaft until the decrease of heat by conduction and radiation has become equal to this increase. Let the specific surface pressure equal p lb. per sq. in. (kg. per sq. cm.) taken as the quotient of load on the journal divided by the projection of the bearing surface, the surface velocity equal w ft. per sec. (meters per sec.) the coefficient of friction $= \mu$, that is the quotient of the total force of friction reduced to the shaft circumference divided by the load on the bearing. The total heat produced per second is

$$Q = A l d \mu p w \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

in which d is the diameter and l the length of the shaft-bearing in inches (centimeters). That Q cannot be decreased by decreasing p was already made known by the experiments of *Tower*, who proposed the approximate law

$$p \mu = \text{constant} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

which means that by decreasing the surface pressure, the coefficient of friction is increased in equal degree, and the total work of friction remains unchanged. The classical work of *Lasche* * and *Stribeck* † first gave the further relationship between friction-ratios and

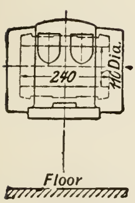
* Zeitschr. 1901, p. 1881.

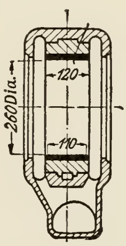
† Zeitschr. 1901, p. 1343.

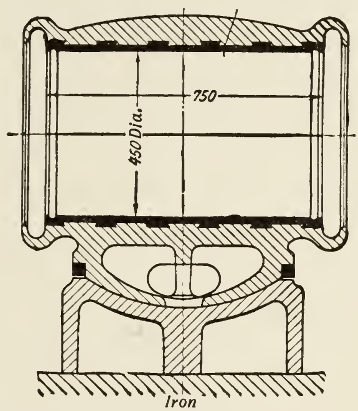
pressure, velocity and temperature. The experiments of the former include especially the higher velocities that are used in turbine construction, and lead to the very simple law

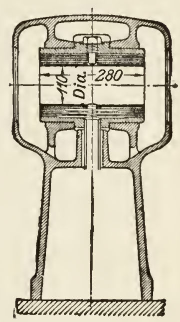
$$\mu p t = \text{constant} = 2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

when the limits of p are from 1 to 15 kilograms per square centimeter (14.2 to 213.37 pounds per square inch) which gives for the temperature t of the rubbing surfaces from 30° to 100° C. (86° to 212° F.). The velocities exert only a very small influence on









Dimensions are given in mm.

Fig. 120. Fig. 121. Fig. 122. Fig. 123.

the value of the constants, so long as they keep within the limits 5 to 20 meters per second (15.4 to 65.6 feet per second).

With very small velocities, μ, according to the experiments of *Stribeck* for Seller's bearings, approaches the value 0.14; that is, it is nearly identical with the coefficient for pure metallic friction, as the film of oil between journal and bearing has a negligibly small thickness. As the speed increases, more oil is carried around by adhesion, and μ decreases, and indeed when $p = 1$ kilogram per square centimeter (14.22 pounds per square inch) with $w = 0.1$ meter (0.328 ft.), or when $p = 25$ kilograms per square centimeter (355.5 pounds per square inch) with $w = 1$ meter (3.28 feet), μ is less than 0.005. Furthermore, the thickness of the oil film seems to increase but slowly, so that, according to Newton's law with increasing velocity, μ is also increased. Beyond 5 meters (16.4 feet) the influence of w is, as has been said, negligible.

Especially important are the experiments of *Lasche* on the rate of heat-radiation from bearings. The bearings shown in Figs. 120–123 were investigated with a rotating shaft, and the total radiation of heat was determined; that is, for bearing-bodies and shaft.

If Δt is the temperature-difference between journal and outer air, *Lasche* expressed the decrease of heat in *work units* in foot-pounds (kilogram meters) per hour, as

$$R = k\,(\pi dl)\,\Delta t \quad . \; . \; . \; . \; . \; . \; . \; . \quad (4)$$

in which l and d are in inches (centimeters). The coefficient k increased very little with the temperature; somewhat, for bearings 120–122, according to the formula,

In English units,

$$K = 42 + 0.2\,\Delta t \quad . \; . \; . \; . \; . \; . \; . \quad (5)$$

In French units,

$$k = 1.62 + 0.0144\,\Delta t.$$

Still, for practical purposes, a law with constant k will suffice; that is,

In English units,

$$\left.\begin{aligned} k &= 51.9 \text{ to } 64.8 \text{ for bearings 120 to 122} \\ k &= 129.6 \text{ to } 155.6 \text{ for bearings 123.} \end{aligned}\right\} \quad . \; . \quad (6)$$

In French units,

$$k = 2 \text{ to } 2.5 \text{ for bearings 120 to 122}$$

$$k = 5 \text{ to } 6 \text{ for bearings 123.}$$

In the latter case it appeared that the relatively large external surface, in combination with the good conductivity of the bearing boxes, seemed to increase the radiation of heat. These values hold good for air at rest, and are no doubt greatly increased by ventilation.

Formula 4 makes possible the calculation of the temperature of a bearing under working conditions. The generation of heat in foot-pounds (kilograms-meters) is, according to Formula 1,

$$R' = ld\mu pw \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

and must be equal to the value of R in Formula 4. If t is the temperature of the bearings, and t_0 that of the air, then,

$$ld\mu pw = k\pi dl(t - t_0) \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

and including equation 3,

$$\mu p = \frac{K}{t}$$

so that

$$kt\,(t - t_0) = \frac{Kw}{\pi} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (9)$$

which serves for the calculation of t.

Lasche determined that with temperatures exceeding 125° C. (257° F.), the lubricating properties of the oil suddenly decrease. If, therefore, Formula 9 shows the likelihood of high temperature, the bearing, or better still the oil, should be cooled. In the latter case, we can, at 3 000 revolutions, utilize the ordinary ring-oiling box. Frequently an oil pump is used, and the oil is cooled in special tanks fitted with cooling-tubes. From the formulæ of *Lasche* is easily determined how many degrees the oil must be cooled, assuming a certain quantity.

The bearing in Fig. 123 is designed to dampen the vibrations of the shaft and to keep them from affecting the foundation. To accomplish this, according to *Parsons*, the box has four concentric bearings, each having a small clearance. The oil is carried in grooves also to the intermediate bearings, its viscosity opposes the forcing out of the oil when vibration occurs, and this yielding resistance acts as a cushion.

The clearance is also important for the total work of friction. A shaft entirely enclosed, with pressure lubrication, even when not carrying an outside load, will experience considerable friction on account of the oil pressure, and run hot.

Recently, the mass-balance of rotating parts has been made so complete that this type of bearing is not being much used. Even in very large machines it seems that good results have been reached with the ordinary bearings, oil-cooling, and pressure-lubrication.

54. STUFFING BOXES.

The stuffing boxes are the most important and delicate part of the steam turbine. As they are subjected to high temperature on account of their proximity to the steam space, the problem of getting rid of their own heat of friction becomes all the more difficult. The advantage of the stuffing box used on reciprocating engines, where the rod for part of the time is exposed to the air, and cools at least its surface by radiation, cannot be considered with the rotating shaft. Water-cooling may be an effective means, but creates considerable loss by condensation in the surrounding steam spaces.

The majority of designers get around this difficulty by avoiding contact between packing and shaft, and secure tightness only by the least possible clearance. This is the principle of the so-called "labyrinth stuffing box" that was first generally used by Parsons. This is shown in Fig. 124, in which A is the shaft, B the stuffing box. The rings on both parts form alternately a narrow space x, and a large space y. The velocity of the steam flowing through this narrow space is destroyed by eddy-currents in the large space, so that for further velocity, a part of the drop in pressure is utilized. With a large number of rings, and with very small spaces x, the loss is greatly decreased. It also seems to have a favorable influence when the steam in leaving this narrow space flows radially inwards, that is, it helps to overcome its centrifugal force.

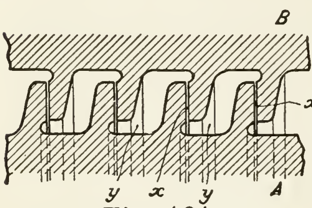
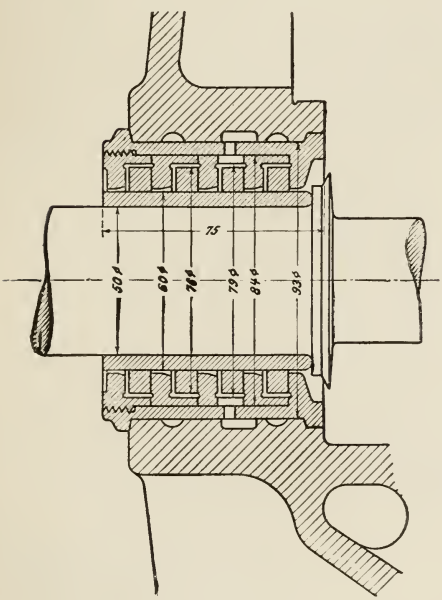


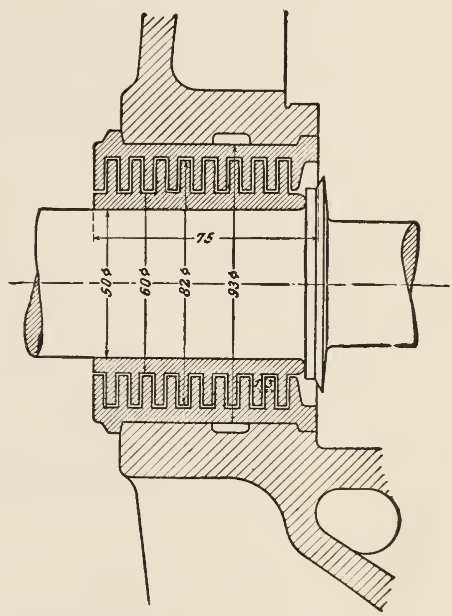
Fig. 124.

Fig. 125 shows the stuffing box of a *Schulz* turbine. No provision is here made for enlarged spaces, but the necessary throttling is accomplished by the great length of the labyrinth path. The designer hoped to limit his clearance to 1 mm. (0.039 in.).

The outer box is made in two parts. Fig. 126 shows a stuffing box by the same designer, built of rings, in which the inner rings are loose, but are made with a neat fit.

Rateau made use of the construction shown in Fig. 127, the main part consisting of a shaft *a* enclosed by a close-fitting box *b*, of suitable metal. The steam leaking through this space flows into the chamber *c*, where a constant pressure of about 0.8 atmosphere (11.8 pounds per square inch) absolute is maintained by





Dimensions here given are in mm.

Fig. 125. Fig. 126.

a reducing valve; from the valve the steam is led to a condenser. The chamber *c* is kept steam-tight from the outside by two bronze rings, *d*, *d*, each made in three parts, which are held against the shaft with slight pressure by spiral springs *e*. A pressure in the axial direction is brought about by springs *f*. The chambers of all the stuffing boxes of a turbine are connected with one another; a part of the steam that leaves the high pressure side can thus be drawn into the low pressure side. When running light, vacuum existing in all stuffing boxes, the reducing valve will allow live steam to enter, thus allowing little or no air to be drawn in.

The *Laval Company* uses a two-part white metal lined box, with spherical seat and spring pressure in the axial direction.

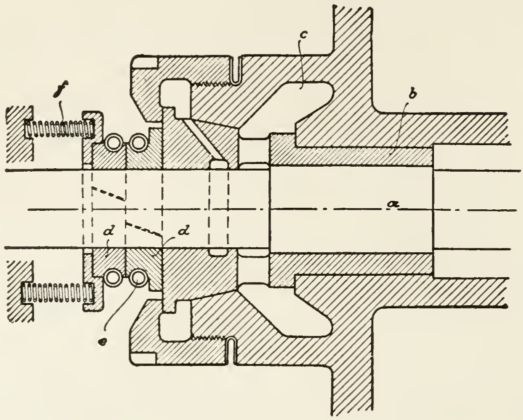


Fig. 127.

The actual steam-tightness is due to an oil-film which is drawn into the vacuum space without making any considerable consumption of oil.

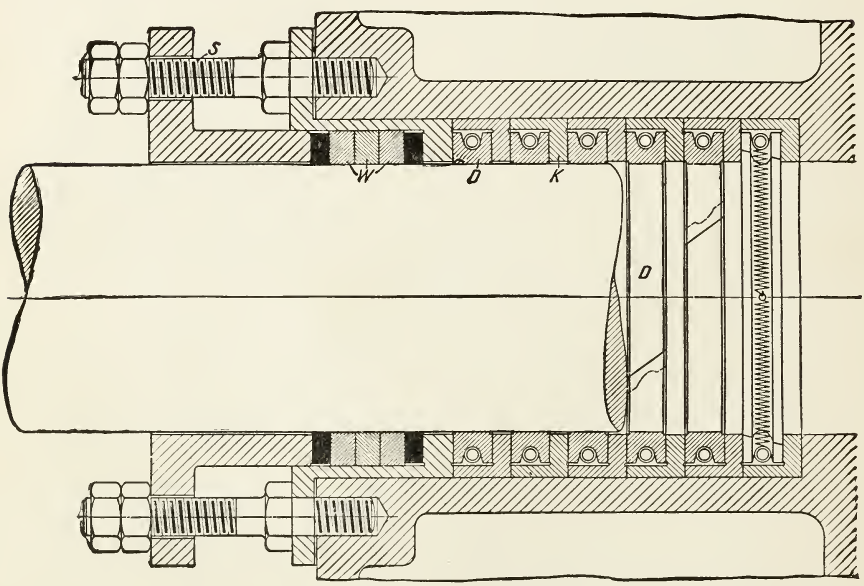


Fig. 128.

If it is only desired to make a vacuum tight fitting, then a labyrinth stuffing box may be used, with considerable clearance space.

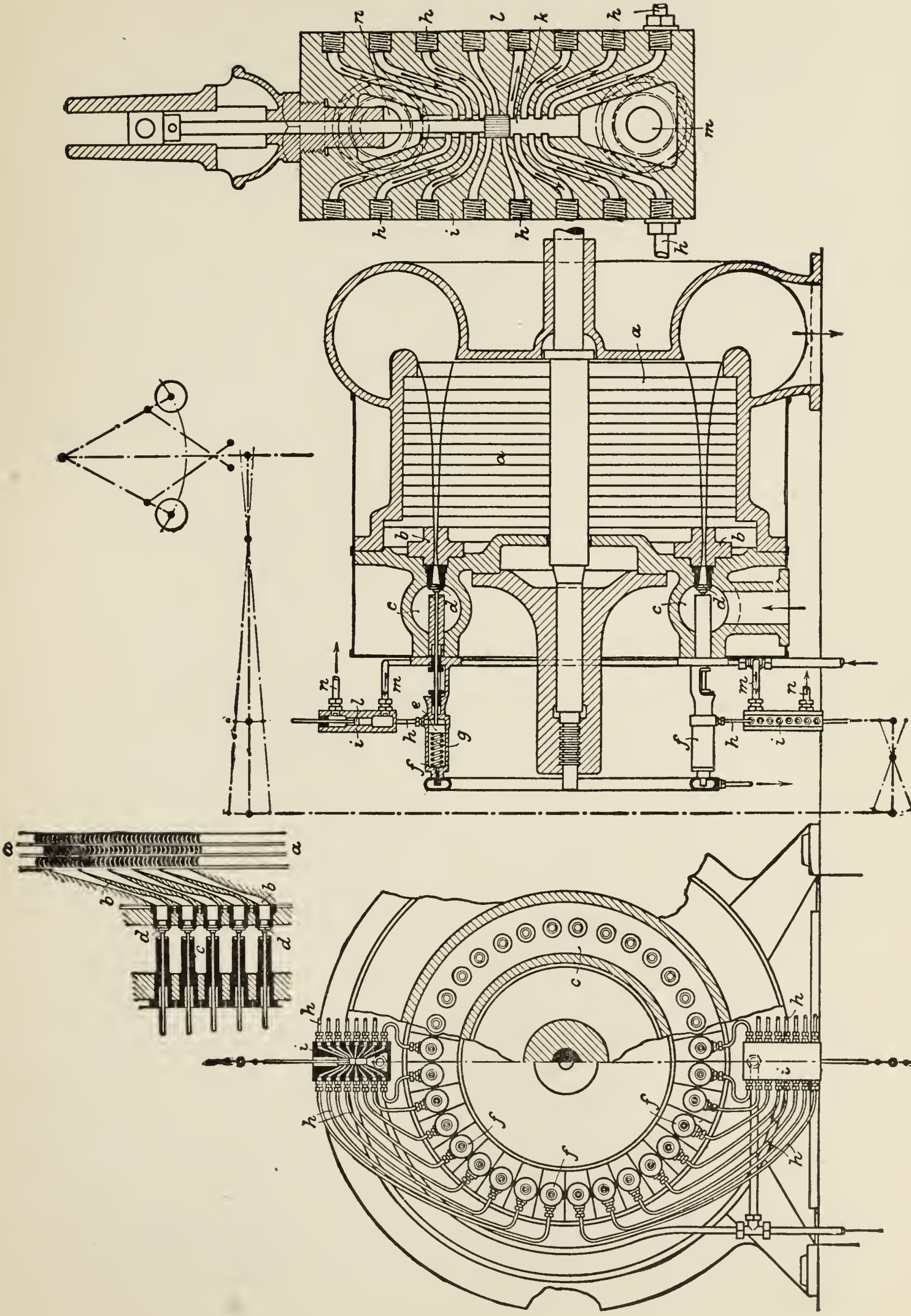


Fig. 129.

Steam is led in Figs. 125 and 126 through the ring passages, and excludes thereby the air, so that the vacuum does not suffer.

The construction of a turbine stuffing box as steam tight as that of the steam engine is still an unsolved problem. For this reason we might add the excellent stuffing box of *Schwabe*, that is used in steam-engine work, shown in Fig. 128. This consists of a large number of rings *D* made in three parts, held together by a circumferential spiral spring. These rings (for the steam engine) press on one another, and should either not touch the shaft at all, or with only the slightest pressure. With turbines, the soft packing at the outer end will of course be omitted, and the rings must be prevented from turning, and so constructed as to be tight against either pressure or vacuum ; the inside and outside ends of the box are provided with means for oiling.

The steam tightness of the intermediate stages of a few stage turbine is secured by much simpler means, on account of the small successive differences of pressure. In Fig. 132, for instance, is shown the packing of the Schulz turbine, which, as may be seen, consists of a short labyrinth of tooth-like profile, and of a loose rotating ring.

55. THE REGULATION OF THE STEAM TURBINE.

The regulation in the majority of different systems is accomplished by simple throttling, thus decreasing, at the very beginning, the available work of the steam, and consequently the economy of the turbine. The loss is measured by the product of the increase of entropy and the absolute temperature of the exhaust steam, which can easily be determined from the entropy tables.

The ideal conditions would be to constantly work with a full initial pressure and to make all cross-sections of steam passages suitable to the power required. Constructively, this idea is most easily applicable to the single stage impulse turbine, in which the nozzles are opened or closed one after another by means of a regulator. *Th. Reuter*, according to the German patent, No. 144 102, made use of such a regulator governing a valve gear (see Fig. 129), which allowed live steam to be admitted to the piston *e*, *e*, which is connected with the valve closing spindle that governs the individual nozzles. If the valve connects the space below the piston

with the atmosphere, the spindle is pressed downwards by means of the spring *g*. With the very small forces that are here exerted, it is sufficient to use small gauge tubing for steam flow, and the valve is so small that the regulator can be attached directly thereto.

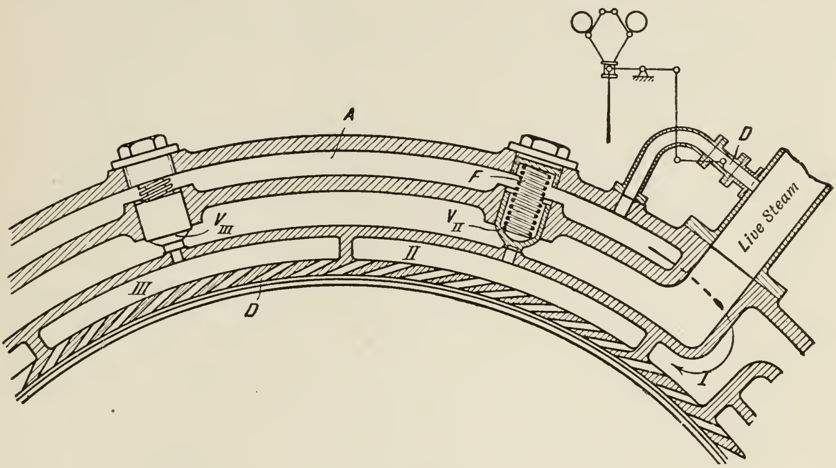


Fig. 130.

Another solution has been attempted by *Stumpf*, in the Swiss patent No. 25 438, class 93, as shown in Fig. 130. The nozzles are divided into groups I., II., III., . . . and receive live steam through the valves V_{II}., V_{III}., . . . The steam admitted at reduced pressure by the throttling valve *D* tends, with the assistance of the spring, to hold down the valves V_{II}., V_{III}., . . . against the pressure of the live steam. In starting up, space *A* has atmospheric pressure, and the admitted steam can lift all valves. When the turbine is un loaded, the regulator allows steam to go into the space *A*, which, in combination with the valve springs, each exerting a different pressure, closes the valves one after another. Finally, only the nozzle in segment I. remains open, which is always connected with space *A*, and is large enough to drive the turbine unloaded. Fig. 131 shows the same with a piston valve as stop valve.

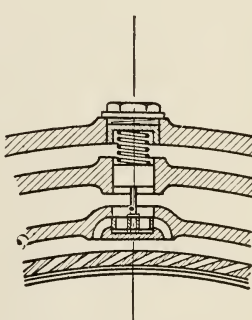
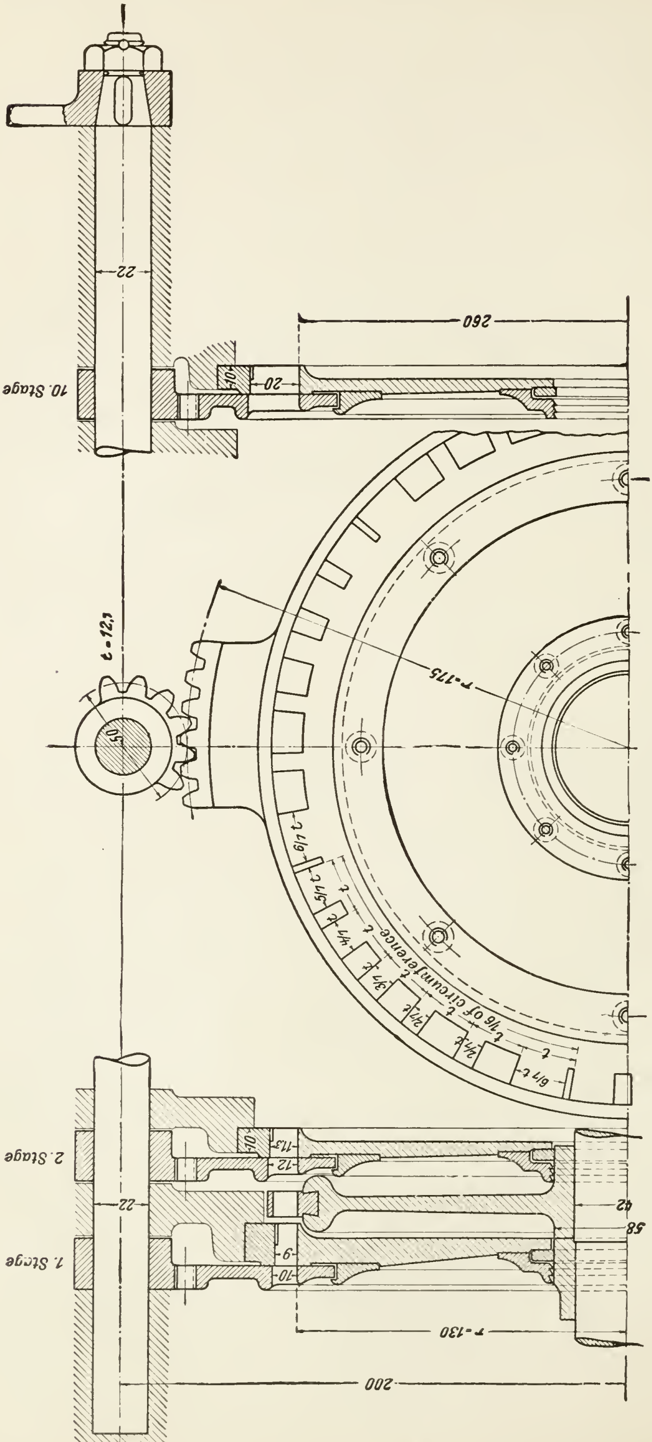


Fig. 131.



Dimensions are given in mm.

Fig. 132.

In a few stage turbine, each guide apparatus must be influenced according to a certain law. A suggestion of this type originates with *Schulz* (German patent No. 132 868), and is shown in Fig. 132 for impulse turbines. The arrangement of valves may be seen from Fig. 133. It consists of a series of bridges having the same depth as the width of the channels, and so spaced that

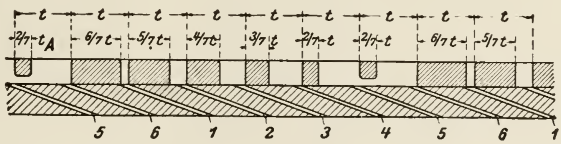


Fig. 133.

by shifting from position *A* the distance of one channel width to the right, one guide channel is closed. Finally, of six channels but one remains open, which means a regulation between wide limits. The disadvantage that the guide channels at their decreased width are too long may be easily taken care of in the design.

Rateau was content to regulate the power of his turbines (German patent No. 143 618) over a considerable interval by throttling, and it was accomplished, as is shown in Fig. 134, by a ground piston valve, *n*. Not before the regulator approached its upper limit was the regulating piston *d* affected, whereby live steam entered the cylinder *t*, and closed the valve *e*. It was not stated on what the relationship between the position of the regulator and that of the piston of cylinder *t* depended. With overload, that is the lowest position of the regulator, the regulating piston *c* admits steam to *w*, which moves the "overload valve" *z'*, admitting live steam to the low-pressure end of the turbine. By this arrangement it is possible to work the turbine at its normal load with the highest economy; that is with full pressure of admission; while occasional overload may be easily taken care of by using the overload valve at a sacrifice of economy. The steam consumption per unit of power is similar to that of a steam-engine, which also works uneconomically when overloaded.

Brown, Boveri & Cie. of Baden patented an idea (Swiss patent No. 25 439) in which with overload, live steam is not only admitted to one, but by degrees to several successive steps of the turbine.

In Fig. 135 the regulator moves a piston valve *K*, that not only governs the normal throttling, but can also open the overload chan-

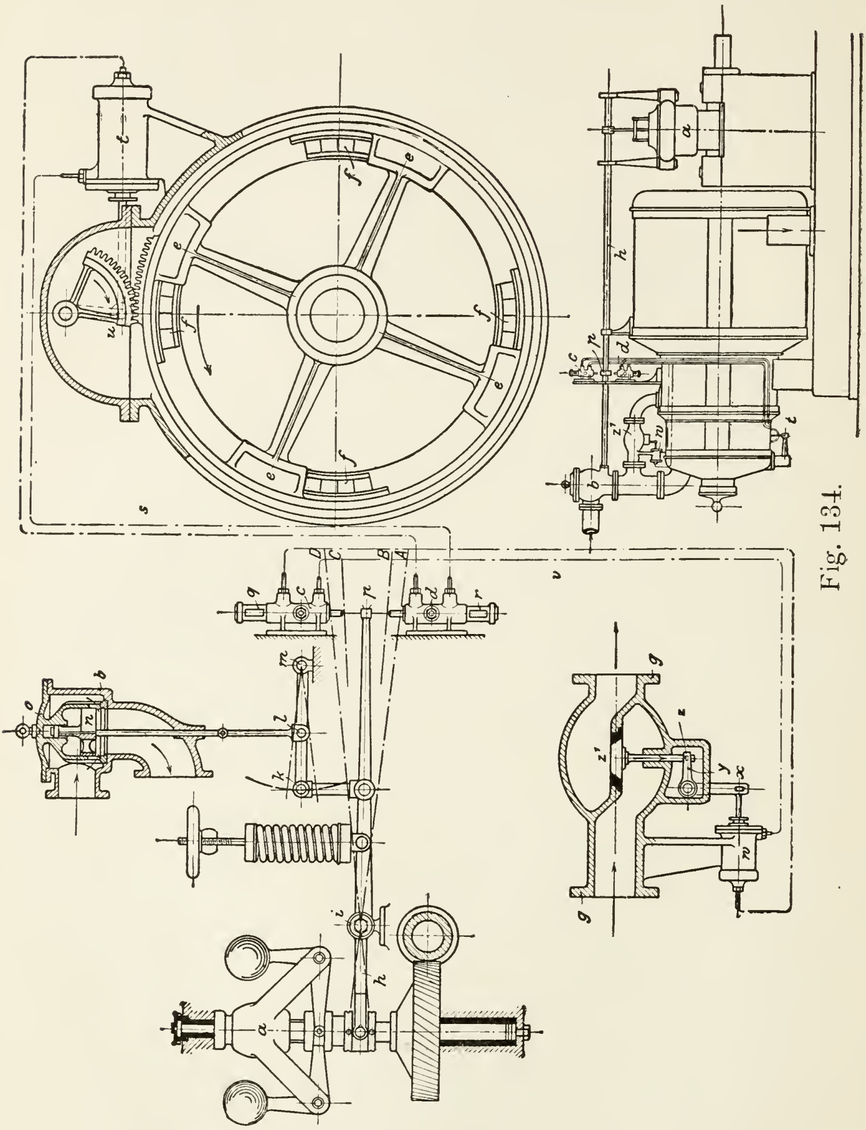


Fig. 134.

nels *a*, *b*, *c*. In Fig. 136 the overload valve *K* is moved by the pressure that exists in back of the ordinary throttle valve *D*, which compresses more or less the spring *F*. *S* is a solenoid with an iron core,

so as to allow of electric regulation. According to this arrangement, full admission pressure at the first guide-wheel remains inseparable from the secondary steam admission; with low powers simple throttling is used.

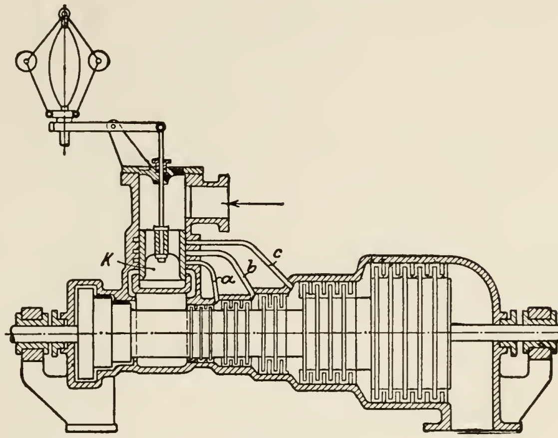


Fig. 135.

In all cases the regulation of the steam turbine is very effective, even with the many stage design, in which it might be feared that the relatively large steam volume in the turbine itself at sudden unloading, in spite of the temporary closing of the inflowing steam, might give up too much work to the rotating wheels from its own energy.

That this is not true is shown by the following short calculation; we will prove later that the steam weight flowing through the turbine in one second is approximately proportional to the initial pressure. Imagine at the time $t = t_0$, the turbine unloaded, and the regulating valve suddenly closed; and let us follow the decrease of pressure. Let the weight of the steam contents between valve and first guide wheel at the start be D_0, at a latter time D pounds (kg.). During the elementary time dt a part flows away, and is

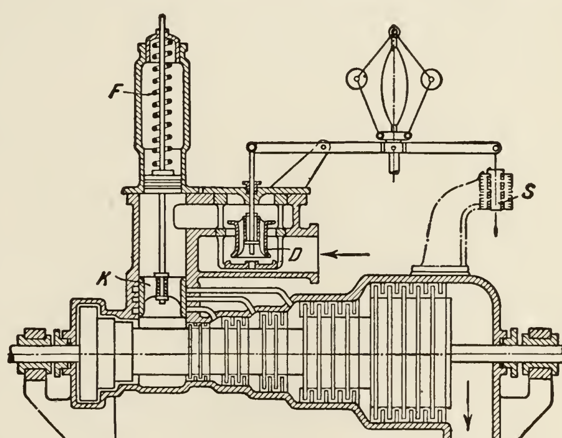


Fig. 136.

$$-dD = \alpha p\, dt.$$

The existing contents can be expressed by assuming the approximate law

$$pv = K,$$

then

$$D = \frac{V}{v} = \frac{V}{k} p,$$

where V is the volume of the space referred to. This value inserted above gives

$$-\frac{V}{k}\frac{dp}{dt} = \alpha p,$$

or, integrating,

$$-\frac{V}{K} \log \frac{p_2}{p_1} = \alpha (t_1 - t_0) \quad . \quad . \quad . \quad . \quad . \quad (1)$$

Here is

$$t_1 - t_0 = \tau,$$

the time required for exhausting from pressure p_1 to p_2; that is, to the pressure of running without load. Inserting the steam weight per second at full load $G = \alpha p_1$, we get

$$\tau = \frac{D_0}{G} \log \left(\frac{p_1}{p_2}\right) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

The increase of velocity is gotten from the energy contained in the weight of steam $D_0 + D$ in the chamber and in the turbine, which is carried over to the turbine masses at about the same efficiency. The work thus obtained may be expressed as

$$L = \left(D_0 + \frac{D_t}{2}\right) L_0 \eta_m \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

in which L_0 is the theoretical power of 1 pound (or kg.) of steam, and η_m a mean value, and D_t is halved because the mean condition of the steam in the turbine corresponds to about one-half of the work L_0. If Θ is the mass moment of inertia of the rotating

parts, ω the angular velocity, then L is the change of kinetic energy, $\frac{1}{2}\,\Theta\,\omega^2$, or approximately,

$$L = \Theta\,\omega\,\delta\,\omega = \Theta\,\omega^2 \frac{\delta\omega}{\omega} \quad . \; . \; . \; . \; . \; . \quad (4)$$

and the *relative change of velocity* is

$$\frac{\delta\,\omega}{\omega} = \frac{L}{\Theta\,\omega^2} \quad . \; . \; . \; . \; . \; . \; . \; . \quad (5)$$

For instance, with a turbine of 1 000 kw. power, D_0 at 10 atmospheres (147 pounds per square inch) initial pressure, is about 0.6 kg. (13.2 pounds), (with the narrowest valve opening). D_t is about 0.75 kg. (1.65 pounds) and AL_0 about 150 calories (592.5 B. t. u.), in which with $\eta_m = 0.5$ and $\omega = 157$; that is, $\eta = 1\,500$ revolutions per minute, with $\Theta = 50$ (moderately estimated),

$$\frac{\delta\,\omega}{\omega} = 0.027\text{ ; } \textit{i.e.},\ 2.7\%\text{ results.}$$

The time taken for exhaust is, with $p_2 = 0.6$ atmosphere (8.8 pounds per square inch), as pressure at no load,

$$\tau = 0.68\text{ sec.}$$

By partial unloading we have naturally to expect much smaller changes. These excellent results were completely affirmed, *e.g.*, with the Parsons' turbine, through all experiments up to the present.

IV.

STEAM TURBINE TYPES.

Any one of the well known types of water turbines, as is evident, can be utilized as steam turbines. However, but slight advantage is derived from this possibility, as the trend in modern water-turbine construction is to utilize the existing small differences of head, and keep the number of revolutions as high as possible. The main problem which each steam-turbine type must solve is, on the contrary, to decrease the revolutions per minute to a practical value, taking into account the necessary constructive limitations and the economy of operation.

Just what speed is practicable must, on account of the present relations between the mechanical and electrical sciences, be chiefly determined by dynamo design, and especially by the demands of the alternating current machine. In Europe, the very general cycle of 50 alternations per second gives a choice between 3 000 and 1 500 r.p.m. for the two and four pole machines, respectively, (with the so called induction type, the latter only can be considered because of the omission of half the poles). The majority of dynamo designers are of the opinion that units of about 1 000 kw. should not exceed 1 500 r.p.m. The lengths of the drums, the difficulty of mass balancing, the possibility of increasing the shaft's vibrations by means of the unsymmetric magnetic field, and the high velocity of the heavily loaded dynamo bearings make the construction of high speed machines seem a very difficult problem, for whose solution no results of any known experiments are at hand.

The ideal of simplicity would, no doubt, be a turbine that would change the total available drop or fall into mechanical work in a single wheel at one operation; that is, a *single-stage impulse turbine*. A solution of this direct change of energy for the smaller

of the practicable number of revolutions, that is, at 1 500 alternations per minute, shows instantly its impossibility, from the turbine builder's standpoint. To secure the correct hydraulic efficiency, we must with the available steam velocity at 1 200 meters (3 937 feet) or more, make the peripheral velocity of the turbine at least one-third as much, or 400 meters (1 312.3 feet). But this demands a wheel diameter of about 5 meters (16.4 feet), which would scarcely be attempted by the designer. Furthermore, such a wheel, according to our formulæ, would require a very large amount of work for running empty. There remain, therefore, if we insist on a *single wheel*, only the following methods:

a. Reduction of speed by use of gearing, as *de Laval* successfully used with powers up to 300 h. p. For larger powers this method is unavailable.

b. Increasing the speed to 3 000 r. p. m. At 400 meters (1 312.3 feet) peripheral speed there is still a diameter of about 2.5 meters (8.2 feet), and this would make the small-powered turbine too expensive; with large powers, such high speeds would hardly be decided, although the construction of the corresponding disc wheels would, according to our formula, offer no difficulty.

c. The application of velocity stages, as was probably first suggested by *Farcot* in his patents. The modern form of velocity stages is found practically applied at moderate speeds and in many ways in the turbines of *Curtis* and *Riedler-Stumpf.*

The most practical means of keeping down the speed is the application of few-stage expansion, of which, as is known, *Parsons* was the successful originator. His turbines work on the reaction principle, with 50 to 70 or more stages. In juxtaposition thereto, we have the impulse-turbine of *Rateau*, with 15 to 25 stages, and partial peripheral admission. On this idea the machines of *Zölly* and *Schulz* are constructed, while the *Lindmark* turbine employs a new principle, the partial retransformation of kinetic into potential energy. These constructions, found partly in practical use and partly in experimental forms, will be discussed later in detail, regardless of their historical order, first the impulse and then the reaction turbines. Old and new suggestions

that have not been developed are briefly discussed in Articles 66 and 67.

56. THE DE LAVAL TURBINE.

The important elements of this turbine have already been mentioned in the discussion of the nozzle, the wheel discs, and the flexible shaft.

In Fig. 137 a wheel is shown of a 10 h. p. turbine; in Fig. 105, p. 168 that of a 200 h. p. turbine. In the former the shaft is reduced before leaving the hub in order to gain flexibility; in the latter is broken so that the discs may be made solid. The blades are caulked but lightly, and can be replaced without injury to the wheel. The general arrangement of a 300 h. p. turbine (Figs. 138 and 139) shows the loose two-part stuffing boxes, and with this construction, the overhanging ball-jointed bearing at the vacuum end. The nozzles are uniformly distributed in a circle, with an angle of slope from 17° to 20°. Of late they have been placed close together in groups, in order not to split up the steam jet. A double seated throttling valve acts as a regulating device, actuated by a spring ball governor placed on the shaft and connected with the valve by a toothed wheel and a metallic steam-tight spindle and lever arm. In the newer constructions is used, besides the above-mentioned valve, an automatic closing device, as shown in Fig. 141. The steam pressure on the ground-in spindle keeps the spring compressed with a slight excess of pressure when the load is greatest. If the load decreases, the regulator starts to throttle, thus allowing the force of the spring to overcome the steam pressure and close the nozzle opening. In this way the

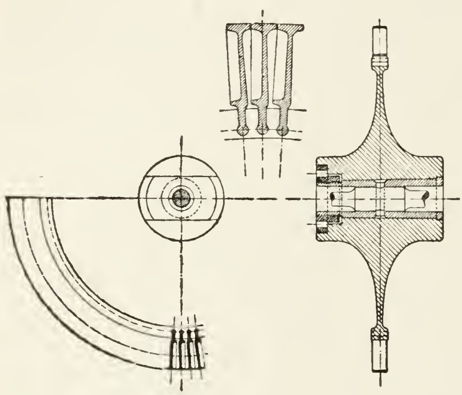


Fig. 137.

strong uneconomical throttling of the steam is avoided and the decrease of pressure is limited to about 1 atmosphere. The toothed

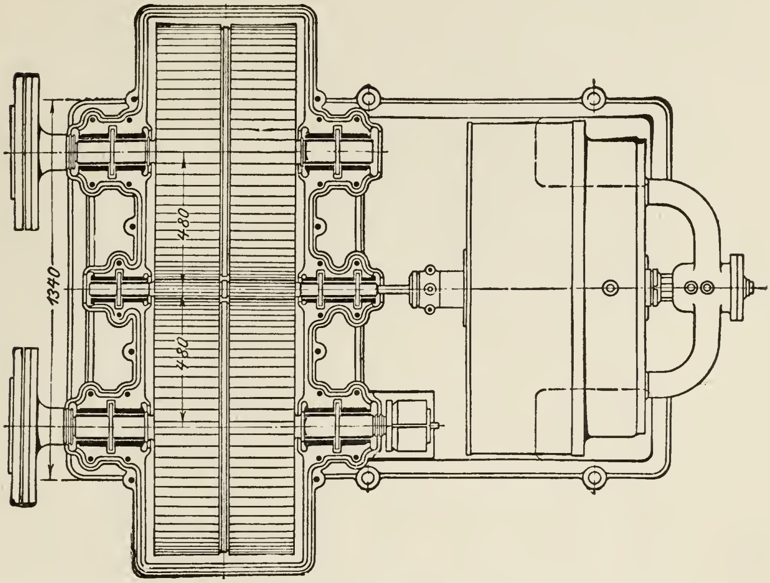


Fig. 138.

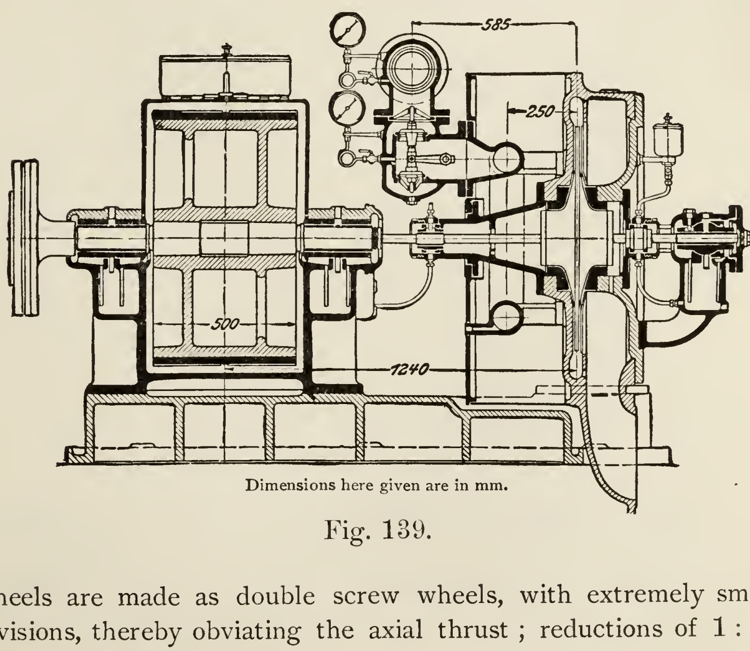
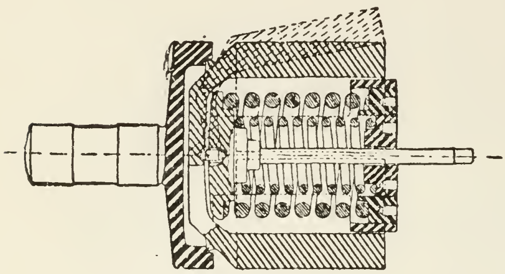


Fig. 139.

wheels are made as double screw wheels, with extremely small divisions, thereby obviating the axial thrust ; reductions of 1 : 10

to 1 : 13 were used. The width of the toothed wheels is 500 mm. (19.7 in.) for the 300 h. p. turbine, as can be seen in Fig. 139.

Fig. 142 represents a 200 h. p. turbine constructed by the Humboldt Machine Works of Kalk, near Cologne. The detail, Fig. 143, shows the ball-jointed bearing of the free end of the shaft, in which the helical oil groove can be seen. Fig. 144 shows the detail of the stuffing box, which by means of its ball-seated support allows the shaft to be out of alignment, and also gives radial play. As the stuffing box is made in two parts, the greatest care must be taken in its construction. In general, the excellent workmanship of the Laval turbine cannot be too highly praised.



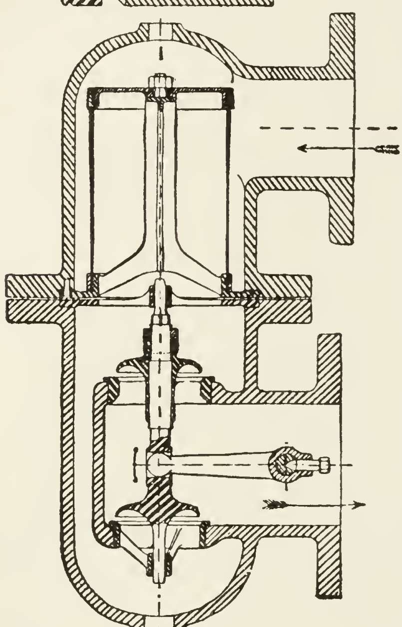


Fig. 140.

The practical results obtained under actual running conditions are, according to all reports, entirely satisfactory. The wear on the blades, with steam flowing through at a velocity up to 800 meters (2624.7 feet) seems for years to have had no great influence on steam consumption. *Sosnowski* in *Revue de Mécanique,* July, 1902, gives for a turbine five years in service, with a vacuum

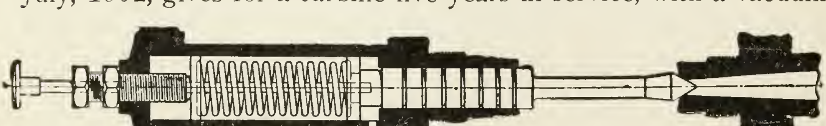
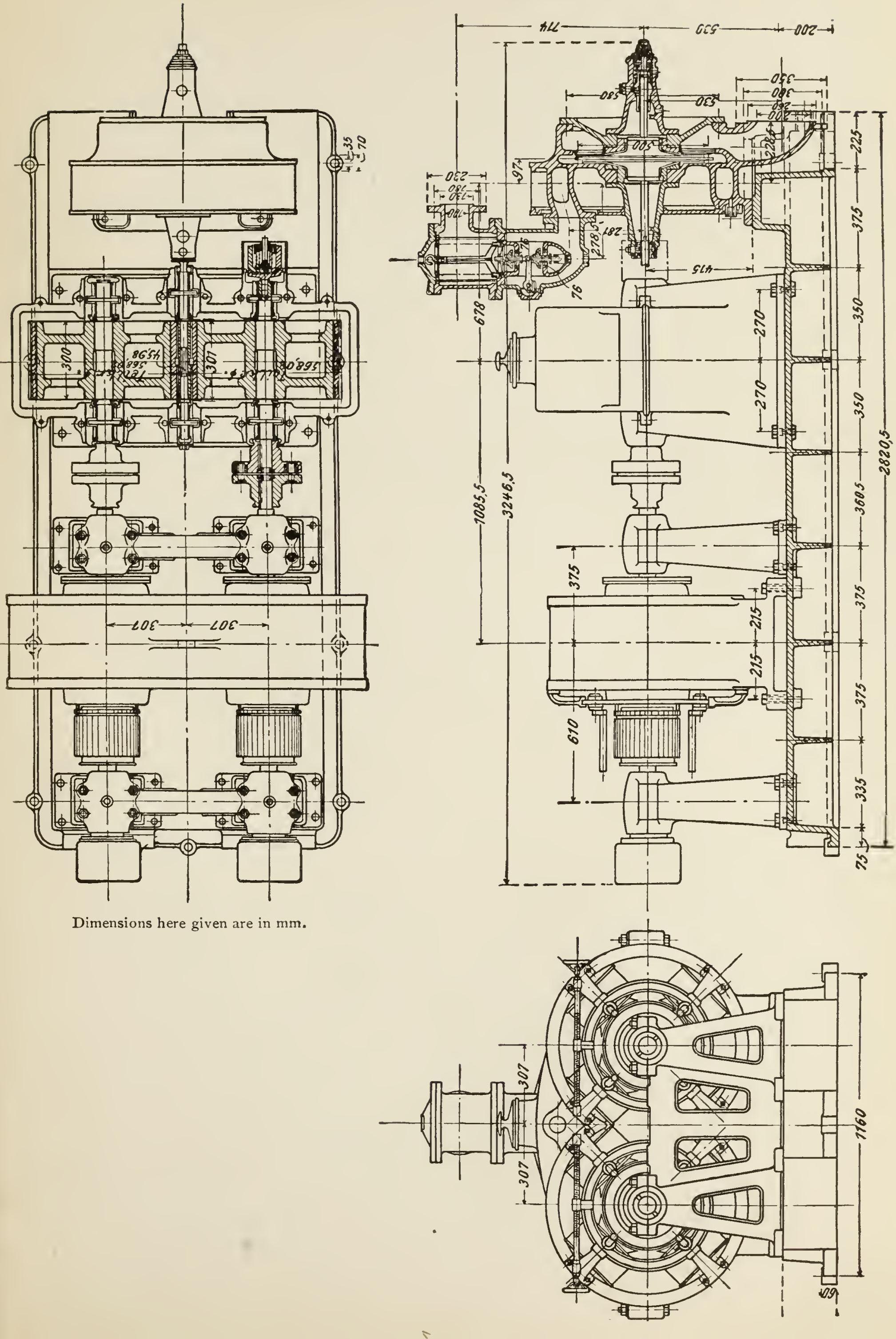


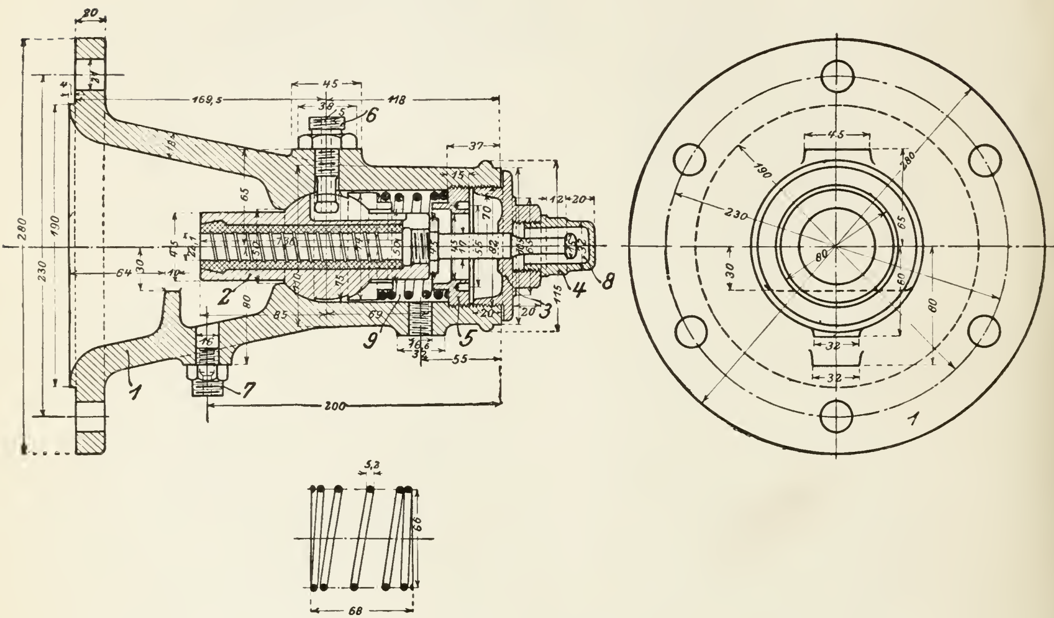
Fig. 141.



Dimensions here given are in mm.

Fig. 142.

64 centimeters (12.4 lb. per square inch absolute), a steam consumption per meter h. p.$_e$ per hour 10.07 kilograms (22.45 lb. English h. p. hour), while the values for an entirely new turbine installed at the same place with 7 centimeters (11 lb. per square inch absolute) better vacuum give a consumption of 9.7 kilograms (21.56 lb. per h. p. effective). The cost of an entire change of blades is said to be slight.



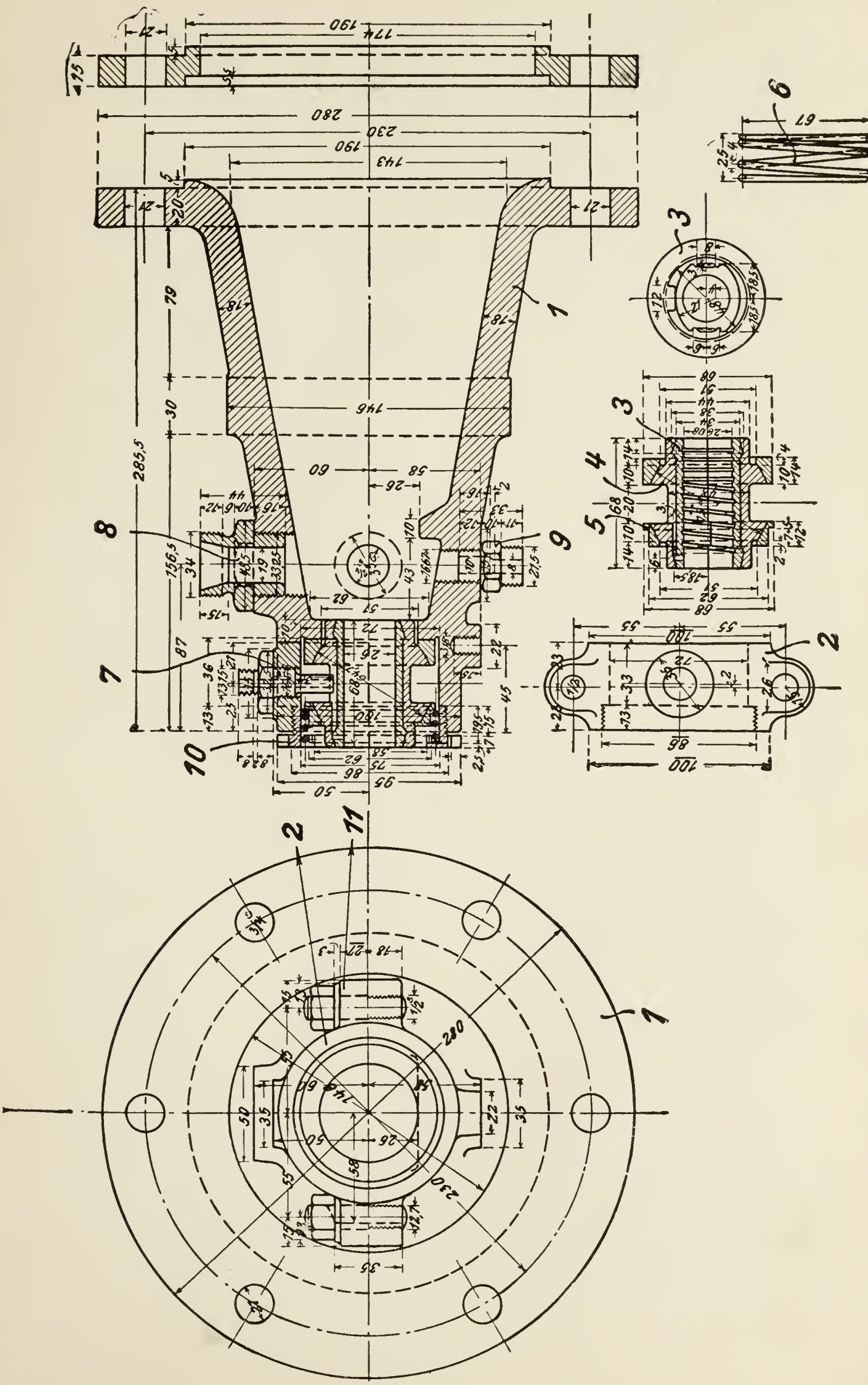
Dimensions are given in mm.

Fig. 143.

Delaporte gives results of extended experiments on the steam consumption (*Revue de Mécanique*, 1902, p. 406). The nozzles in the 200 h. p. turbine under investigation were placed close together in two groups, differing from the ordinary arrangement, so that a fairly compact steam jet resulted. The data of experiment No. 10 are as follows:

In the French units:

$p_1 = 10.72$ kg. per sq. cm. absolute;
$p_2 = 0.166$ kg. per sq. cm. absolute;
$N_e = 197.5$ h. p.
Consumption of saturated steam = 6.9 kg. per h. p. hour.



Dimensions here given are in mm.

Fig. 144.

In English units:

$p_1 = 152.2$ lb. per sq. in. absolute;
$p_2 = 2.36$ lb. per sq. in. absolute;
$N_e = 194.7$ h. p.
Consumption of saturated steam = 15.4 lb. per h. p. hour.

The harmful resistances were:

Wheel friction = 10.2 h. p. (10.06 h. p. English).
Bearing friction = 2.5 h. p. (2.43 h. p. English).
Gearing friction = 2.0 h. p. (1.97 h. p. English).

A further loss, which is caused by the refilling of the emptied cells on the rotating wheel by the surrounding steam, *Delaporte* estimates as 1.1 h. p. (1.08 h. p. English). The harmful resistances, according to his calculation, were 15.8 h. p. (15.58 h. p. English), and the clear or indicated steam work would be $N_i = 197.5 + 15.8 = 213.3$ h. p., (or $194.7 + 15.58 = 210.28$ h. p. English). Referred to 1 h. p. of indicated steam work, the consumption in one hour would be $6.9 \frac{197.5}{213.3} = 6.39$ kg., or in English units, $15.4 \frac{194.7}{210.28} = 14.26$ lb. An analysis of the experiment, which was also performed mathematically by *Delaporte*, gave the following conditions:—

	In French Units, per Kilogram of Steam.		*In English Units, per Pound of Steam.*	
Available heat-energy	**154.0**	cal.	277.8	B. t. u.
Losses in the nozzle, according to Delaporte 5.2%	**8.0**	cal.	14.1	B. t. u.
Effective outflow velocity . . c_1 =	**1 102**	meters	3 614.5	ft.
Peripheral velocity, according to Delaporte . . . . u =	**343**	meters	1 124	ft.

The construction of a velocity diagram with $\alpha = 20^0$ gives $w_1 = 787$ meters (2 581.4 ft.), and with the assumed trial value $w_2 = 0.74\, w_1 = 582$ meters (1 909 ft.), we have finally $c_2 = 326$ meters (1 069 ft.). The balance of the turbine is as follows, giving the losses in per cent. of theoretical available energy:

	French Units, per Kilogram Steam.	*English Units, per Pound Steam.*	Per Cent.
		B. t. u.	
Loss in the Nozzle	8.0 cal.	1.44	5.2
Loss in the Blades	$\left(\frac{787}{91.2}\right)^2 - \left(\frac{582}{91.2}\right)^2 = 33.7$ cal.	$\left(\frac{2\,581.4}{223.7}\right)^2 - \left(\frac{1\,909}{223.7}\right)^2 = 60.28$	21.9
Loss at Exit . .	$\left(\frac{326}{91.2}\right)^2 =$ 12.8 cal.	$\left(\frac{1\,069.3}{223.7}\right)^2 =$ 22.85	8.3
Total Loss . .	. . . .	. . . .	35.7

With 154 calories per kg. the ideal turbine required $\frac{632.4}{154} =$ 4.10 kg. of steam per h. p. hour; or, in English units with 277.8 B. t. u., the ideal turbine requires $\frac{2\,544.65}{277.8} = 9.16$ lb. per h. p. hour. Therefore the efficiency of the indicated steam work is $\eta = \frac{4.00}{639} = 0.642$, or (English units) $\eta = \frac{9.16}{14.26} = 0.642$. The loss, therefore, is 35.8%, corresponding closely to the above assumptions. If we consider the assumed small loss in the nozzle as given by *Delaporte* to be correct, the analysis gives an exceedingly high value for the loss in the blades, namely $1 - (0.74)^2$, that is 45% of the kinetic energy existing at the entrance to the wheel. If, on the other hand, we assume the nozzle loss as 10%, $w_2 =$ about $0.83\ w_1$, therefore the loss in the blades would be about 30% of the initial energy.

These losses appear to be still more unfavorable from the experiments of *Jacobson* with a 300 h. p. *turbine* at the *Pötsch Mill.* (Zeitschr., 1901, p. 150). For an overload turbine delivering 342.1 h. p. (337.3 Eng. h. p.), *Jacobson* found a steam consumption of 7.01 kg. per h. p.$_e$ hour (15.63 lb. per Eng. h. p. hour), with $p_1 = 11.28$ kg. per sq. cm. (160.4 lb. per sq. in.) absolute, and $t_1 = 192.3°$ C. (378.1° F.) at the valve; with $p_1' = 9.61$ kg. per sq. cm. (136.7 lb. per sq. in.) absolute, at the nozzle, the temperature is $t_1' = 189.8°$ C. (373.7° F.) the expansion to $p_2 = 0.101$ kg. per sq. cm. (1.436 lb. per sq. in.) absolute, gives an available heat energy per kg. of steam, of 164.4 calories, (or in English units, per lb. of steam 296.5 B. t. u.); the consumption of the ideal turbine is $\frac{632.4}{164.4} = 3.84$ kg. per h. p. hour, or in English units the ideal turbine

requires $\frac{2\,544.65}{296.5} = 8.58$ lb. per h. p. hour. If we estimate the power for running without load as given by *de Laval* at 30 h. p. (29.58 Eng. h. p.), then the indicated steam work $= 3\,721$ h. p. (366.88 Eng. h. p.). The corresponding consumption per h. p.$_i$ per hour = 6.44 kg. (14.37 lb. per Eng. h. p.$_i$ per hour), having an efficiency $\eta = \frac{3.84}{6.44} = 59.6\%$, or in English units, $\eta = \frac{8.58}{14.37} = 59.6\%$, and the losses are therefore about 40%. In order to account for these losses it would require an assumption of much greater friction in the nozzle, as *Delaporte* found. In agreement with our own experiments we set the losses in the nozzle at about 15%, and find $c_1 = 1\,078$ meters (3 536.7 ft.). The peripheral velocity may be estimated, according to a table of *de Laval*, at 400 meters, (1 345.1 ft.). Graphically, we found $w_1 = 720$ meters (2 362.2 ft.), $w_2 = 0.666\, w_1 = 480$ meters (1 574.8 ft.), and also find the following:

	In French Units, per Kilogram Steam.	*In English Units, per Pound Steam.*	Per Cent.
Losses in the Nozzle,	23.7 Calories	42.8 B. t. u. =	1.50% of Available Energy
Losses in the Blades,	34.6 "	62.3 " =	21.0 " " "
Losses at Exit . .	7.5 "	13.5 " =	4.6 " " "
. . . .	. . . .	Total Loss =	40.6% of Available Energy.

This corresponds closely to values given above. With these experiments we must assume as the losses in the blades, the very large value, $1 - (0.666)^2 =$ about 56% of the added kinetic energy in order that the actual total results may be in harmony.

These great losses may be explained by the following:

a. The cylindrical steam-jet issuing from the nozzle is cut by the surface of the wheel into a very flat ellipse, whose ends do not completely fill the blade channels, and therefore works at a disadvantage. In the newer turbines, this fault is partly remedied.

b. Due to the churning of the disc, and of the blades which at the time being are not having steam admission, there is, as *Baumann* has noticed, a tangential steam-flow under and between the nozzles, that impinges against the nozzle-jet and is absorbed as eddy-currents. This evil is minimized by moving the nozzles closer together.

c. The eddy-currents and the ensuing steam shocks in the blade channels have an especial influence, as was explained on page 97.

Further experiments are necessary in order to explain the values of the individual losses. As *Delaporte's* turbine only differs from the one used in the Pötsch Mill by having the nozzles placed closer together, it seems that the losses under case *a* are of especial importance.

Referring to the effective power, the thermodynamic efficiency with these experiments is for those of

Delaporte,

$$\eta_e = \frac{4.10}{6.90} = 0.594 \text{ (expressed in English values, } \frac{9.16}{15.4} = 0.594\text{);}$$

Jacobson,

$$\eta_e = \frac{3.84}{7.01} = 0.548 \text{ (English values, } \frac{8.58}{15.63} = 0.548\text{).}$$

The consumption of 6.9 and 7.0 kg. per h. p.$_e$ hour (15.4 and 15.63 lb. per Eng. h. p.$_e$ hour) corresponds to values of a good compound engine.*

57. THE SEGER TURBINE.

The Seger turbine is designed with one pressure and two velocity stages, without a second guide apparatus, by having the

* In the Zeitschrift des Vereins deutscher Ingenieure, 1903, *Lewicki* tells of experiments with highly superheated steam up to 460° C. (860° F.), in which he has shown that the Laval turbine, if only the nozzles are made of steel, can be worked at such high temperatures without further consideration. An application of these results for a thermodynamical investigation was not possible at the time, as the exact value of the specific heat was not known.

steam on leaving the first rotating wheel immediately enter a second rotating wheel running in the opposite direction. Seger carried the power of both wheels by a single belt to the main shaft, which was placed at right angles (see Fig. 145), and by taking suitable sizes of pulleys obtained the desired reduction.

The steam consumption was derived from the following experiment: *

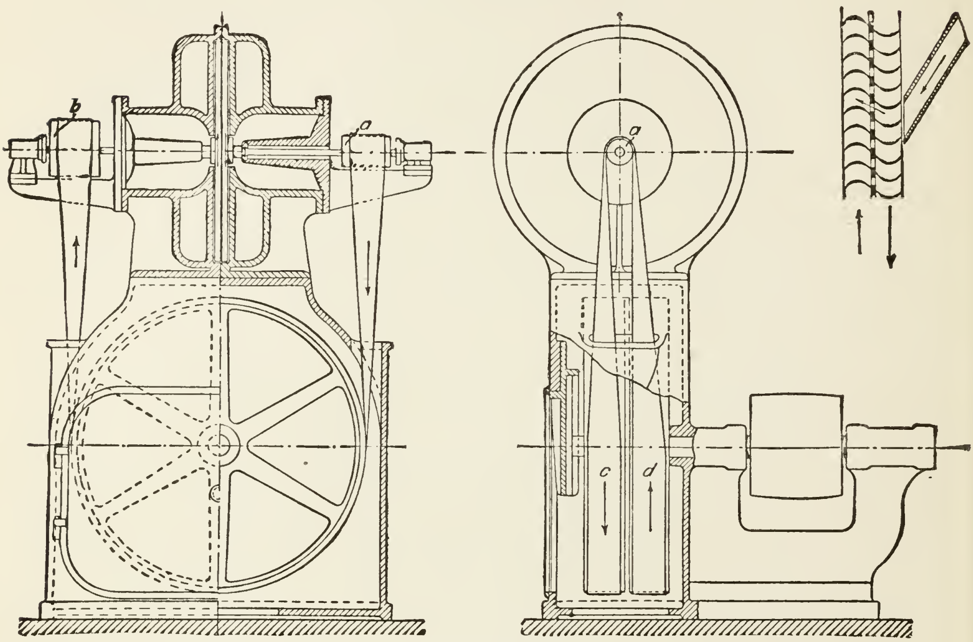


Fig. 145.

R. p. m. of first wheel . . .	8 400
" " second wheel . .	4 200
" " main shaft. . . .	700
Pressure at entrance	$p_1 =$ 7.5 kg. per sq. cm. (106.7 lb. per sq. in.)
Condenser pressure.	$p_2 =$ 0.111 kg. per sq. cm. (1.58 lb. per sq. in.)
Brake load	$N_e =$ 60.85 h. p.$_e$ (60 Eng. h. p.$_e$)
Consumption per h. p. hour . .	$D_e =$ 10.5 kg. 23.42 lb. per Eng. h. p. hr.

* Zeitschr. d. Ver. deutsch. Ing., 1901, p. 641.

Exhausting into the atmosphere the steam consumption, according to other information, with 6 600 and 3 300 r. p. m. respectively, 7.79 kg. per sq. cm. (110.8 lb. per sq. in.) pressure at entrance, 61.37 h. p.$_e$ (60.51 Eng. h. p.$_e$) is 16.7 kg. per h. p.$_e$ hour (37.25 lb. per h. p.$_e$ hour).

The manufacture of the Seger turbine has been given up on account of the liquidation of the makers; the principal idea has been already applied in other makes.

58. THE RIEDLER-STUMPF TURBINE.*

The essential characteristics of the Riedler-Stumpf turbine are the peculiarly formed Pelton buckets that were discussed on page 147; also the rectangular nozzles that allow a homogeneous steam

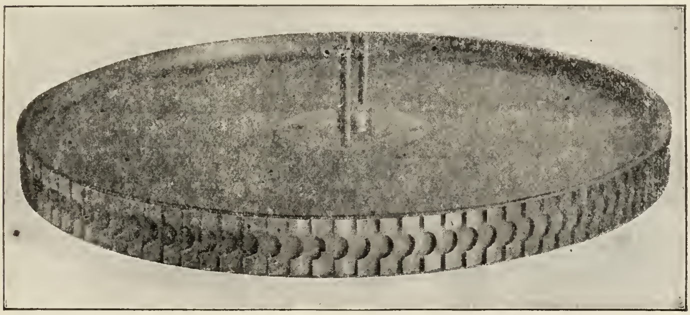
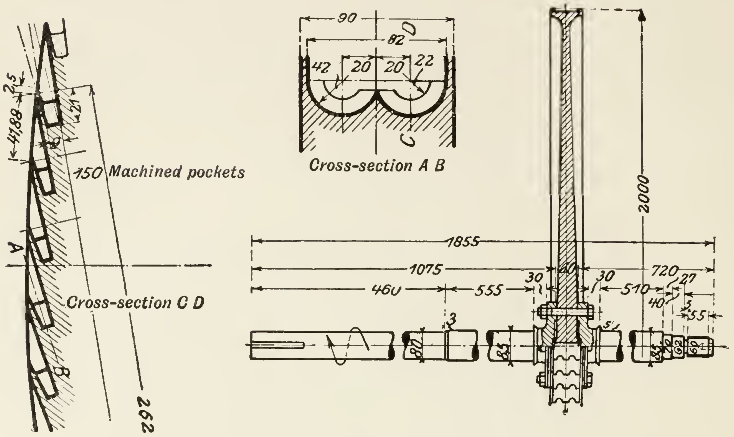


Fig. 146.

jet to be directed against the wheel. Fig. 146 shows a wheel with one-sided discharge; Fig. 147 shows sections of a wheel with symmetrical double buckets, that are again shown in perspective in Fig. 148. A turbine of this type is built by the Allgemeine Elektrizitäts-Gesellschaft in Berlin, which returns the steam to the

* Taken from a lecture of Prof. Riedler in the reports of the Schiffbau Technischen Gesellschaft, V. Vol., 1904, from which also the Figures 147, 149 to 153, 156, and 157 have been taken, and according to the reports of the Allgem. Electriz. — Gesellschaft, Berlin.

same or to a second bucket system of the same wheel ; that is, it works as an impulse turbine with one pressure and two velocity



Dimensions here given are in mm.

Fig. 147.

stages. Fig. 149 shows the construction of a double-bucket rim, by which the jet is split into two symmetrical parts by the sharp middle partition. The jet is turned in both directions, and then returned to the middle plane of the wheel by the reverse blades, and is again brought to the wheel as a united jet. In Fig. 150 the first peripheral admission takes place in a small one-sided bucket, and the steam is returned to a second bucket-rim of the same wheel, also one-sided, but wider.

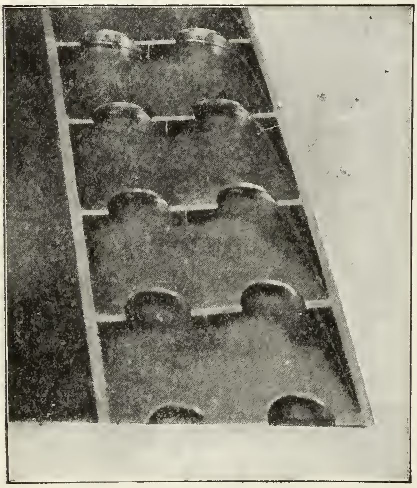


Fig. 148.

In the first case the principle of the united steam jet is given up because the reverse blades require a certain space between every two nozzles.

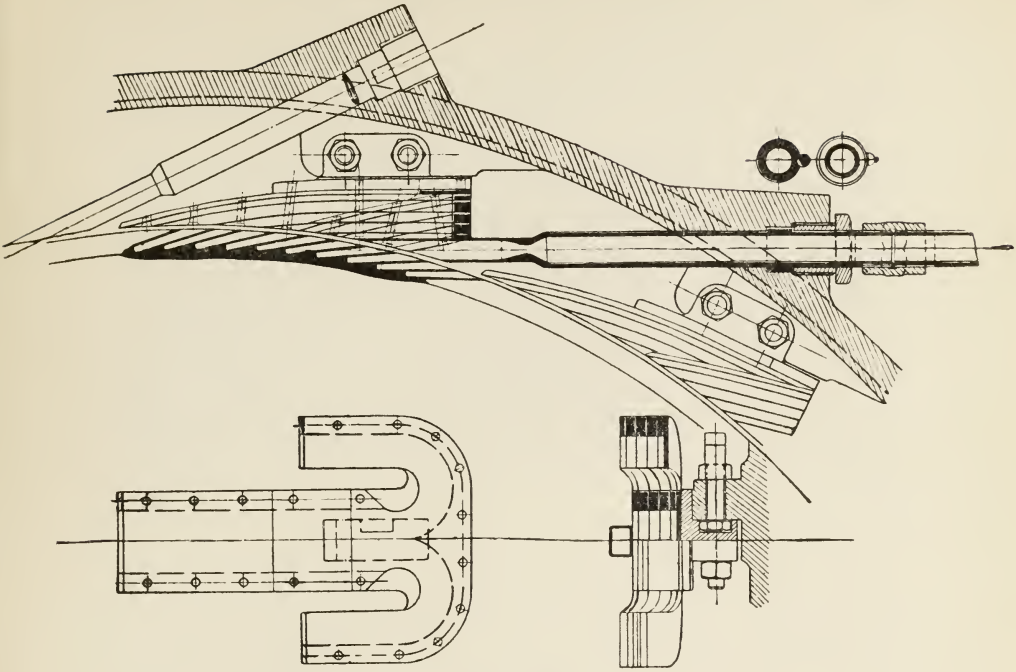


Fig. 149.

Nearly the entire periphery has primary or secondary admission, and hereby the fan-work of the idle blades has been reduced to a minimum. The steam stream is also always turned in the same direction, which is of importance if we take into account what was said in Article 27 concerning the distribution of pressure. The reverse blades must have a somewhat more obtuse entrance angle for the outflowing steam, and a very acute angle

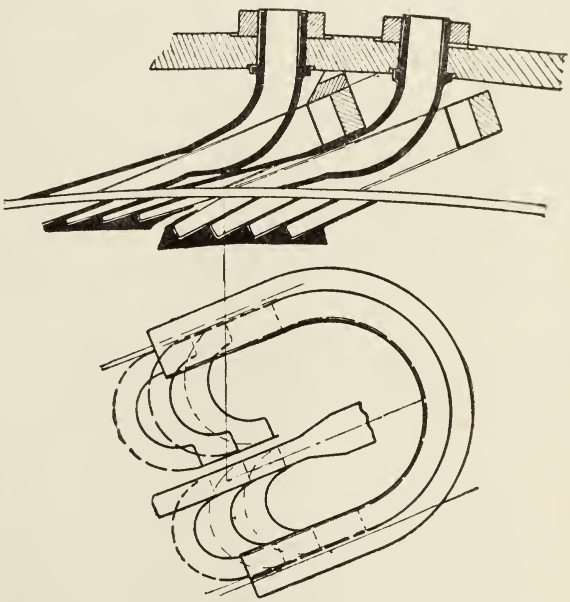


Fig. 150.

for the reëntering steam, which requires a peculiarly twisted form.

If we construct the wheel with two bucket rims, the reverse blades can remain even, and the second wheel buckets will have the necessary obtuse angle of slope. As Fig. 150 shows, the reverse blades are led between the outside of the turned-away nozzle-necks so that the nozzle ends still touch each other and deliver a nearly homogeneous jet. Fig. 151 shows the method of attaching of the reverse blades. The somewhat long path of the return can be shortened if we decide to let the steam jet describe a cross-loop,

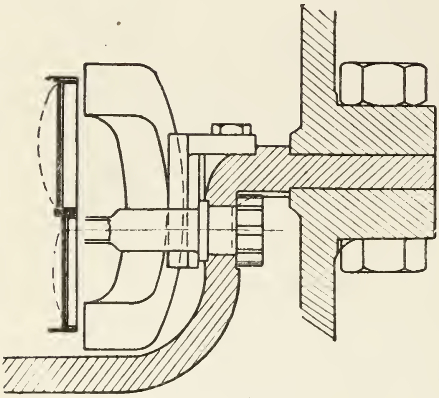


Fig. 151.

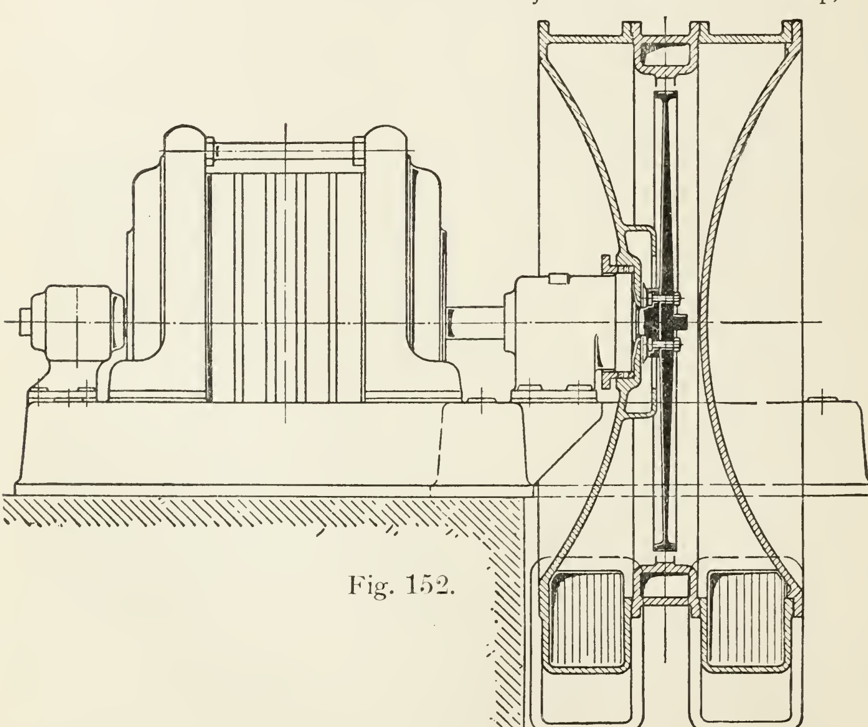


Fig. 152.

but designers still properly keep to a bending in the same direction.

Fig. 152 shows the general arrangement of a 2 000 h. p. tur-

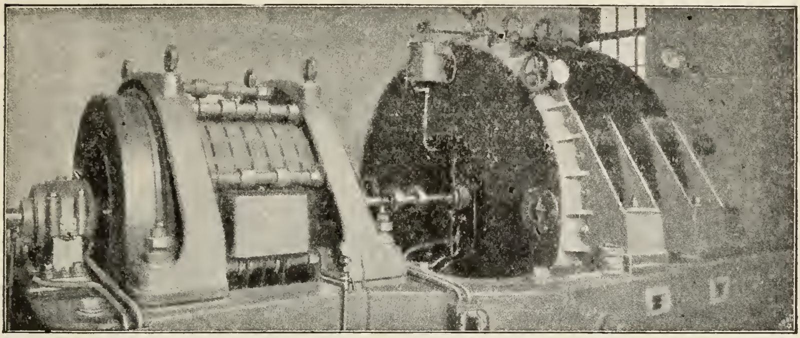
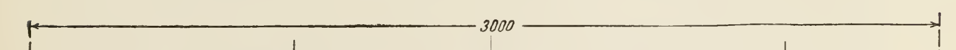


Fig. 153.

bine making 3 000 r. p. m., and this machine is about as simple as could be made. This turbine has the same dimensions as the one erected in the Moabit Elektrizitätswerk for experimental purposes.



Dimensions here given are in mm.

Fig. 154.

The inner parts are made accessible by using a cover, dished in for strength, and by overhanging the rotating wheel for machines up to 5 000 kw. capacity. The casing is made steam-tight at

the bearing by a flexible stuffing-box; air is prevented from entering at the shaft by box-bearings, to which sufficient oil is fed to entirely fill the space between shaft and bearing. The oil is drawn into the turbine and is again pumped out; the loss of oil while the turbine is working was undetermined. Fig. 153 gives an outside view of the Moabit turbine with dynamo. Fig. 154 shows a section, Fig. 155 an outside view, of a very compactly constructed turbine for marine use, built by the Allgemein Elek.-Gesellschaft for powers up to about 100 kw.

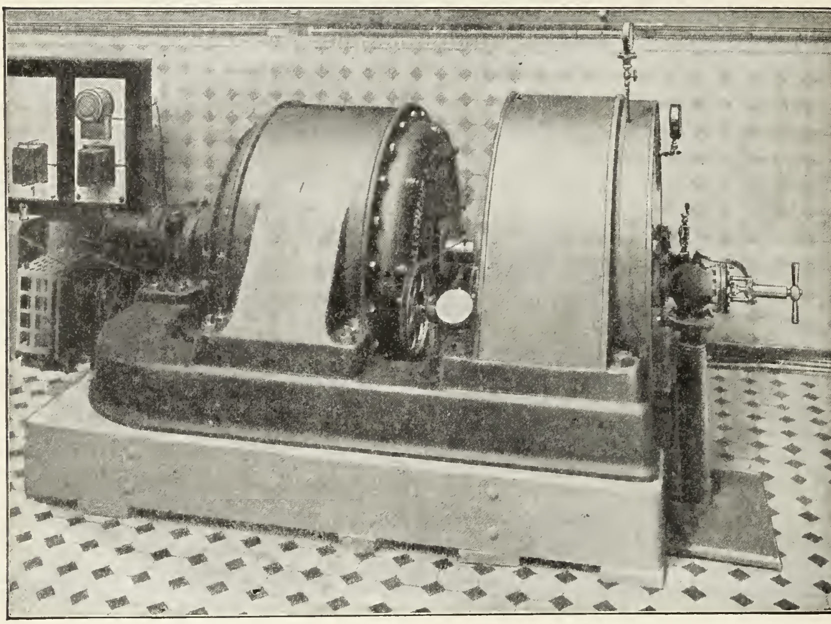


Fig. 155.

When the single-pressure stage with velocity stages is not sufficient to reduce the revolutions to the desired speed, we plan for the application of few-pressure stages. Fig. 156 shows the design of a 500 kw. turbine with four pressure stages, each with two velocity stages, making only 500 r. p. m. Fig. 157 shows a vertical turbine of 2 000 kw. with two pressure stages, each with two velocity stages, whose r. p. m. is 750. The peripheral velocity of the design in Fig. 156 is, according to the scale of the drawing, about 53 meters (173.9 ft.), while that of Fig. 157 is about 118

meters (437.1 ft.). The centrifugal condenser mounted on the lower end of the turbine shaft in the latter design is worthy of notice, and its trial is said to have been successful.

Experiments have been made with the 2 000 h. p. turbine at the Electrical Works, and the following results obtained: —

α. With 1 365 kw. load, 3 000 r. p. m., 9 atmospheres (147 lb. per sq. in. absolute) nozzle pressure; 13.25 atmospheres (207.9 lb. per sq. in. absolute) at the turbine entrance;

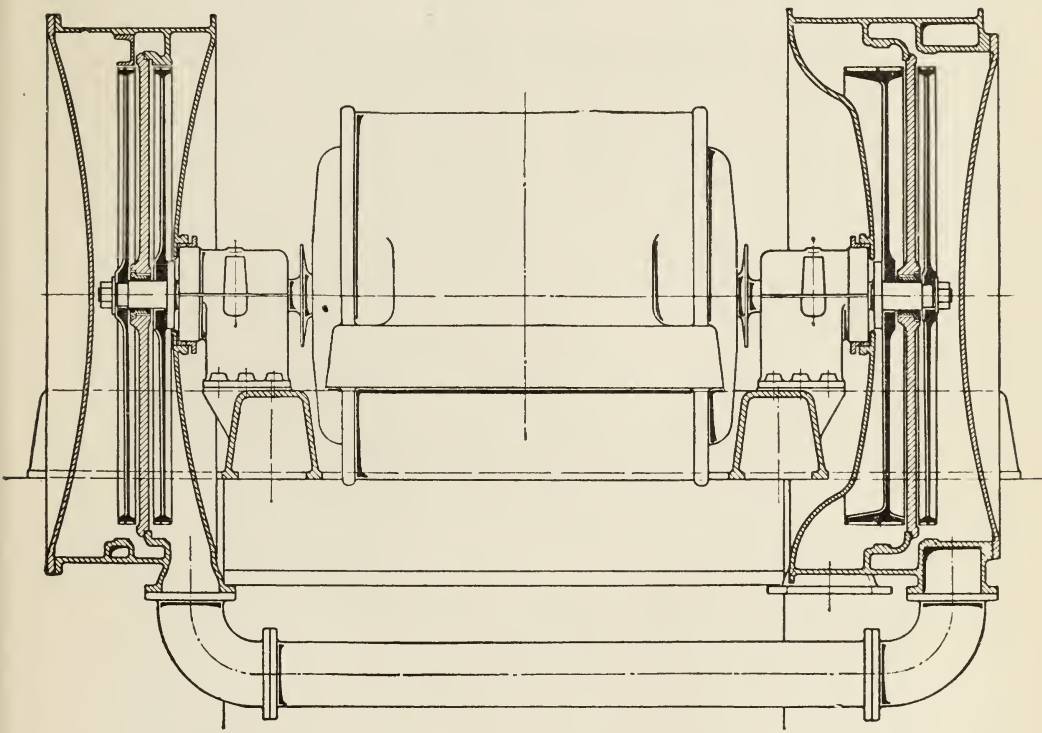


Fig. 156.

294.5° C. (562.1° F.) temperature of steam; 0.15 atmospheres (2.13 lb. per sq. in.) absolute condenser pressure, we get a steam consumption of 8.89 kg. (19.56 lb.) per kw. hour.

β. With 1 917 h. p. (1 890 Eng. h. p.) (taken with a hydraulic brake); at 3 800 r. p. m.; 12 atmospheres (185.4 lb. per sq. in. absolute) steam pressure; 300° C (572° F.) tem-

perature of steam ; 0.0855 atmosphere condenser pressure (1.22 lb. per sq. in. absolute), we have a steam consumption of 7.9 kg. (17.38 lb.) per kw. hour.

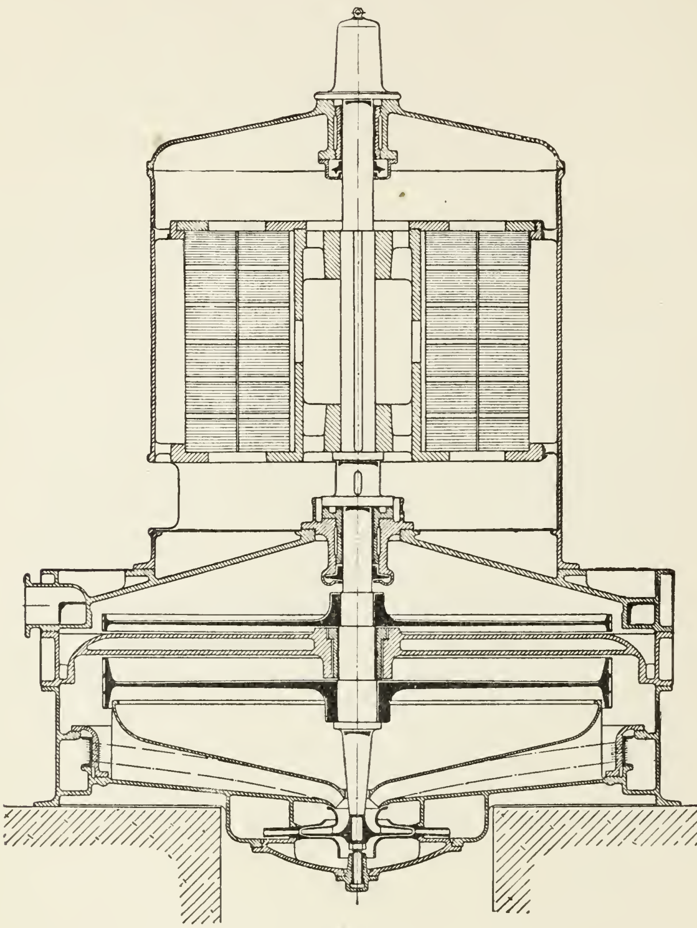


Fig. 157.

It would be of especial interest to determine the thermodynamic results of the last experiment, even when this is only approximately possible because of incomplete data. The power is 1 917 h. p., which we can consider as the "effective" power, as a brake was used. This will give with 0.95 as efficiency of dynamo, $1\,917 \times 0.736 \times 0.95 = 1\,340$ kw. useful electric work. The loss by free expan-

sion from 12 atmospheres (185.4 pounds per square inch absolute) and 300° C. (572° F.) temperature to 0.0855 atmosphere (1.22 pounds per square inch absolute) vacuum corresponds to an available work of about 198 calories per kg. steam (357.2 B.t.u. per pound), therefore a theoretical consumption of $\frac{632.4}{0.736 \times 198} = 4.34$ kg. per kw. hour; $\left(\frac{2\,544.65}{0.746 \times 357.2} = 9.55 \text{ lb. per kw. hour.}\right)$. The total efficiency then is

$$\eta = \frac{4.34}{7.9} = 0.549,$$

or in English units,

$$\eta = \frac{9.55}{17.38} = 0.549.$$

To investigate the losses individually, the pressure at the entrance to the nozzle must be known so that the outflow velocity can be calculated. An approximation of the same can be made from the data, that with 850 kw. power and 8 to 8.1 atmospheres (132.3 to 133.8 pounds per square inch absolute) pressure at the nozzle entrance with equally high superheating, as in experiment β, the consumption of steam was 9.2 to 9.4 kg. (20.24 pounds to 20.68 pounds) per kw. hour. If we compare this with the data of experiment α, it seems likely that the probable minimum value should be about 8.7 atmospheres (142.6 pounds per square inch absolute) to which we shall imagine the steam at 13 atmospheres (185.7 pounds per square inch) absolute, throttled. If we estimate the work of running light at 150 h. p., which is sufficiently accurate, we would obtain for the "indicated" power, 2 065 h.p.$_i$ and 5.12 kg. per indicated h. p. hour (11.42 pounds per indicated Eng. h. p. hour) as the steam consumption. But referring to the condition at the entrance to the nozzle, the consumption per h. p. hour was 3.39 (7.56 pounds per Eng. h. p. hour), therefore the "indicated" degree of efficiency tor the same initial condition is

$$\eta_i = \frac{3.39}{5.12} = 0.662,$$

or in English units,

$$\eta_i = \frac{7.56}{11.42} = 0.662.$$

This efficiency we can derive from the velocity diagram, by taking the wheel dimensions and angles from Fig. 147 and the slope of the nozzles from Fig. 96. We must take 10% as the loss in the nozzles, and choose the ratio

$$\frac{w_2}{w_1} = 0.7$$

in order to harmonize with the above. The loss due to friction in the wheel buckets is 20.6%, and that due to the velocity of exit, 3.2%. If we take 15% as the loss in the nozzle, we must choose the ratio $\frac{w_2}{w_1} = 0.78$, and this value is no doubt its highest limit. The thermodynamic results therefore stand in very close agreement with those which the Laval turbine reached at 400 meters (1 312.3 ft.) peripheral velocity.

Interesting reports have been made concerning the steam consumption of a 20 h. p. turbine exhausting into the atmosphere, with a rotating wheel 800 mm. (31.5 in.) diameter, and 3 500 r. p. m. This consumption with single stage expansion and without reverse blades was 26 kg. (58 pounds); with reverse blades, only 17 kg. per effective h. p. hour, (37.9 pounds per effective Eng. h. p.). It was not stated whether saturated or superheated steam was used. Whatever assumptions we make, analysis will show that with the steam flowing through the rotating buckets but once, under every circumstance large loss must always occur; but on returning the steam for a second peripheral admission, the wheel seems to work with a very high efficiency, because such a large reduction of the consumption would not otherwise be possible.

The *Riedler-Stumpf* constructions for ship propulsion will be discussed later.

59. THE ZÖLLY TURBINE.

The new turbine of *Zölly* shown in longitudinal view in Fig. 158, and in Fig. 159 as an outside view, is a many stage impulse turbine, and lies to a certain extent at the boundary between the

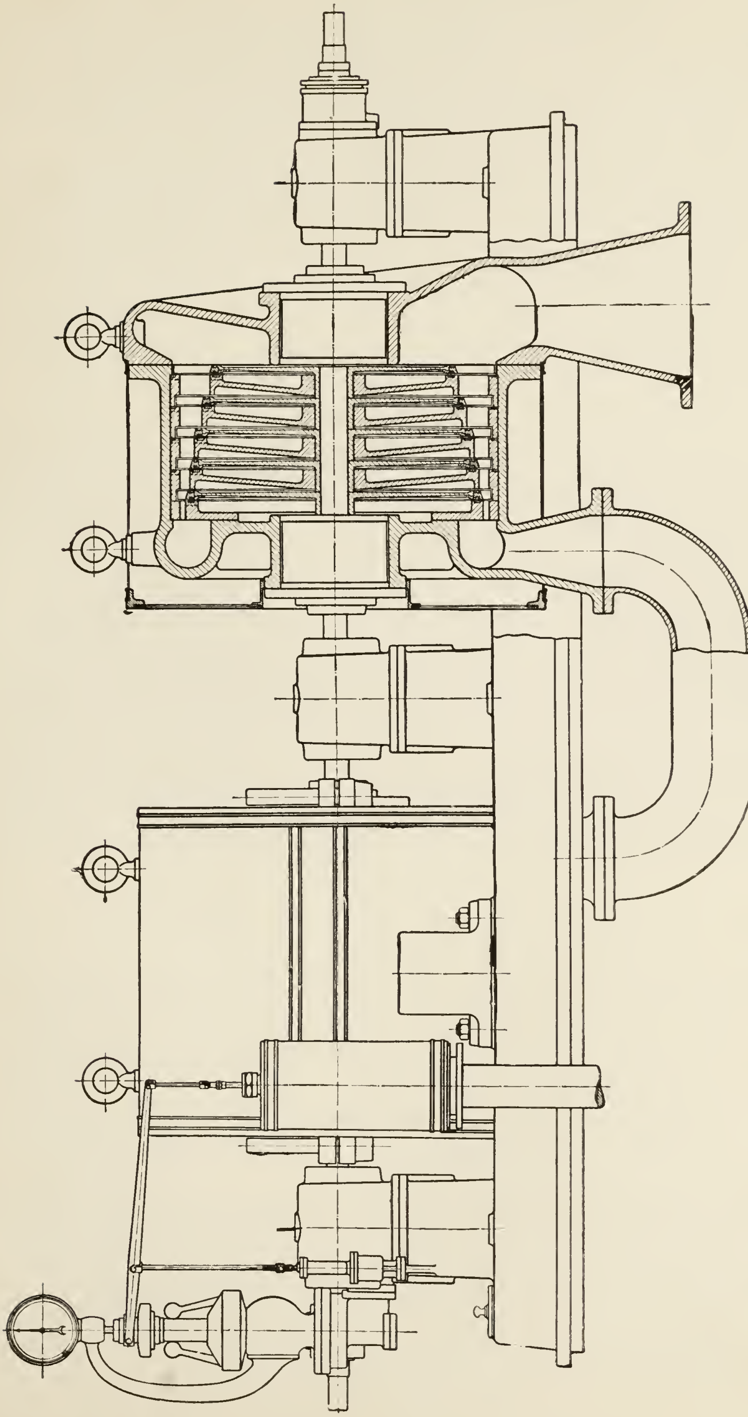


Fig. 158.

"nozzle" and the "blade" types. In this turbine there are only so many stages chosen that the guide arrangements may be constructed from ordinary blades without using the nozzle form, whose divergence is looked upon as harmful.

The experimental turbine was made with ten wheels with true axial admission,* divided into two groups; Fig. 158 shows the low pressure part in cross-section. The small number of wheels gives also a small number of blades, and these can be made by machine-work with the greatest accuracy. The forged steel wheels are turned smooth on both sides to decrease friction. The construction

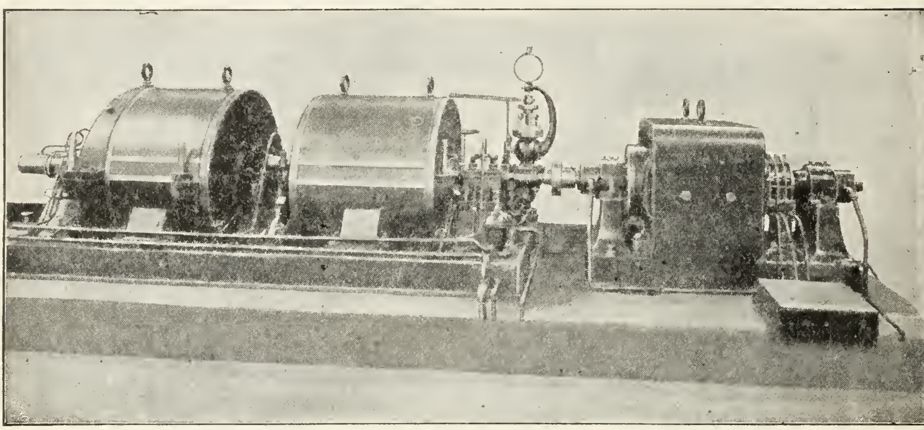
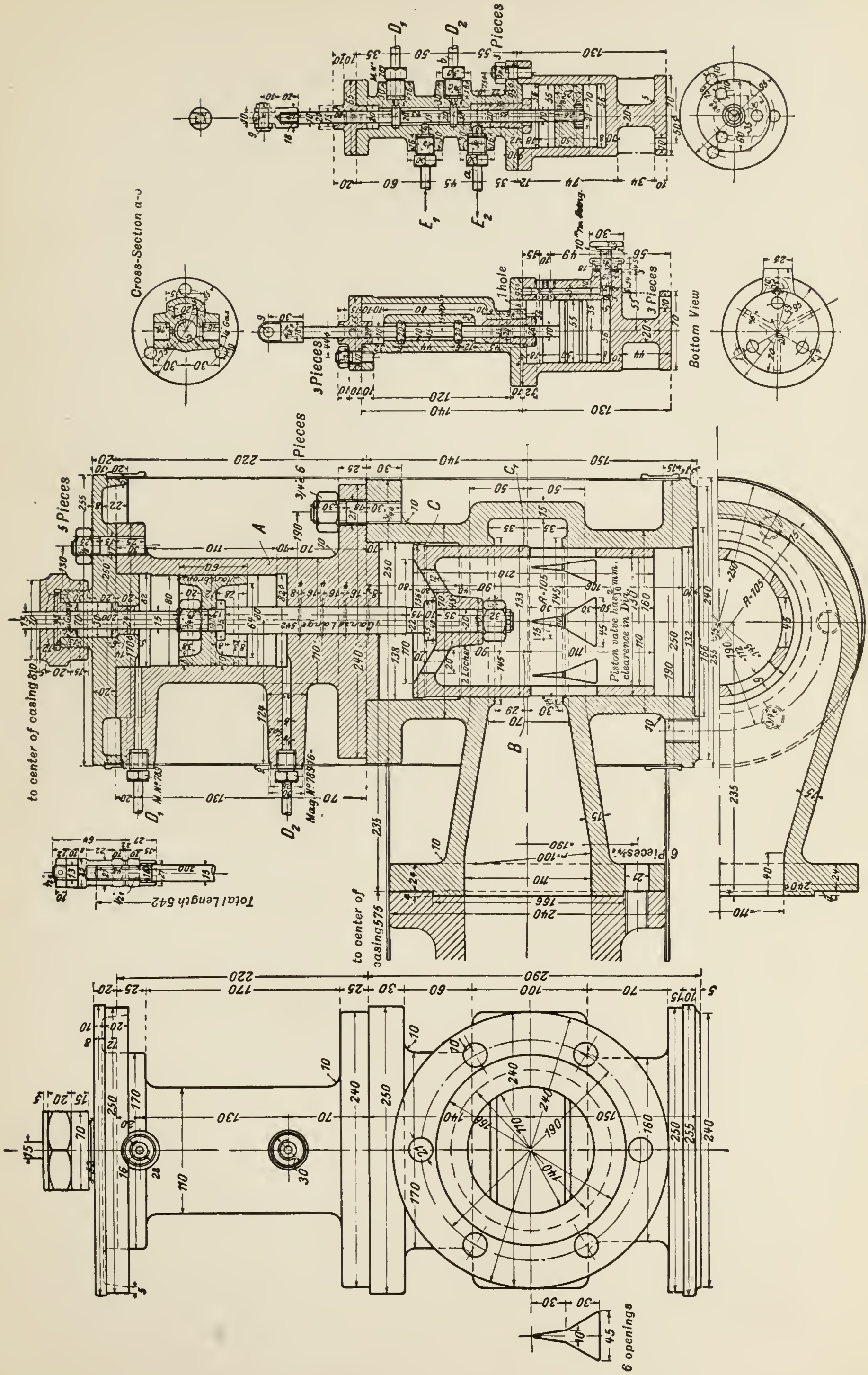


Fig. 159.

of the blades is discussed on page 145. The guide arrangements, made with the same high degree of accuracy, are shown on page 154. A distinguishing feature is the radial *divergence* of the rotating blades, which makes possible the use of smaller exit angles, and *Zölly* deserves the distinction of being the first to draw away from the usual *Laval* form of equal entrance and exit angles.

The shaft is supported in three bearings, and is held axially by a trust bearing placed at the low-pressure end. At the high-pressure end is found the driving mechanism of the governor.

* The Pelton buckets of the first design were not here used.



Dimensions here given are in mm.
Fig. 160.

The regulation is brought about indirectly by interposing a steam cylinder A (Fig. 160) whose piston is directly connected with the throttling valve B. The latter has triangular slots, so that the regulation is sufficiently sensitive with small loads. The ground-in valve has a steam-tight face $C\,C_1$, to prevent racing due to leakage. The movement of the steam-piston is controlled by an auxiliary valve (Fig. 160), which receives oil under pressure through tube E_1, and leads it through the tubes D_1 and D_2 that are connected to the ends of the cylinder. From Fig. 158 can be seen the application of the "floating lever" to this governing device, by having the middle of the regulating lever attached to the governing valve, one end to the steam piston, and the other end to the governor proper. The oil-brake is connected with the regulating valve. Fig. 159 shows clearly the pipe arrangement for cold water and for oil under pressure. The oil is delivered by a rotary pump driven from the governor spindle. The frame serves at the same time as a reservoir and as a cooler for the lubricating oil, and contains a system of cooling tubes for the latter purpose. The bearings and caps are separately cooled.

A steam turbine set up in the shops of the *Escher Wyss & Cie.* Machine Works, system *Zölly*, for a normal load of 500 h. p.$_e$ with 10 atmospheres boiler pressure (161.7 pounds per square inch absolute) and 3 000 r. p. m., was tested by the author in conjunction with *H. Wagner*, director of the municipal electric works of Zürich, and *Prof. Weiss* of the Polytechnikum (who attended to the electrical measuring instruments). The resulting data is given collectively in the following table. The turbine delivered its power to a direct-connected *Siemens & Halske* three-phase dynamo, whose field current was derived from an outside source; the corresponding power (product of field-current amperage and voltage at the binding-posts of the machine) was deducted from the total power of the generator. For condensing the steam a surface condenser was used with an independently steam-driven air pump. The cooling water was taken partly from the city water supply and from an electrically driven circulating pump drawing water from the well at the works. The consumption of power by the condenser was difficult to determine, and was not taken into account in the steam consumption figures.

The pressure and the temperature of the steam was measured

directly in front of the steam separator at the turbine because, for ordinary reasons, the observation at the entrance (throttle-valve) was useless. The vacuum was measured directly with a mercury column whose readings were reduced to 0° C. (32° F.) temperature, as this correction was found necessary on account of the high temperature of the engine room. Measuring the feed water was useless, as the boiler was being used for other purposes at the same time; and for these reasons we limited ourselves to weighing condensed steam from the air pump, which flowed into an elevated reservoir and thence to a reservoir on a scale.

That we had reached the condition of steady running was proved by finding an equal quantity of condensed steam during equal times, and also by the steadiness of temperature of certain exposed parts of the turbine, at the foot of the housing of the high-pressure end, and at a sight hole at the low-pressure end. These measurements were an externally fine indication of the steadiness of the internal temperatures.

Experiments 1 to 8, inclusive, were taken with decreasing load with as nearly constant r. p. m. and steam pressure as was possible. The experiments were conducted in the reverse order; that is, beginning with no load; and the temperature at the foot of the high-pressure housing showed a steady rise; that is, full steady-running conditions were not reached. Such a condition would only be arrived at after hours of running, as is especially the case when running light, experiment No. 8. In this experiment, the machine was run for about 20 minutes at half-load in order to heat it up, and it showed after two hours that the temperature at the foot was still decreasing. With higher loads the balancing would take place sooner, and with only 15 minutes preliminary running, the actual experiment could begin.

Experiments 9, 10 and 11 were conducted with admission pressure kept constant, and with the greatest possible variations in speed to determine how and in what quantities the volume of steam flowing through per hour changes with the revolutions. The voltage of the dynamo could not be brought to the desired value on account of the low velocity, and hence the load had to be decreased. It was found that the through-flowing steam volume was practically independent of the number of revolutions.

Experiments 12 to 15 were used to determine the influence of

EXPERIMENTS WITH A

No.			Saturated Steam.					
1.	Experiment No.		1	2	3	4	5	6
2.	Date		21 D 03	25 Ja.04	25 Ja.04	25 Ja.04	25 Ja.04	18 Ja.04
3.	Began		3 hr. 10	3 hr. 15	3 hr. 55	2 hr. 45	1 hr. 30	4 hr. 00
4.	Ended		6 hr. 10	4 hr. 35	4 hr. 45	3 hr. 35	2 hr. 20	5 hr. 00
5.	Duration	Minutes	180	80	50	50	50	60
6.	Total power	kw.	363.78	388.47	335.31	240.78	182.85	80.62
7.	Excitation, volt amperes	kw.	0.72	0.82	0.80	0.68	0.63	0.49
8.	Useful power (subtracting excitation, but not subtracting work of air-pump)	kw.	363.06	387.65	334.51	240.1	182.22	80.13
9.	No. of revolutions	Per minute	2 967	2 967	2 977	2 983	2 984	2 995
10.	Pressure (At entrance to steam separator)	Atm. absolute	11.16	11.16	10.90	11.01	10.97	11.04
10a.	*Pressure* (At entrance to steam separator)	*Lb. per sq. in. abs.*	164.0	164.0	160.2	161.8	161.2	162.26
11.	Temperature (At entrance to steam separator)	C.°	187.2	187.6	184.7	185.3	185.1	184.9
11a.	*Temperature* (At entrance to steam separator)	*F.°*	369.0	369.7	364.5	365.5	365.2	364.8
12.	Temp. of saturation (At entrance to steam separator)	C.°	183.7	183.7	182.6	183.1	182.9	183.2
12a.	*Temp. of saturation* (At entrance to steam separator)	*F.°*	362.7	362.7	360.7	361.6	361.2	361.8
13.	Superheat (11) — (12) (At entrance to steam separator)	C.°	3.5	3.9	2.1	2.2	2.2	1.7
13a.	*Superheat (11a) — (12a)* (At entrance to steam separator)	*F.°*	6.3	7.0	3.8	4.0	4.0	3.1
14.	Pressure (At entrance to I. guide wheel)	Atm. absolute	(10.1)?	10.11	9.03	6.92	5.47	3.07
14a.	*Pressure* (At entrance to I. guide wheel)	*Lb. per sq. in. abs.*	(148.4)?	148.6	132.7	101.7	80.39	45.12
15.	Temperature (At entrance to I. guide wheel)	C.°	179.9	180.0	175.1	164.9	156.6	136
15a.	*Temperature* (At entrance to I. guide wheel)	*F.°*	355.8	356.0	347.2	328.8	313.9	276.8
16.	Temp. of saturation (At entrance to I. guide wheel)	C.°	178.9	179.4	174.5	163.6	154.4	133.6
16a.	*Temp. of saturation* (At entrance to I. guide wheel)	*F.°*	354.0	354.9	346.1	326.5	309.9	372.5
17.	Superheat (15) — (16) (At entrance to I. guide wheel)	C.°	1.0	0.6	0.6	1.3	2.2	2.4
17a.	*Superheat (15a) — (16a)* (At entrance to I. guide wheel)	*F.°*	1.8	1.1	1.1	2.3	4.0	4.3
18.	Pressure at exit from I. guide wheel	Atm. absolute	6.03	6.32	5.59	4.29	3.44	1.84
18a.	*Pressure at exit from I. guide wheel*	*Lb. per sq. in. abs.*	88.62	92.89	82.16	63.05	50.56	27.04
19.	Pressure in connecting pipe	Atm. absolute	1.068	1.11	0.982	0.739	0.58	0.32
19a.	*Pressure in connecting pipe*	*Lb. per sq. in. abs.*	15.7	16.31	14.43	10.86	8.524	4.703
20.	Pressure in exhaust pipe	Atm. abs.	0.0715	0.0721	0.0679	0.0657	0.0661	0.0521
20a.	*Pressure in exhaust pipe*	*Lb. per sq. in. abs.*	1.051	1.059	0.9979	0.9656	0.9714	0.7656
21.	Temp. in exhaust pipe	C.°	39.1	39.9	38.9	37.1	36.6	32.7
21a.	*Temp. in exhaust pipe*	*F.°*	102.4	103.8	102.0	98.8	97.9	90.9
22.	Pressure in condenser	Atm. abs.	. . .	0.046	0.0471	0.051	0.053	0.044
22a.	*Pressure in condenser*	*Lb. per sq. in. abs.*	. . .	0.6761	0.6921	0.7495	0.7789	0.6467
23.	Temp. of condensed steam — Pipe	C.°	22.5	22.4	22.2	22 8	24.1	. . . .
	Temp. of condensed steam — Tank	C.°	23.9	23.9	24.8	26.2	26.8	23.6
23a.	*Temp. of condensed steam — Pipe*	*F.°*	72.5	72.3	72.0	73.0	75.4	. . . .
	Temp. of condensed steam — Tank	*F.°*	75.0	75.0	76.6	79.2	80.2	74.5
24.	Barometer reading	Mm. Hg.	736	731	730	730	730	733
25.	Total Steam Consumption per hour	Kg.	3 585	3 776.6	3 368.5	2 621.0	2 124.2	1 202.0
25a.	*Total steam consumption per hour*	*Lb.*	7 903.5	8 325.8	7 426.4	5 778.4	4 682.6	2 649.9
26.	**STEAM CONSUMPTION PER USEFUL KW. HOUR**	Kg.	9.874	9.742	10.070	10.916	11.657	15.00
26a.	Steam Consumption per useful kw. hour	*Lb.*	21.768	21.477	22.201	24.065	25.699	33.069
27.	Theoretical steam consumption per kw. referred to condition of steam at entrance to steam separator and vacuum in exhaust pipe	Kg.	4.885	4.887	4.873	4.835	4.85	4.702
27a.	*Theoretical steam consumption per kw. as above*	*Lb.*	10.769	10.774	10.743	10.659	10.692	10.366
28.	Thermodynamic efficiency	%	52.3	50.2	48.4	44 3	41.6	31.3

500 H. P. ZÖLLY TURBINE.

		Variable Number of Revolutions.							Poor Vacuum.		Superheated Steam.			Poor Vacuum.
		Low Power.			Normal Power.									
7	8	9	10	11	12	13	14	15	16	17	18	18a	19	20
25Ja.04	25Ja.04	26Ja.04	26Ja.04	25Ja.04	26Ja.04	26Ja.04	26Ja.04	26Ja.04	26Ja.04	26Ja.04	5 F.04	5 F.04	5 F. 04	5 F.04
11hr.25	10hr.35	1 hr. 45	11hr.35	10hr.10	4 hr. 50	5 hr. 02	5 hr. 15	5 hr. 32	5 hr. 55	6 hr. 19	3 hr.50	3 hr.5 0	11 hr.15	5 hr. 35
12hr.25	11hr.10	2 hr. 35	12hr.35	11hr.10	4 hr. 55	5 hr. 12	5 hr. 23	5 hr. 42	6 hr. 10	6 hr. 30	5 hr.00	4 hr.10	12 hr.35	5 hr.45
60	35	50	60	60	5	10	8	10	15	11	70	20	80	10
....	....	296.4	280.03	243.15	397.4	400.6	404.4	375 2	289.25	319.42	392 5	390.41	391.2	306.21
0.497	....	0.498	0.511	1.09	(0.8)	(0.7)	(0.5)	(1.1)	0.55	0.74	0.81	0 806	0.816	0.78
....	....	295.9	279.52	242.06	(396.6)	(399.9)	(403.9)	(374.1)	288.7	318.68	391.66	389 6	390.4	305.43
2 995	3 000	3 229	2 430	1 890	3 048	3 122	3 229	2 649	2 982	2 982	2 972	2 973	2 968	2 960
11.03	11.19	11.12	10.61	11.00	10.87	11.03	11.13	10.71	10.54	10.48	11.81	13.13	11.26	(10.23)
162.1	164.5	163.4	155.9	161.7	159.8	162.1	163.6	157.4	154.9	154.0	188.3	193.0	165.5	(154)
184.9	185.7	188.5	188.2	192	189.1	190.0	190.6	184 9	184.6	183.7	247.1	258 .5	226.6	247.7
364.8	366.3	371.3	370.8	374.4	372.4	374.0	375.1	364.8	364.3	362.7	476.8	497.3	439.9	477.9
183.15	183.8	183.5	181.57	183.05	182.5	183.15	183.68	181 9	181.2	180.95	189 95	191.02	184.1	179.9
361.67	362.8	362.3	358.83	361.49	350.5	361.67	362.62	359.4	358.2	357.71	373.91	375.84	363.4	355.8
1.8	1.9	5.0	6.7	7.2	6.6	6.9	7.0	3.0	3.4	2.8	57.2	67.5	42.5	67.8
3.2	3.4	9.0	12.1	13.0	11.9	12.4	12.6	5.4	6.1	5.0	103.0	121.5	76.5	122.0
1.22	0.747	7.96	7.96	7.96	10.08	10.08	10.08	10.08	9 41	9.48	9.72	9.72	9 80	9.43
17.93	10.98	117.0	117.0	117.0	148.1	148.1	148.1	148.1	138.3	139.3	142.9	142.9	144.0	138.6
108.8	102.9	171.2	172.0	172.2	180	180.1	180.2	179 2	176.7	176 9	216.5	219	216.5	224.5
227.8	217.2	340.2	341.6	342.0	356.0	356.2	356.4	354.6	350.1	350.4	421.7	426.2	421.7	436.1
104.7	91.2	169.2	169.2	169.2	179.2	179.2	179 2	179.2	176 3	176.6	177 6	177.6	178.0	178.9
220.5	196.2	336.6	336.6	336.6	354.6	354.6	354.6	354.6	349.3	349.9	351.7	351.7	352.4	354.0
4.1	11.7	2.0	2.8	3.0	0.8	0.9	1.0	0.0	0 4	0.3	38.9	41.4	38.5	45.6
7.4	21.1	3.6	5.0	5.4	1.4	1.6	1.8	0.0	0.7	0.5	70.0	74.5	69.3	82.1
0.652	0.383	4.76	4.95	4.95	6.36	6.34	6.30	6.35	5 93	6.0	6.23	6.212	6.28	6.15
9.582	5.629	69.95	72.75	72.75	93.48	93.17	92.56	93.33	87.16	88.18	91.56	91.30	92.30	90.39
0.197	0.176	0.84	0.87	0.862	1.12	1.14	1.15	1.12	1.05	1.06	1 07	1.056	1.09	1.06
2.895	2.587	12.35	12.78	12.69	16.46	16.75	16.90	16.46	15.43	15.58	15.73	15.52	16.02	15.58
0.051	0.0514	0.0683	0.0665	0.0682	0.0696	0.0695	0.0692	0 0690	0.1922	0 137	0.0653	0.0664	0.0692	0.213
0.7495	0.7554	1.004	0.9772	1.002	1.023	1.021	1.023	1.014	2.825	2.013	0.9596	0.9759	1.017	3.130
32.2	42.1	38.5	38.0	38.5	39.6	39.5	39.1	39.2	59.3	51.8	38.0	38.8	38.0	61
90.0	107.8	101.3	100.4	101.3	103.3	103.1	102.4	102.6	138.7	125.2	100.4	101.8	100.4	141.8
0.044	0.046	0.051	0.046	0.048	....	....	....	....	....	....	0.040	0.042	0.042	0 203
0.6467	0.6761	0.7495	0.6761	0.7053	....	....	....	....	....	....	0.5879	0.6172	0.6172	0.2983
16.5	16.5	23.3	21.8	21.1	....	....	....	....	....	....	20.2	20.5	20.4	44.25
26.2	27.1	25.3	23.2	23.3	....	....	....	....	....	....	22.4	22.4	23.7	34.15
61.7	61.7	73.9	71.2	70.0	....	....	....	....	....	....	68.4	68.9	68.7	111.65
79.2	72.8	77.5	73.8	73.9	....	....	....	....	....	....	72.3	72.3	74.7	93.47
730	731	731	731	731	731	731	731	731	731	731	715	715	715	715
465.0	295.4	2 980.1	2 978.4	2 974.9	(3 770)	(3 770)	(3 770)	(3 770)	(3 500)	(3 516)	3381.1	3327.0	3 505.7	(3 225)
1 025.2	651.24	6 569.9	6 566.1	6 558.6	(8 311.3)	(8 311.3)	(8 311.3)	(8 311.3)	(7 716.2)	(7 751.4)	7 454.0	7 334.7	7 728.8	(7 109.8
....	....	10.07	10.653	12.29	(9.50)	(9.43)	(9.33)	(10.08)	(12.12)	(11.03)	8 633	8.339	8.98	(10.56)
....	....	22.20	23.486	27.094	20.94	20.79	20.57	22.222	26.719	24.317	19.032	18.384	19.797	23.281
....	....	4.825	4.876	4.8[illegible]6	4.867	4.855	4.843	4.897	5.87	5.60	4.46	4.41	4.683	5.642
....	....	10.637	10.749	10.683	10.730	10.703	10.677	10.796	12.941	12.346	9.833	9.722	10.324	12.438
....	....	47.9	45 8	39 4	(51.2)	(51.5)	(51.8)	(48.5)	(48.4)	(50 8)	51.7	51.3	52.2	(53.4)

increased or decreased number of revolutions on the efficiency at normal load. The experiments can, from what has been previously stated, be limited to a short interval; it was sufficient to keep the admission pressure constant, and to read the electrical power and number of revolutions. The steam consumption might be determined by interpolation. If we plot the derived points as a function of the number of revolutions, we would get a tangent to the so-called power-parabola, from which the parabola itself may be constructed.

Experiments 16 and 17 refer to an artificially lowered vacuum which was decreased 87% and 81%. Because of lack of time, the steam consumption was also determined by interpolation, according to Zeuner's formula.

Experiments 18 to 20 were conducted with superheated steam. From experiment 18, an interval of 20 minutes was taken as experiment 18*a*, during which the highest mean temperature of 258.5° C. (497.3° F.) existed. Experiment 18 is a mean of observations taken over an interval of 70 minutes.

Experiment 20 was conducted with superheated steam but poor vacuum, and the steam consumption must again be calculated by interpolation. The values that were not directly observed are put in parentheses in the table.

The following may be pointed out as the most important results of these experiments:

If the turbine at constant boiler pressure and constant number of revolutions is loaded more and more, the useful power at the brushes of the dynamo increases in almost direct ratio with the absolute pressure of admission measured at the first guide wheel. It reaches the 0 value (if we neglect the consumption of work for producing the heat of the field current, about 0.5 kw.) with admission pressure at running light (with excitation) *i.e.*, at 1.22 kg. per sq. cm. (17.34 lb. per sq. in.), absolute.

The quantity of steam flowing through the turbine per unit time increases with the absolute admission pressures, only approximately directly. Rather, the value of the ratios of the volume of steam per hour to the admission pressure may be taken as linear, for it decreases in the ratio of about 106% running light (without excitation) to 100% at full load. In experiment No. 1 the pressure of admission to the first guide wheel was not taken into consideration,

as it may have been too large. If the ratio of the steam weight per hour to the pressure existing after the first guide wheel is taken, the derived values agree satisfactorily with each other. If the steam is superheated, at equal pressure of admission and vacuum, a less steam weight flows through than if it were saturated.

The consumption of steam running light, with dynamo connected but without excitation, was only 7.84% of the consumption of saturated steam for full load. Including excitation, the consumption was only about 12.5%.

If the vacuum decreases about 0.01 kg. per sq. cm. (0.142 lb. per sq. in.), in round numbers 1%, the consumption of saturated steam increases about 1.8% of its value, beginning at 0.06 kg. per sq. cm. (0.853 lb. per sq. in.). With superheated steam this increase is less, and is about 1.5%.

The steam volume per hour hardly changes more than 1% in going from 3 229 r. p. m. to 1 800 r. p. m., provided the admission pressure remains constant. It is therefore, practically speaking, constant.

The speed of 3 000 r. p. m. at normal admission pressure (10.0 kg. per sq. cm., 142.23 lb. per sq. in. absolute) lies somewhat be low the most favorable value. An increase of the r. p. m. from 3 048 to 3 229, other conditions remaining the same, increases the power from 396.6 kw. to 403.9 kw.; that is, a gain of 7.3 kw., or about 1.9%.

The best results were obtained from experiment No. 18*a* with 8.539 kg. steam consumption per effective kw. hour (18.825 lb. per kw. hour) at 258.5° C. (497.3° F.) steam temperature; that is, 67.7° C. (121.8° F.) superheat. Comparing this with the consumption, using "dry" saturated steam, and equal initial pressure, gives the superheat gain as 1% in steam consumption for each 5° C. (9° F.) difference between the true and saturation temperatures of the steam.

All experiments took place without interference. The vibrations of the turbine shaft were minimum, practically nil. The bearings were fed with oil at 30° to 35° C. (86° to 95° F.) temperature, and the oil flowed away at 40° to 50° C. (104° to 122° F.).

The consumption of power for the circulating pumps and the

air pump at 0.06 kg. per sq. cm. (0.853 lb. per sq. in.) back pressure was estimated by the builder of the turbine from self-made measurements, at 3% of the normal power.

60. THE CURTIS TURBINE.

The original patent of *Curtis* consisted of various construction forms of a 4 to 6 stage-impulse turbine, which practically consisted of as many successive Laval turbines. Since then the construc-

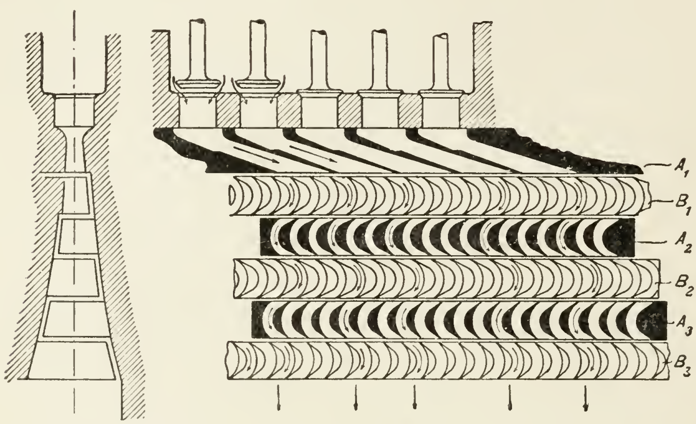


Fig. 161.

tion of this system has been taken up by the General Electric Co. (Schenectady, N. Y.) and further developed by them.* Fig. 161 shows a development of the nozzle and blade profiles. The steam leaving axially the nozzle A_1, strikes next the first rotating wheel B_1; at exit from the latter it possesses still such a considerable velocity that it is worth while sending it through a second guide wheel system A_2 to a second rotating wheel B_2, and to again repeat this process with guide wheel A_3 and rotating wheel B_3. The rotating blades B_1 B_2 B_3 can be fastened to the rim of a single disc wheel. In order that the wheel may not be subject to an excess

* According to an article in the Electrical World and Engineer, 11 April and 23 May, 1903, as well as reports from the builders.

pressure there must exist equilibrium of pressure on both its sides. The steam must therefore expand to this pressure in the nozzle. The flow through the blade channels occurs then with constant pressure, the steam exerting only its kinetic energy. The velocity decreases greatly, first because work is being delivered, and again because of friction; the blade channel cross-section must increase

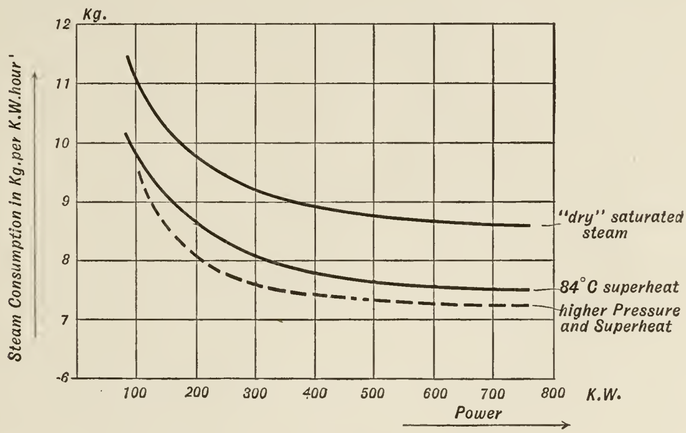


Fig. 162.

in inverse ratio. By drawing a velocity diagram it can be shown that this increase is quite considerable, and cannot be brought about in any other way than by widening the blade channels. Even then we are forced to work with comparatively large angles, or the widening would be too great, and there may be doubt as to the steam remaining in contact with the diverging side walls.

The turbine consists of two or more equal groups mounted successively. A 600 kw. machine works at 1 500 r. p. m. and about 130 meters (426.5 feet) peripheral velocity. The nozzles are arranged closely in groups so that, according to reports, the steam flows to the wheels in one wide strip. The regulation is accomplished by closing individual nozzles, but only for entrance into the first wheel; the flow to the other groups remains unchanged.

Fig. 162 represents the steam consumption for saturated and

slightly superheated steam per kw. hour, taken from the reports of the builders of the 600 kw. machine. The dotted curve shows the results the builders expect with higher pressures and greater degree of superheat. It was mentioned that the curve for slight superheat was not found for the 600 kw. machine, but was calculated from results derived from smaller units. A consumption of about 8.6 kilograms (18.96 pounds) saturated steam per kw. hour would be a performance that would place this turbine in the same class as other systems.

The *British Thomson-Houston Co.* in *Rugby* give the following results for their make of Curtis turbine, rated at 500 kw. At maximum power:

Steam pressure at Turbine . . .	10.55 kg. per sq. cm. (150.05 lb. per sq. in.)
Superheat . . .	64° C. (115.2° F.)
Vacuum pressure, absolute . . .	0.0516 kg. per sq. cm. (0.734 lb. per sq. in.)
Revolutions per minute . . .	1 800
Power	660 kw.
Steam consumption per kw. hour .	8.35 kg. (18.41 lb.)

It was not stated whether the work of running the condenser was included in the above. At 470 kw. load the air pump needed only 1.8 kw. and the circulating pump 7.1 kw.; in all, only 1.9% of the consumption of power.

The reports of the *increase of steam consumption with decrease of vacuum* are of especial importance. Between about 0.025 and 0.10 kg. per sq. cm. (0.36 to 1.42 lb. per sq. in.) absolute condenser pressure, this increase was 2.3% of the initial value for each 0.01 kg. per sq. cm. (0.142 lb. per sq. in.) decrease of vacuum; between about 0.1 and 0.35 kg. per sq. cm. (1.42 to 4.98 lb. per sq. in.), about 1.5% for each 0.01 kg. per sq. cm. (0.142 lb. per sq. in.)

The decrease of steam consumption with increase of superheat is very nearly proportional to the difference between the actual tem-

perature and that of saturation, and is almost exactly 1% for each 5° C. (9° F.) superheat.

A comprehensive analysis of the steam consumption will be possible only when we know more about the value of the blade friction. If the same losses are assumed as occur in the Laval blade, then the given steam consumption cannot be reached. In general, it may be said that *Curtis* works with higher flow velocities, and hence also greater friction, than if the decrease of pressure were divided into as many stages as there are rotating wheels. On the other hand, the wheel friction is decreased by combining 2 or 3 rotating wheels into one single disc, which rotates in steam of lower density than would be the case if the wheel were divided into pressure stages as above suggested. The work of running light is further decreased by having a vertical construction with the dynamo placed above, as is the preference of the General Electric Co., so that the bearing friction, which is nearly proportional to the area of the bearing surfaces, does not need to be considered.

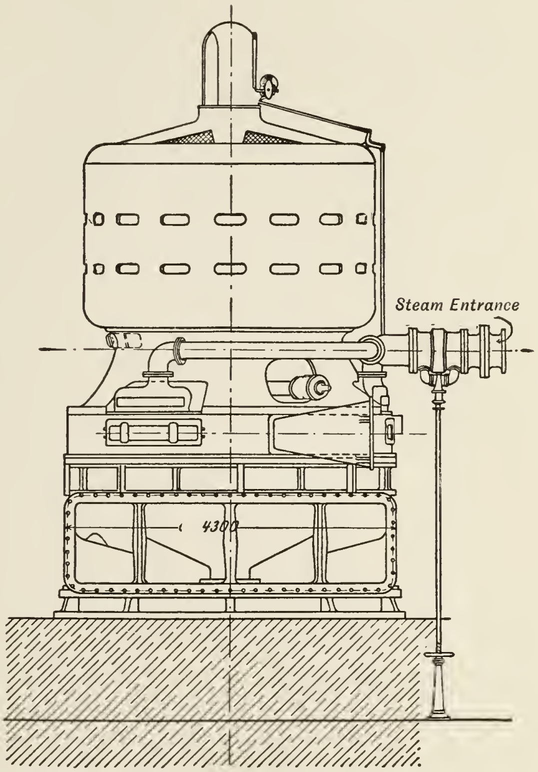
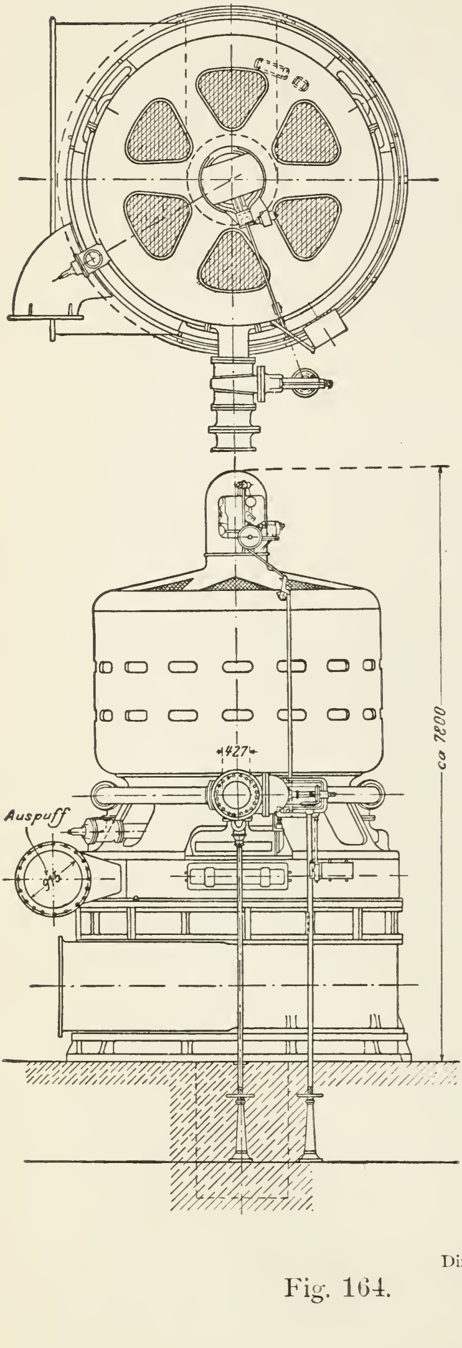
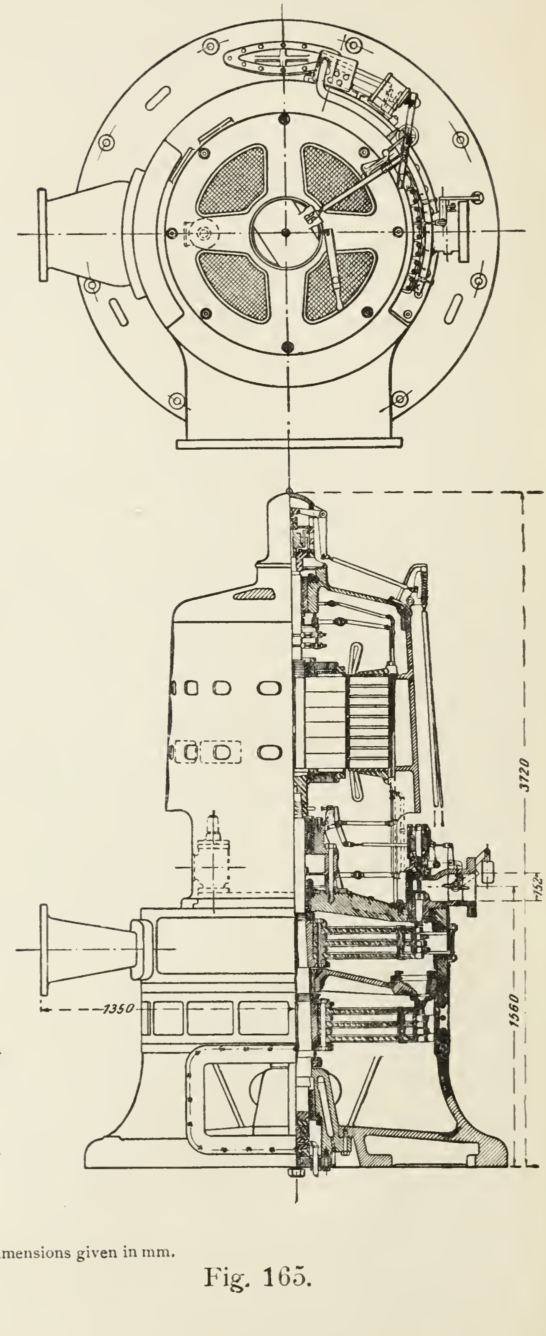


Fig. 163.

The illustrations show the construction; Figs. 163 and 164 are outside views of a 5 000 kw. turbine giving a few principal dimensions, from which the very small space required by the machine





Dimensions given in mm.

Fig. 164. Fig. 165.

may be seen. The weight of this turbine with generator is said to be about $\frac{1}{8}$ of the weight of a steam engine and generator of approximately equal capacity, in the power house of the Manhattan Railway Co. of N. Y. City.

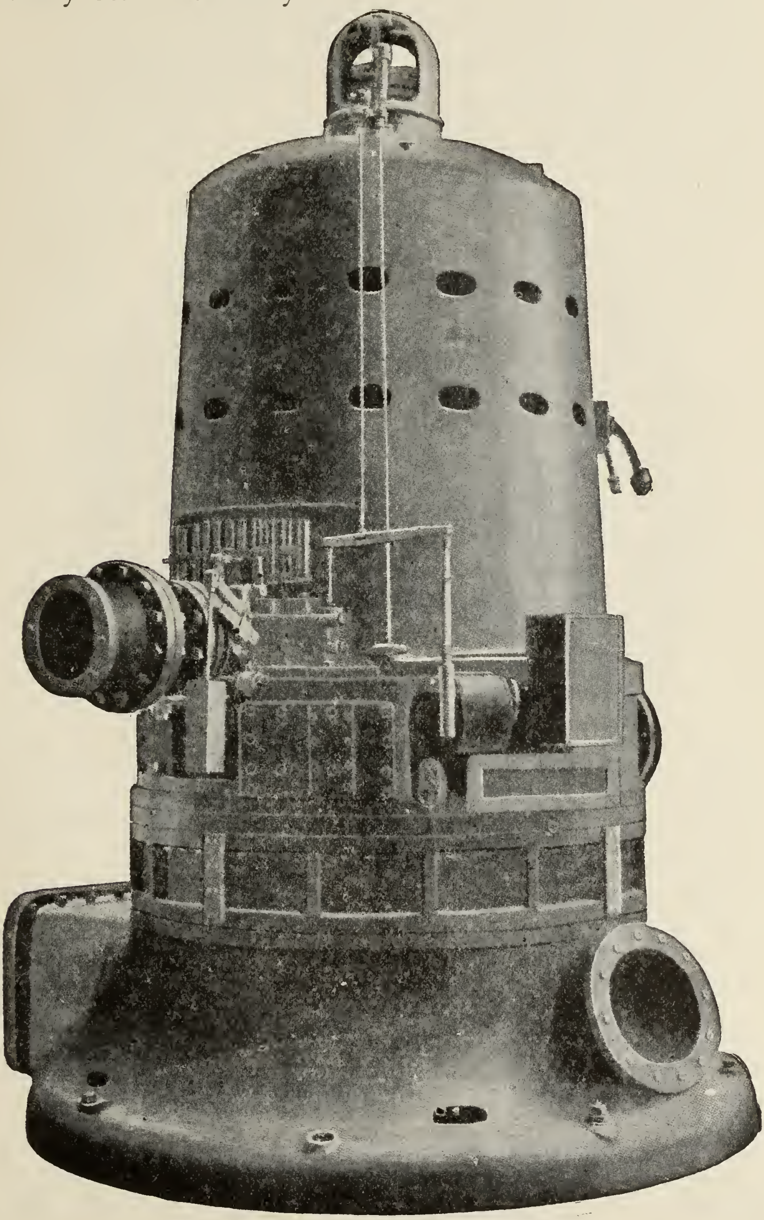


Fig. 166.

The weights and the speeds of some units are given as follows:—

Power .	kw.	15	500	1 500	1 500	5 000
Revolutions per minute, .		3 000	1 800	1 800	800	500
Wt. with dynamo	Kg.	830	16 400	43 000	55 000	55 000–175 000
	Lb.	**1 830**	**36 160**	**94 800**	**121 250**	**121 250–385 800**

The weight, referred to unit power, is exceedingly small. The vertical section, plan view, and outside view, shown in Figs. 165

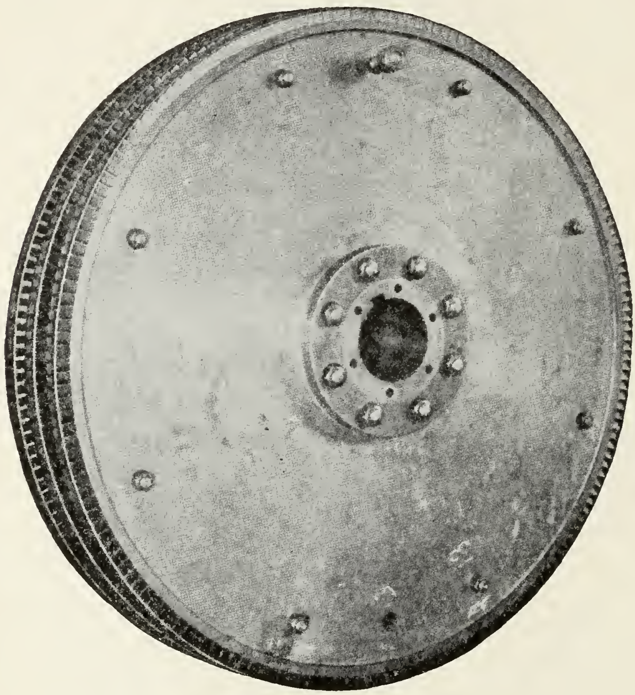


Fig. 167.

and 166 are of a small unit (about 500 kw.), which is constructed in two main stages, each with three velocity stages. The wheels, according to the drawing, may be conceived as discs of uniform thickness which are attached to the hub by axial bolts. The illustration shows clearly with what freedom the steam plays around the intermediate guide wheels, as any especial steam tightness would be useless. The construction of the blades was explained on page 148. Fig. 167 represents a rotating wheel with three blade-systems. The governor balls for the high pressure nozzles

are loaded with opposing pistons, and are moved by a solenoid that regulates the admission of steam to the operating piston. The governor is placed in a casing above the generator. Another and simpler safety governor closes a valve placed in the steam pipe when the normal velocity is exceeded. The coupling between turbine and dynamo shafts, consisting of a cone fit and a cross-key, is peculiar to this machine. This construction is not used on account of any lack of strength in the particularly strong shaft, but aids considerably in saving vertical space. The total weight of the rotating parts is carried by a step bearing with cast-iron friction plates, with abundant pressure lubrication. This construction is used a great deal in water turbines, but is required to satisfy even greater demands for steam turbines, on account of their higher velocities. Fig. 165 shows a foot-plate made of several parts, probably with the intention of decreasing the friction velocities by having several plates turn one upon the other, which is perhaps possible, provided oil is freely supplied. Curtis also uses, as Fig. 168 shows, foot-bearing constructions with the usual simple bearing plate. In both cases the oil under pressure also lubricates

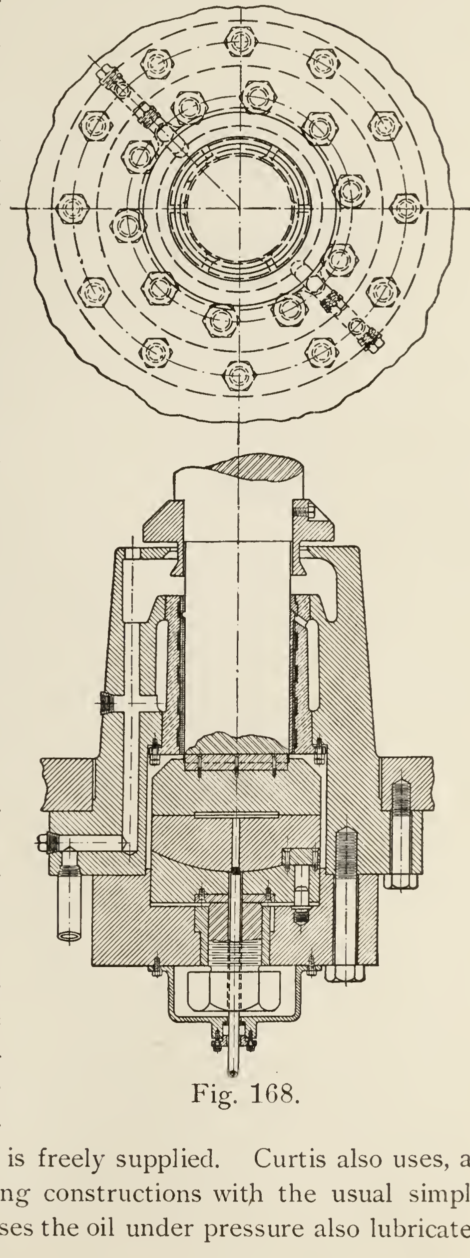


Fig. 168.

the adjoining journal bearings which in Fig. 165 is shown provided with an outer water cooling device.

Of late the multiple-disc wheels are being supplanted by single discs with decided increase of thickness towards the hub, by which

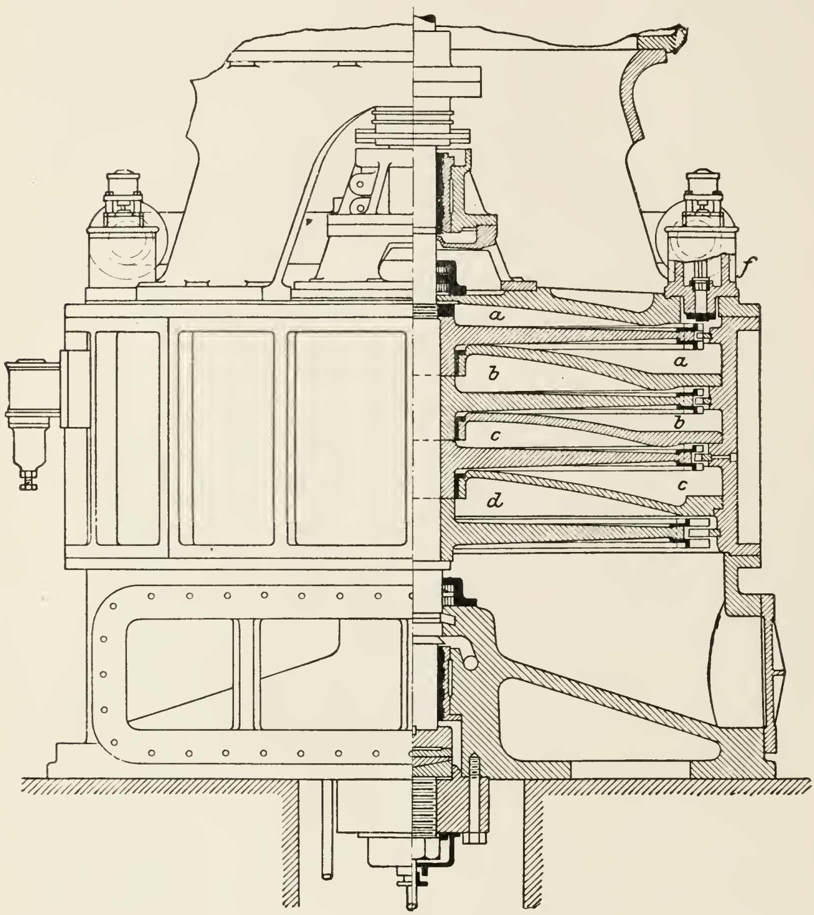


Fig. 169.

with equal weights, according to the formulæ of stability in Article 39, a decided decrease of stress is obtained. The blade rings are separately attached, as may be seen in Fig. 169 (taken from *Engineering*). The important question as to how much the discs deflect due to their own weight, and how much they again straighten out during rotation, due to centrifugal force, will be investigated in

Chapter V. The considerably complicated construction of the casing only partly shown in Fig. 169 can be more fully seen in Fig. 170. It will be noticed that the chamber is built up of rings, which are in turn constructed of segments.

According to a lecture by *Emmet*, there is, for vertical turbines,

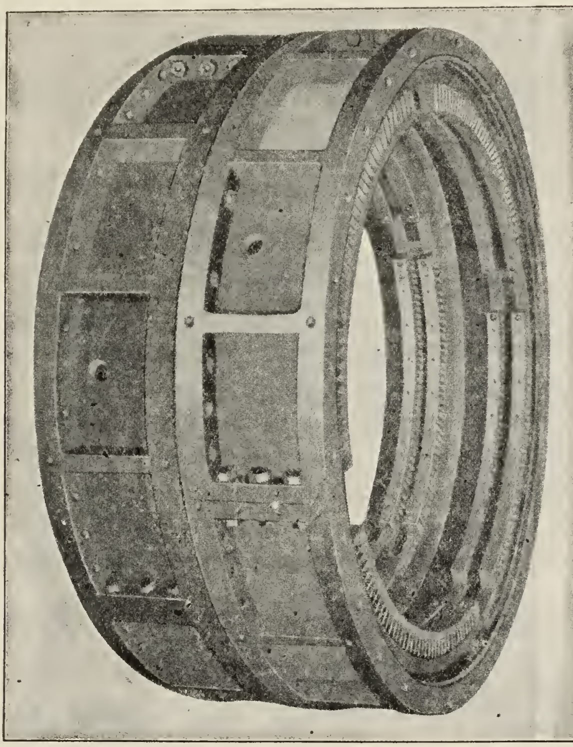


Fig. 170.

a cast-iron protection-plate, on which the rotating parts may sink and be brought to rest by friction in case the step bearing is worn down too far. For oiling other parts of the step bearing, oil cups are provided. Recently, large condensing turbines have been constructed, having four pressure stages for every two velocity stages.

According to pamphlet notices, the Allgemeine Elektrizitats-gesellschaft of Berlin have obtained the right of manufacturing the Curtis turbine for Europe.

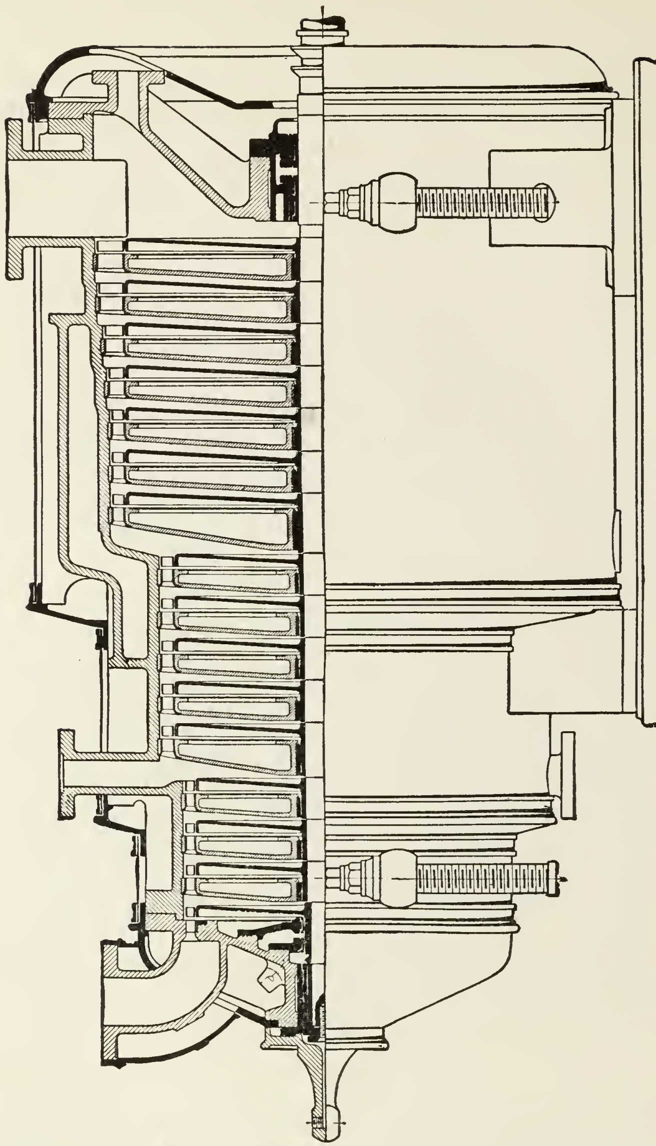


Fig. 172.

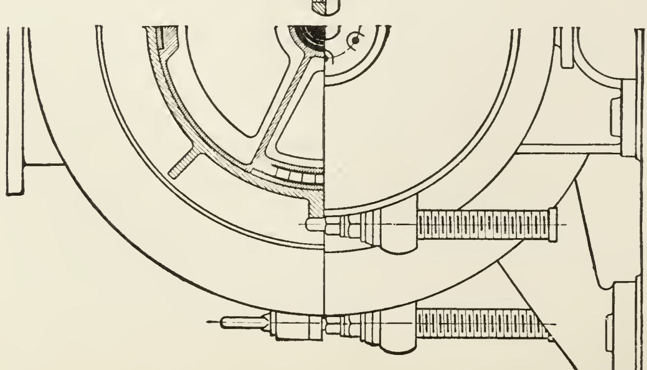


Fig. 171.

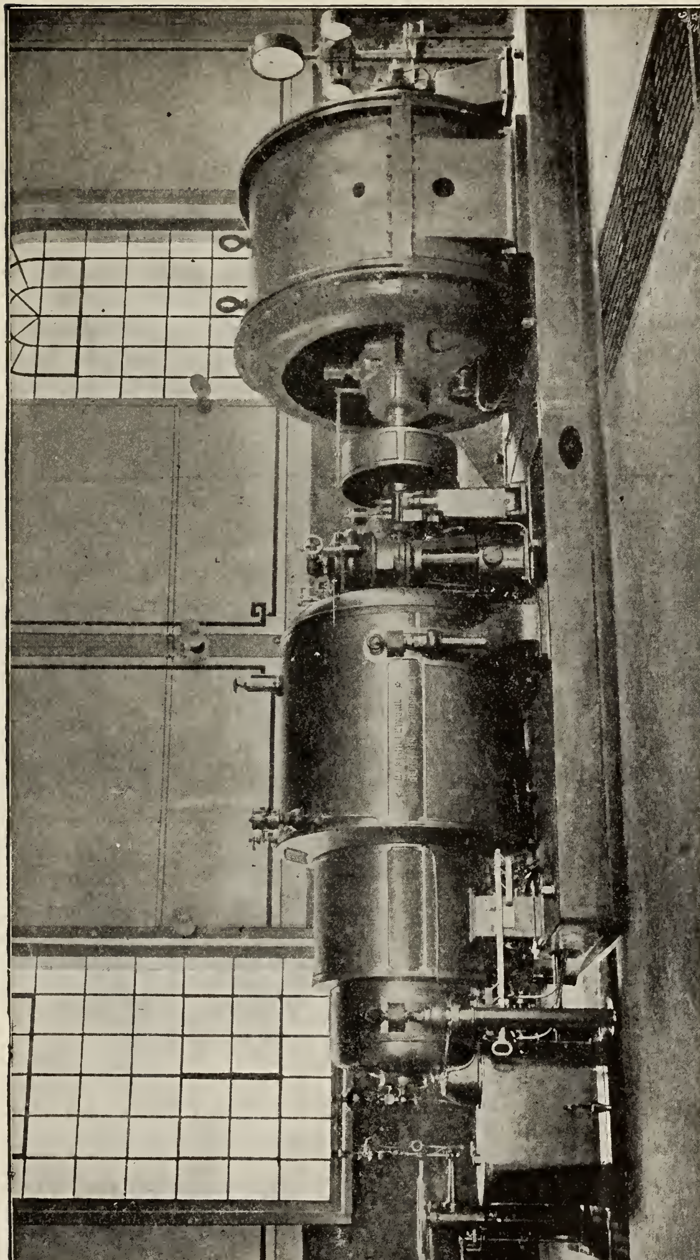


Fig. 173.

61. THE RATEAU TURBINE.

The *Rateau* turbine is purely an impulse turbine, using wheels of thin plates pressed into a slightly conical form. These are mounted on a common shaft and separated from each other by division walls. The first wheels have partial peripheral admission, so that the peripheral velocity may be high from the very beginning, without using too short blades. The guide blades are set

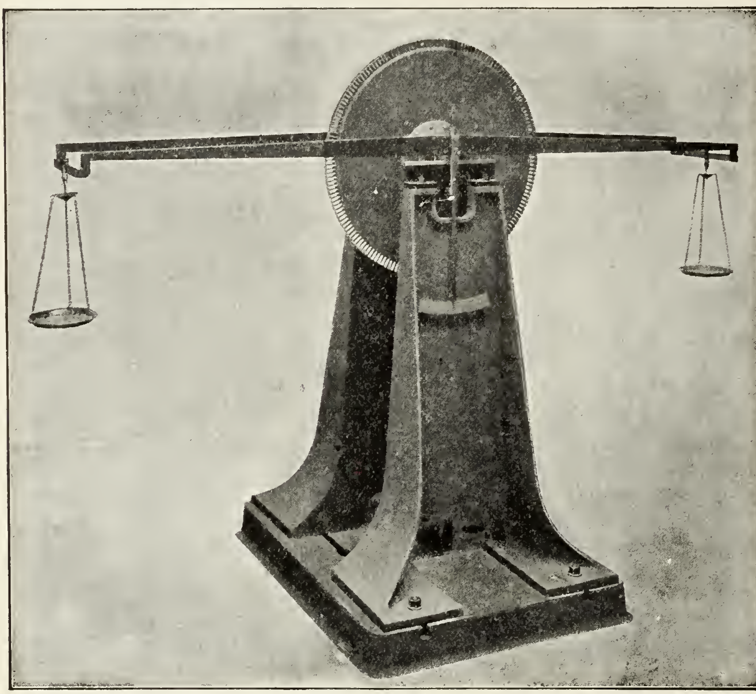


Fig. 174.

into division walls, the rotating blades are bent from one piece of bronze or steel plate, and riveted to the double turned rim of the wheel-disc. The shaft bearings were originally built as part of the cover of the turbine, but lately are made independent. At the low pressure end, the shaft is made steam-tight, by a simple stuffing box, into which enough water is allowed to flow to secure complete steam tightness. As the same pressure exists on both sides of

each rotating wheel, the axial thrust has only the small value due to the pressure on the area of the end of the front journal. Figs. 171 and 172 show sections through the machine in which it is to be noted that the wheel discs are riveted to their hubs. Fig. 173 gives an outside view of the turbine with generator and oil arrange-

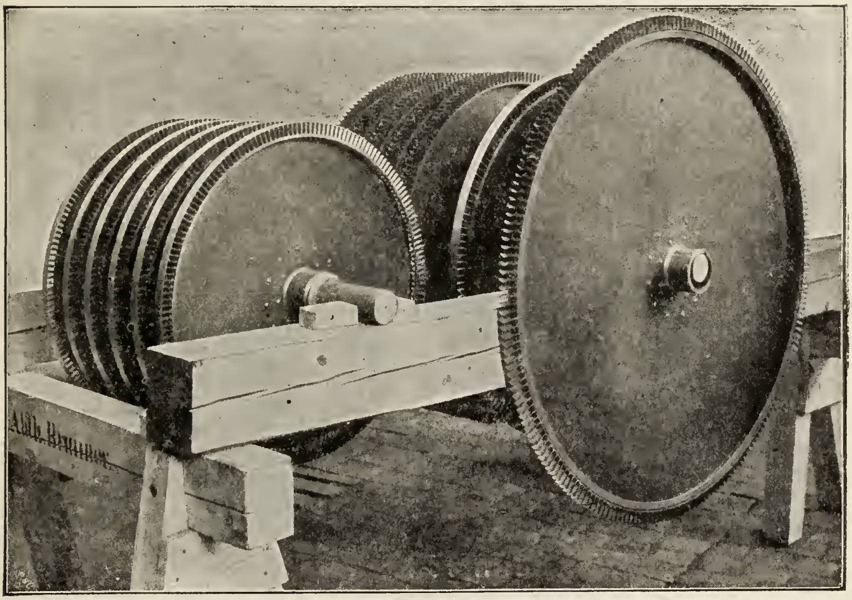
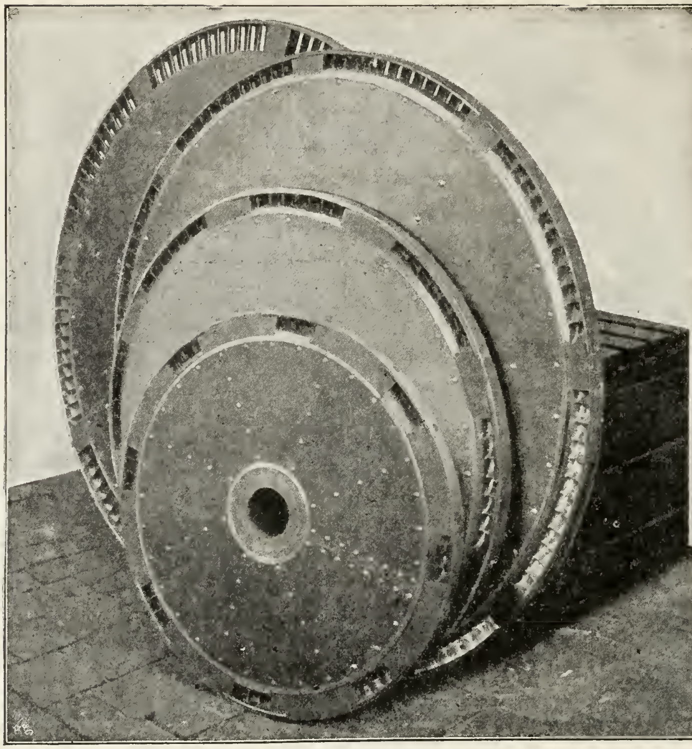
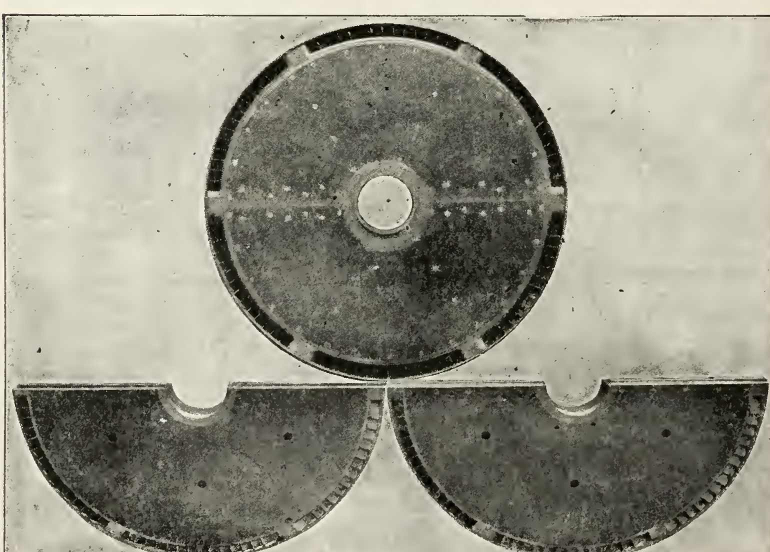


Fig. 175.

ment. The mass-balancing of the individual wheels is accomplished by means of the scale shown in Fig. 174, by balancing the discs in two positions 90° apart. The construction of the wheels and division walls can easily be seen in Figs. 175, 176, and 176*a*. The construction according to the latter figure, with division walls made in sections is preferred, because after taking away the casing cover, all inner parts can be freely gotten at.

The turbine as constructed by *Sautter, Harlé & Cie.* of Paris, is shown in Fig. 177, a 500 kw. machine. The shaft, both at the high pressure end and elsewhere is made steam tight by the stuffing box described on page 203; Figs. 178 and 178*a* show the governing device used with the so-called *Denis* compensator. K is a pendulum spring governor with a central spring K_1 and an





Figs. 176 and 176*a*.

TABLE 1.

EXPERIMENTS AT THE ORLIKON MACHINE WORKS WITH A 1 000 KW. RATEAU TURBINE.

	Power in Kilo-watts.	Pressure. In Boiler.		Pressure. At Entrance to First Guide Wheel.		Pressure. In Condenser.		Temperature at Entrance to First Guide Wheel.		Actual Steam Consumption per Kw. Hour D_{el}.		Theoretical Steam Consumption per Kw. Hour, D_0.		$\eta = \frac{D_0}{D_{el}}$.
		Kg. per Sq. Cm.	*Lb. per Sq. In.*	Kg. per Sq. Cm.	*Lb. per Sq. In.*	Kg. per Sq. Cm.	*Lb. per Sq. In.*	C.°	*F.°*	Kg.	*Lb.*	Kg.	*Lb.*	
1	194	13.1	186.32	2.14	30.44	0.078	1.11	148	298.4	14.5	31.97	7.36	16.22	0.504
2	425	10.9	155 03	4.06	57.74	0.083	1.18	155	311.0	11.3	24.91	6.22	13.71	0.552
3	659	11.3	162.72	5.99	85.30	0.140	1.99	162	323.6	10.8	23.81	6.31	13.91	0.583
4	871	12.7	180.64	7.89	112.22	0.222	3.15	175	347.0	11.2	24.69	6.48	14.29	0.578
5	1024	12.6	179.21	8.19	116.49	0.171	2.43	176	348.8	9.97	21.98	6.05	13.34	0.607

auxiliary spring *N*, used to regulate the revolutions. The connection between the regulating arm *L* and the throttle valve *E* would be direct if *S U V* were neglected. *S* (see Fig. 178*b* to a larger scale) has a constant rotation, which revolves the bevel gears *V* and *X* in opposite directions. The projecting key *U* on the spindle *P* is capable, for instance, of engaging with the slot on *V*, when spindle *P* is lifted, thereby causing it to rotate, so that the spindle screws into the nuts *T* and *Q* which are threaded left and right hand respectively, and this closes the throttle valve further than the governor desired. The velocity is therefore too greatly decreased, the governor returns to its former position, and the machine reaches (on account of the shortening due to the compensating gear) a new normal condition at the same number of revolutions as existed previously. The governor must be so built as to be extremely sensitive. The regulation for constant normal velocity is excellent.

The *Orlikon* Machine Works notified the author that they conducted in their experimental laboratory a steam consumption test of a Rateau turbine of 1 000 kw., shown in Fig. 173. Table 1, page 261, gives the resulting data.

The mean number of revolutions was 1,500. The theoretical consumption is referred to the condition of the steam at entrance to the first rotating wheel of the turbine. The slow increase of efficiency is noteworthy, and the reason is no doubt because at small powers the turbine is filled with steam at less pressure, and the work of fan resistance of the wheels decreases. At a better vacuum the *Orlikon* Works hope to reach a steam consumption of 8.4 kilograms (18.52 pounds) per kw. hour.

Sautter, Harlé & Cie. of Paris have lately installed a low pressure turbine which was tested by the experts *Sauvage* and *Picon* of Paris, according to whose reports (received 19th April, 1902) they worked in connection with the interesting and highly promising *Rateau* heat-accumulator. This accumulator is a sufficiently large iron casting which condenses the steam intermittently delivered from the preceding machines, and during the pauses the accumulated heat again evaporates the steam so that the turbine is kept constantly running. The turbine consists of seven wheels, each 800 millimeters (34.65 inches) in diameter. The experimental results and the calculated thermodynamic efficiency referred to the electric power are given collectively in the following table:

TABLE 2.

EXPERIMENTS OF SAUVAGE AND PICON WITH A RATEAU TURBINE.

	Number of Revolutions per Minute.	Power.			Pressure.				Temperature of Admission.		Actual Steam Consumption per Electrical H. P. Hour, D_{el}.		Theoretical Steam Consumption per Electrical H. P. Hour, D_0.		Thermodynamic Efficiency.
					At Turbine Entrance.		In Condenser.								
		Kw.	French h. p.	*English h. p.*	Kg. per Sq. Cm. Absolute.	*Lb. per Sq. In. Absolute.*	Kg. per Sq. Cm. Absolute.	*Lb. per Sq. In. Absolute.*	C.°	*F.°*	Kg.	*Lb.*	Kg.	*Lb.*	$\eta_{el} = \frac{D_0}{D_{el}}$.
1	1 610	Running Light, without Excitation.			0.136	1.934	0.087	1.237	111.4	232.5	(570 per hr.)	(1274.)	. . .	. . .	. . . .
2	1 589	70.3	95.6	94.29	0.381	5.419	0.088	1.252	111.0	231.8	23.26	51.99	11.8	26.38	0.506
3	1 600	140.9	191.4	188.8	0.659	9.373	0.128	1.821	135.0	275.0	19.14	42.78	10.1	22.58	0.526
4	1 591	202.0	274.4	270.6	0.902	12.83	0.163	2.318	137.0	278.6	18.03	40.30	9.66	21.59	0.535
5	1 598	232.5	315.8	311.5	1.034	14.71	0.197	2.788	147.0	296.6	17.88	39.97	9.80	21.91	0.548

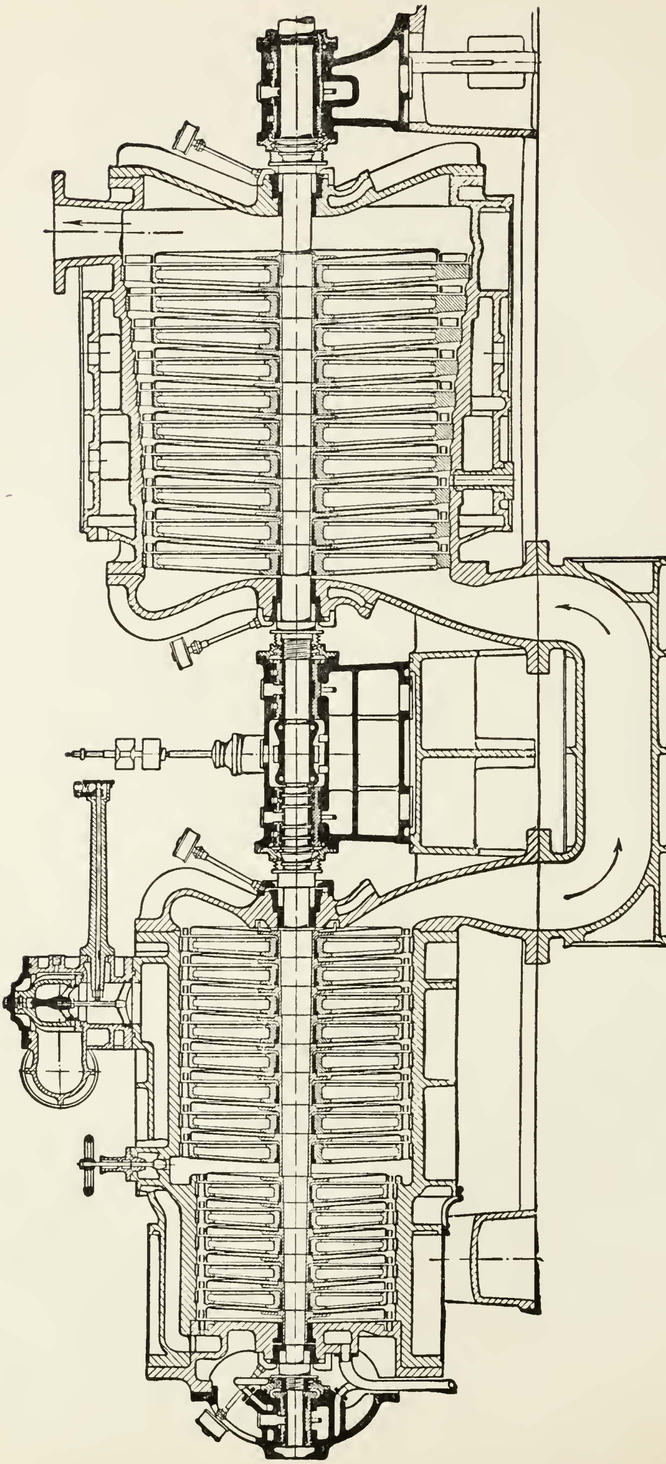


Fig. 177.

In Feb., 1903, the author, in conjunction with Professor *Wyssling* and Professor *Farny* who looked after the electrical measurements, were invited by Messrs. *Sautter, Harlé & Cie.* of Paris, to

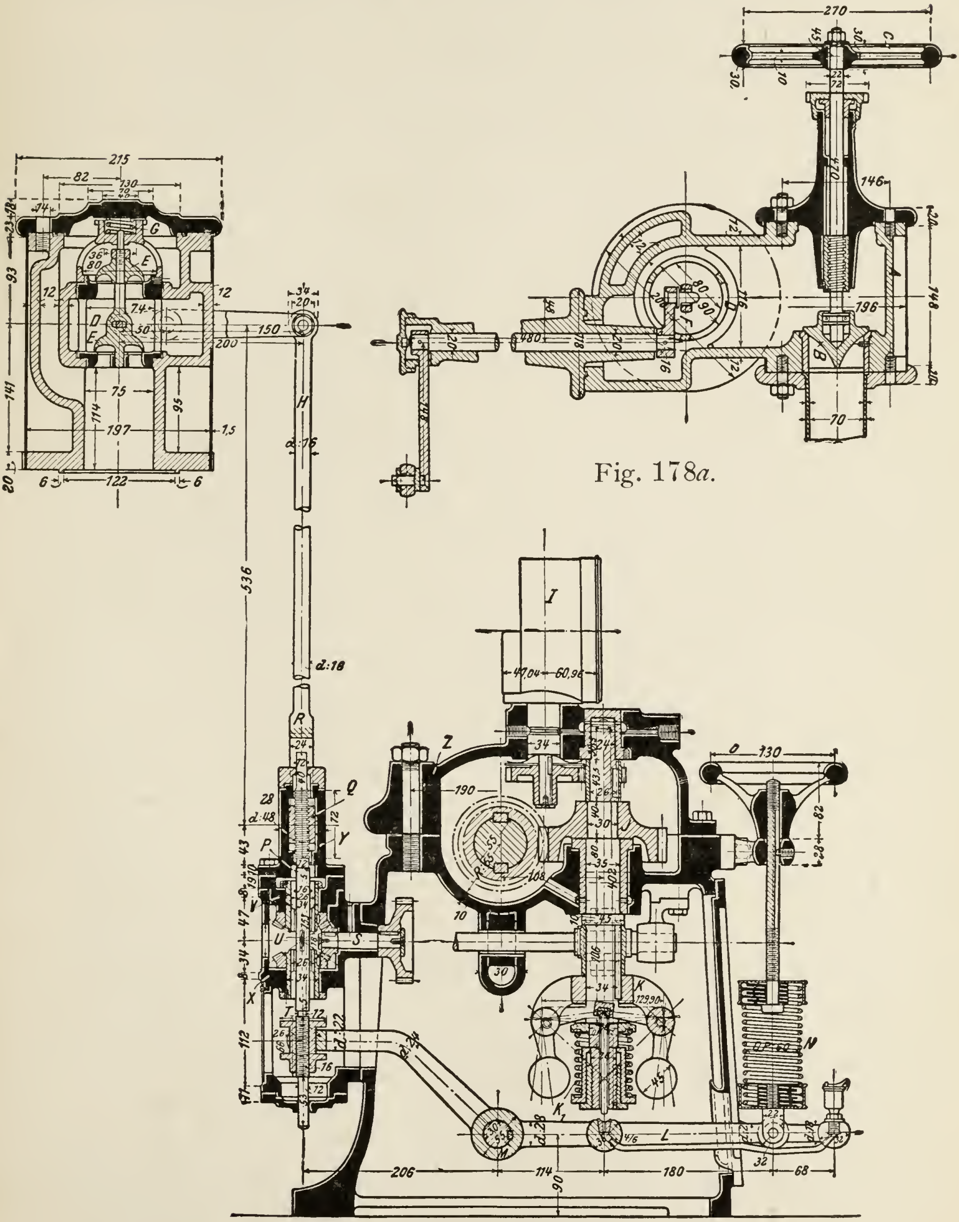


Fig. 178*a*.

Dimensions are given in mm.

Fig. 178.

TABLE 3. (In French Units.)

EXPERIMENTS WITH THE RATEAU TURBINE OF SAUTTER, HARLÉ & CIE., OF PARIS.

Experiment No.		I.	II.	III.	IV.	V.	VI.	VII.	VIII.	IX.	X.	XI. •	XII.	XIII.
1. Power at the Dynamo Brushes	kw.	Light with't excit'n	Light with excit'n	58.45	107.5	172.35	279.9	127.9	366.0	440.1	436.5	344.7	462.9	470.27
2. Revolutions	per. min.	2 196	2 181	2 186	2 184	2 181	2 190	1 054	2 101	2 200	2 200	1 998	2 360	2 310
3. Duration	min.	30	18	25	40	50	35	20	180	30	26	10	22	30
4. Absolute steam pressure at entrance to admission valve	Kg. per sq. cm.	12.33	12.66	12.26	12.38	12.31	11.99	10.91	11.84	12.73	11.36	11 45	15.73	15.20
5. Temp. at entrance to admission valve	C.°	188.2	189.6	190.9	191.2	193.2	195.1	188 6	197.5	197.7	195.9	(195.9)	212.6	209.6
6. Temp. of saturation at ent. to adms. valve	C.°	188.2	189.3	190.2	188.3	188.2	186.9	182.6	186.4	189.6	184.5	184.8	199.5	197.8
7. Superheat at entrance to admission valve	C.°	0.0	0.3	0.7	2.9	5.0	8.2	6.0	11.1	8.1	11.4	(11.4)	13.1	11.8
8. Absolute pressure at entrance to 1 guide wheel	Kg. per sq. cm.	0.66	0.875	2.28	3.14	4.49	6.71	4.54	8.43	10.1	8.68	8.65	10.71	10.32
9. Temp. at entrance to 1 guide wheel	C.°	118.3	124.6	141.5	152.4	164.9	174	165.3	182.1	185.9	185.1	182.1	193.9	192.1
10. Temp. of saturation at ent. to 1 guide wheel	C.°	83.7	95.4	123.7	134.3	147.0	162.4	147.4	171.6	179.3	172.3	172.3	181.8	180.3
11. Superheat at entrance to 1 guide wheel	C.°	34.6	29.2	17.8	18.1	17.9	11.6	17.9	10.5	6.6	12.8	9.8	12.1	11.8
12. Absolute pressure at intermediate pipe	Kg. per sq. cm.	0.120	0.140	0.266	0.383	0.545	0.802	0.546	0.999	1.20	1.26	0.99	1.24	1.27
13. Absolute pressure at exhaust pipe	Kg. per sq. cm.	0.106	0.103	0.088	0.091	0.0935	0.106	0.091	0.115	0.131	0.141	0.128	0.151	0.13
14. Temp. of water for condenser, entrance	C.°	12.9	11.5	12.2	17.5	16.5	18.2	16.72	15.8	16.04	. . .	. . .	21.5	13.9
15. Temp. of water for condenser, exit	C.°	14.3	13.2	16.6	22.9	24.0	28.0	24.4	26.8	27.7	. . .	. . .	33.8	27.2
16. Temp. of condensed steam	C.°	. . .	23.0	20.0	21.4	22.5	27.0	23.7	29.8	32.5	33	37	40	33
17. Total steam consumption per hour	Kg.	338.0	445.0	1003.2	1483.5	2044.8	2976.0	2085.0	3754.0	4385.0	4592.3	3768.0	4640.5	4647.0
18. Steam consumption per kw. hour exclusive of work of air pump	kg.	. . .	. . .	17.16	13.80	11.86	10.63	16.30	10.25	9.96	10.52	10.93	10.02	9.88
19. Efficiency of Dynamo	%	. . .	. . .	74.0	84.0	90.2	92.0	86.0	92.4	93.0	92.9	92.3	93.3	93.4
20. Effective power of Turbine	h. p.	. . .	. . .	107.3	174.0	260.2	422.5	202.0	538.2	643.0	637.7	507.4	674.1	684.0
21. Steam consumption per effective h. p. hour (exclusive of work of Air Pump)	kg.	. . .	. . .	9.35	8.52	7.86	7.04	10.32	6.97	6.82	7.20	7.42	6.88	6.79
22. Thermodynamic efficiency referred to the useful electrical work at the brushes of the dynamo, and to the steam condition at entrance to the 1 guide wheel	%	. . .	. . .	43.3	49.5	52.4	54.4	37.7	54.3	54.9	54.6	51.6	55.2	54.9

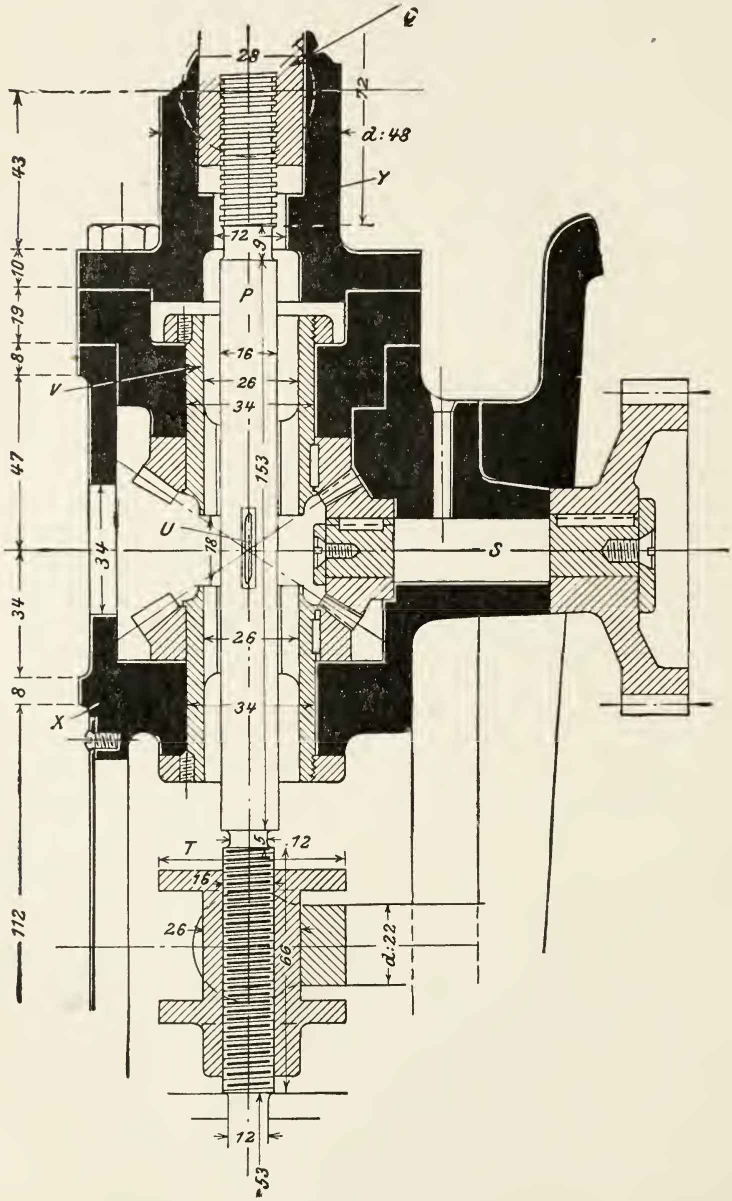
TABLE 3. (In English Units).

EXPERIMENTS WITH THE RATEAU TURBINE OF SAUTTER, HARLÉ & CIE., OF PARIS.

Experiment No.		I.	II.	III.	IV.	V.	VI.	VII.	VIII.	IX.	X.	XI.	XII.	XIII.
1. Power at the dynamo brushes	kw.	Light without excit'n.	Light with excitation.	58.45	107.5	172.35	279.9	127.9	366.0	440.1	436.5	344.7	462.9	470.27
2. Revolutions	per Min.	2196	2181	2186	2184	2181	2190	1054	2101	2200	2200	1998	2360	2310
3. Duration	Min.	30	18	25	40	50	35	20	180	30	26	10	22	30
4. Absolute steam pressure at entrance to admission valve	Lb. per sq. in.	175.4	180.1	174.4	176.1	175.1	170.5	155.2	168.4	181.1	161.6	162.9	223.7	216.2
5. Temp. at entrance to admission valve	F.°	370.8	373.3	375.6	376.2	379.8	383.2	371.5	387.5	387.9	384.6	384.6	414.7	409.3
6. Temp. of sat'n at ent'e to adm'n valve	F.°	370.8	372.7	374.4	370.9	370.8	368.4	360.7	367.5	373.3	364.1	364.6	391.1	388.0
7. Superheat at ent'e to admission valve	F.°	0.0	0.5	1.3	5.2	9.0	14.8	10.8	20.0	14.6	20.5	20.5	23.6	21.2
8. Absolute pressure at entrance to 1 guide wheel	Lb. per sq. in.	9.387	12.45	32.13	44.66	63.86	95.44	64.57	119.9	143.7	123.5	123.0	152.3	146.8
9. Temp. at entrance to 1 guide wheel	F.°	244.9	256.3	286.7	306.3	328.8	345.2	329.5	359.8	366.6	365.2	359.8	381.0	377.8
10. Temp. of sat'n at ent'e to 1 guide wh.	F.°	182.7	203.7	254.7	273.7	296.6	324.3	297.3	340.9	354.7	342.1	342.1	359.2	356.5
11. Superheat at entrance to 1 guide wheel	F.°	62.3	52.6	32.0	32.6	32.2	20.9	32.2	18.9	11.9	23.0	17.6	21.8	21.2
12. Absolute pressure at intermediate pipe	Lb. per sq. in.	1.707	1.991	3.783	5.448	7.752	11.41	7.766	14.21	17.07	17.92	14.08	17.64	18.06
13. Absolute pressure at exhaust pipe	Lb. per sq. in.	1.508	1.465	1.252	1.294	13.30	1.508	1.294	1.636	1.863	2.006	18.21	2.148	1.849
14. Temp. of water for condenser entrance	F.°	55.2	52.7	54.0	63.5	61.7	64.8	62.10	60.4	60.87	...	...	70.7	57.0
15. Temp. of water for condenser exit	F.°	57.7	55.8	61.9	73.2	75.2	82.4	75.9	80.2	81.9	...	...	92.8	81.0
16. Temp. of condensed steam	F.°	...	73.4	68.0	70.5	72.5	80.6	74.7	85.6	90.5	91.4	98.6	104.0	91.4
17. Total steam consumption per hour	Lb.	745.17	981.05	2211.6	3270.5	4508.0	6560.9	4596.7	827.60	9667.2	10123.0	8307.0	10230.0	10245.0
18. Steam consumption per kw. hour exclusive of work of air pump	Lb.	...	...	37.83	30.42	26.15	23.43	35.93	22.60	21.96	23.19	24.10	22.09	21.78
19. Efficiency of dynamo	%	...	...	74.0	84.0	90.2	92.0	86.0	92.4	93.0	92.9	92.3	93.3	93.4
20. Effective power of turbine	h. p.	...	...	105.8	171.6	256.6	416.7	199.2	530.8	634.2	629.0	500.5	664.9	674.7
21. Steam consumption per effective h. p. hour (exclusive of work of air pump)	Lb.	...	...	20.90	19.04	17.57	15.74	23.07	15.57	15.24	16.09	16.585	15.38	15.18
22. Thermodynamic effi. referred to the elec. useful work at the brushes of the dynamo and to the steam condition at entrance to the 1 guide wheel	%	...	...	43.3	49.5	52.4	54.4	37.7	54.3	54.9	54.6	51.6	55.2	54.9

test a 500 h. p. Rateau turbine. The results are given in Table 3, pages 266 and 267.

The measurement of the steam consumption was made by col-



Dimensions are given in mm.

Fig. 178*b*.

lecting the condensed steam from a surface condenser in two equal vessels alternately used. The boilers were located at some distance, and in order to dry the steam, high pressures were used and the steam throttled at the turbine. The table shows that it was possible to even superheat the steam a few degrees. The power necessary to drive the air pump is not deducted from the total power at the brushes, but should not exceed several per cent.,

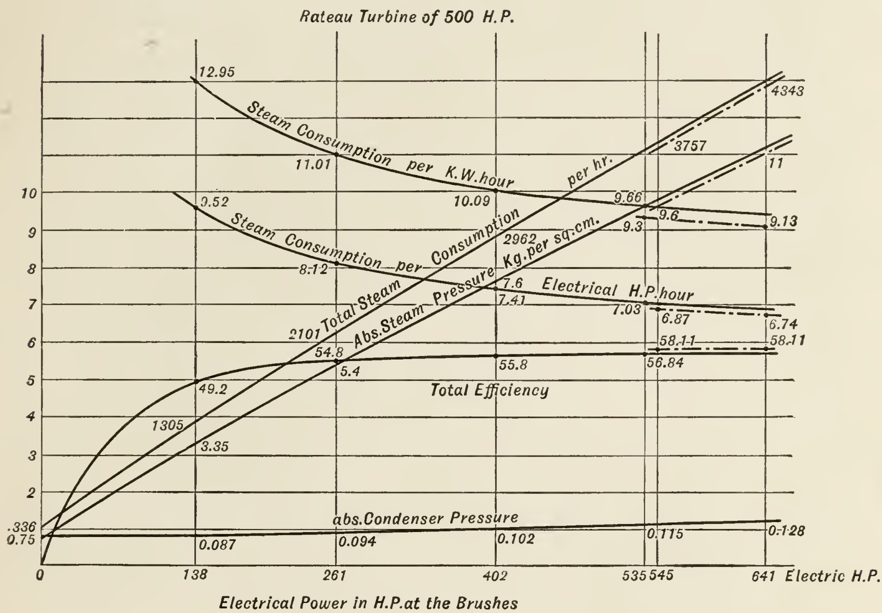


Fig. 179.

judging from other data. Two direct coupled direct current dynamos were driven by the turbine, which was kept at an exceedingly constant load by having the dynamo work on the metallic resistances with which the laboratory was abundantly supplied. The readings of the condensed steam volumes were taken every 5 to 10 minutes, thus determining in the least time the normal running condition, and minimizing greatly the actual time of the experiment. Only at normal load (No. VIII.) did the experiment extend over three hours. During experiment No. X. the overload valve was partly open.

Of especial interest is experiment No. VII. conducted at one-half the number of revolutions, which gave, in comparison with No. V., the steam flowing through per hour at equal admission pressures, and which was here also independent of the number of revolutions.

The consumption of only 9.9 kilograms (21.83 pounds) per kw. hour, with only 0.13 kilogram per square centimeter (1.85 pounds per square inch) vacuum, and 10.3 kilograms per square

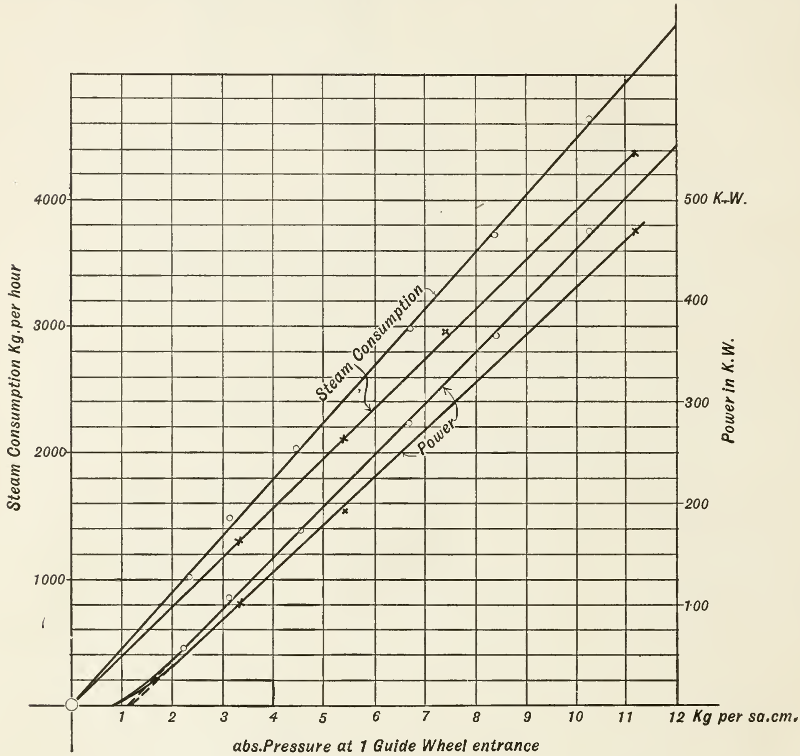


Fig. 180.

centimeters (146.5 pounds per square inch) absolute admission pressure, must be considered very favorable.

The firm since then have built a third turbine of almost equal size, in which besides other improvements, a decrease of the guide-wheel cross-sections made possible an increase of admission pressure at equal power. The experiments conducted by the builders with this turbine gave the results shown graphically in Fig. 179. The full curves refer to saturated steam, and the dotted curves to

steam of about 10° C. (18° F.) superheat. The improvement is considerable, and besides increasing the admission pressure, allows a speed of 2400 r. p. m. Fig. 180 represents the steam weight per hour and the power in kw. as function of the absolute entrance pressure; the small circles refer to my own experiments with this new turbine, the crosses to those of *Sautter, Harlé & Cie.* The steam volume as well as the power increases almost exactly linearly with the pressure. If from Fig. 179 we estimate the consumption of saturated steam as 9.3 kilograms (20.5 pounds) per kw. hour at 11.1 kilograms per square centimeter (157.88 pounds per square inch) absolute pressure at the first guide wheel, and 0.128 kilogram per square centimeter (1.82 pounds per square inch) vacuum, we obtain as the thermodynamic efficiency 57.8%. This figure will decrease at increase of vacuum, because the losses of exhaust increase; still, these results remain unusually good. The efficiency determined by the author for the former turbine might represent the highest value that could be reached with such slight superheat for motors of that size.

62. THE PARSONS TURBINE.

The construction of this oldest practically tried turbine is shown diagrammatically in longitudinal cross-section in Fig. 181. The steam enters through the opening shown by the dotted circle and flows towards the right, through the successive guide and rotating wheels. The drums are constructed in steps, with abrupt increases of peripheral velocity. The blade lengths also increase in smaller and in larger stages.

At the left are found the pistons* for balancing pressure, made

* The axial pressure exerted upon a certain rotating wheel is expressed by the formula:

$$P = F(p' - p'') - M(c_{02} - c_{01}),$$

in which p' is the pressure in the clearance space in front of the rotating wheel; p'' the pressure back of the rotating wheel; F the area of the ring between the outer and the inner blade radii; M the steam mass per second; c_{01} and c_{02} the axial components of the absolute velocities at entrance to, and exit from, the rotating wheel. To the sum of the forces P are added the pressures which the steam exerts upon the ring areas between two drums, and finally the end pressure on the last wheel. It is worthy of notice that the balancing of pressures by the labyrinth pistons, when the balancing exists for a certain initial pressure, holds good even with a change of load. As will be later proved, the pressure at any one place varies nearly proportionately to the initial pressure; the pressures on the wheels, the drums, and on the balancing is also assisted by the increase of vacuum corresponding to a drop in the load.

steam tight by the "labyrinth" system previously described, one for each drum. The space in front of each piston is connected with the steam inlet to the corresponding drum, for the twofold reason of causing equal pressures in both spaces, and also to at least partly utilize the steam leaking through the labyrinth packing. Imagine the overload valve at A, and as is shown in the figure, when an overload occurs this may be opened by hand or by a governor, thus admitting live steam to the succeeding drum (see page 211). By so doing, a counter-current is exerted against the first drum,

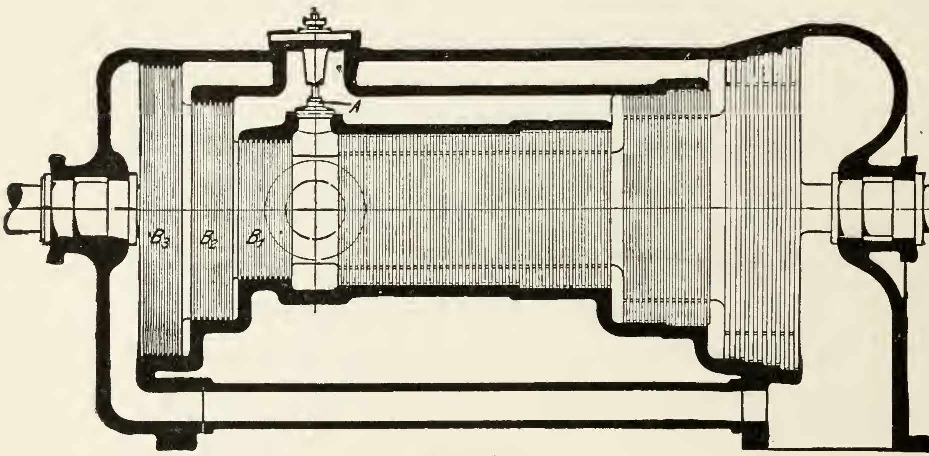


Fig. 181.

and the efficiency of the steam drops; but this disadvantage is more than offset by the advantage that the turbine at normal load will work with full boiler pressure at the first rotating wheel, because otherwise a strong throttling would be necessary. The extending shaft is made steam tight by successive labyrinth grooves, which are connected to the vacuum side of the regulating gear in order to prevent the drawing in of air.

Fig. 182 shows the rotating drums and the balancing pistons of a 3 000 h. p. turbine built by the Westinghouse Machine Co., of Pittsburg.* The total weight is about 12 600 kg. (28 000 lb.), the distance between bearings 3.75 m. (13.75 feet) the largest diame-

* According to an address by *Fr. Hodgkinson* in the Proceedings of the Eng. Soc. of Western Pennsylvania, Nov., 1900.

ter 1.83 m. (6 ft.). Figs. 183 and 184 show an outside view of this exceedingly well constructed machine, which is installed at the Electric Works in Hartford, Conn. The conspicuous piping would of course have been placed underground with us. European constructors divide the turbine of large capacity into two parts, as in Fig. 185, a *Brown, Boveri & Co.* 5 000 h. p. turbine for Frankfort.* This arrangement, of course, has the disadvantage of too great shaft length, which caused *Parsons*, for instance, to construct between the high and low pressure groups of the turbine at Elberfeld a movable coupling, so that the labyrinth pistons of each group can be adjusted by means of bolts to the small clearance required. In

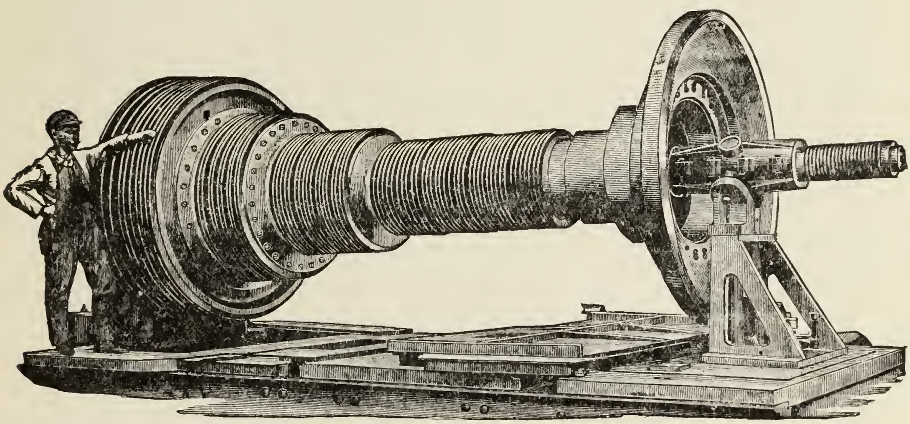


Fig. 182.

using superheated steam, this precaution is doubly necessary because, due to the great expansion of the shaft, the rings of the packing pistons either press against one another or cause an open joint. It was not stated whether this expansion is taken into account in the movable stuffing box. According to latest reports, 10 000 h. p. turbines of the Hartford type are built with only two bearings.

The bearings of small turbines only consist of multiple boxes, as originally used by *Parsons*. For large machines, water-cooled bearings are used, with ball-boxes, and all bearings are fed with oil

* Schweiz Bauzeitung, 1902. Vol. I., p. 240, etc.

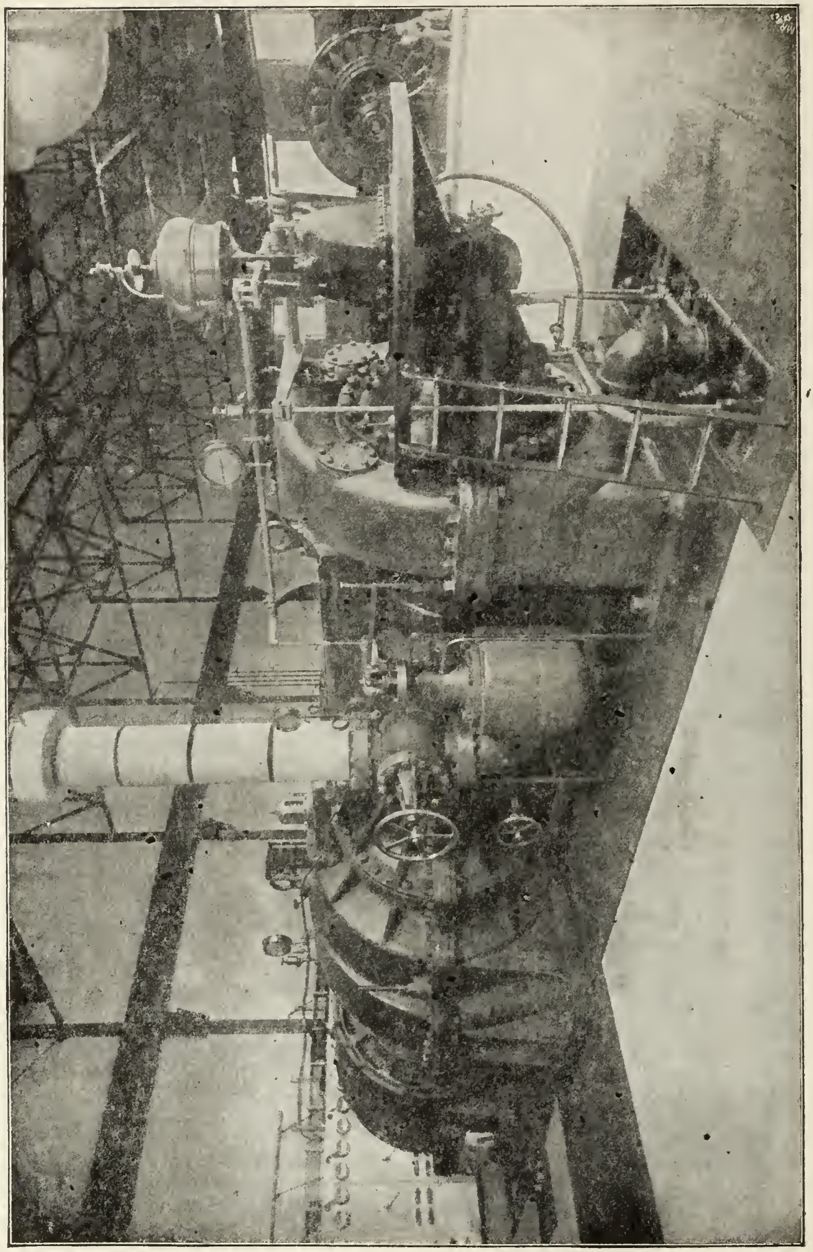


Fig. 183.

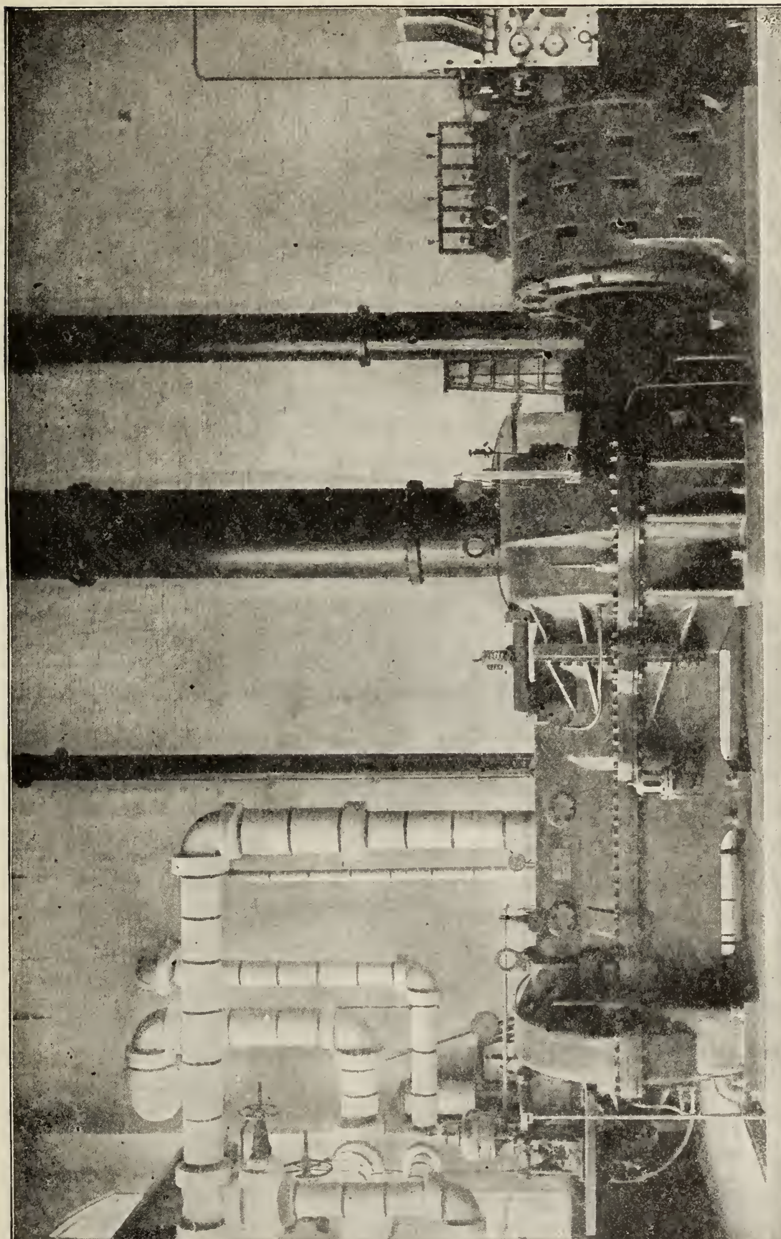


Fig. 184.

under pressure from special pumps. The oil is led through tube arrangements, cooled by water, and again used.

The *Österreichische Dampfturbinen-Gesellschaft*, of *Brünn*, uses the regulating devices shown in Figs. 186 and 186*a*. *A* is the stop valve (for high superheat) provided with a nickel seat. *B* is a double-seated regulating valve, which according to Parsons' process is kept in periodic up-and-down vibration by means of a piston *C* in a steam cylinder. The piston is loaded by the spring *B*, and receives live steam beneath, through the valve *E*. The regulating device moves the small piston valve *F*, that alternately closes and opens the lower cylinder space to an exhaust pipe, by which the

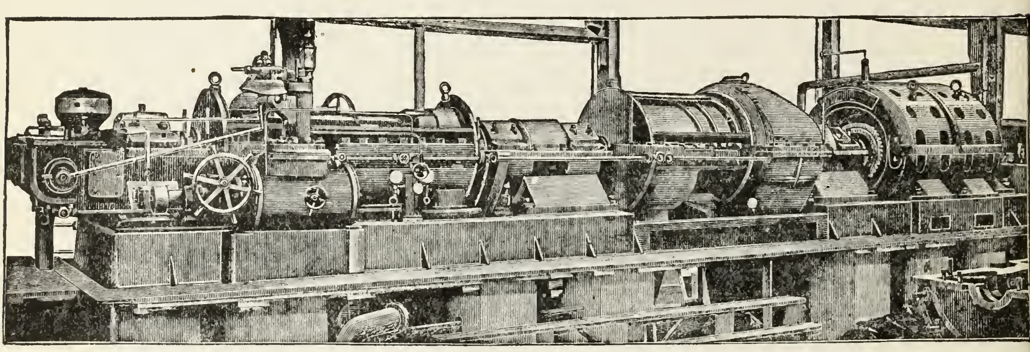


Fig. 185.

main piston is moved upwards or is allowed to fall. The piston valve also moves the regulator by an eccentric hub *G*, which is shown enlarged in Fig. 186*b*. Casing *H* contains the spring for fixing the number of revolutions. The stop-valve is arranged as a safety-valve in case the velocity is too high. The auxiliary regulator *A* in Fig. 186*a* in opening carries with it the loose worm *B* on the regulating shaft, and by means of the spindle *Z* turns the collar *K*, Fig. 186, which allows the weighted lever *L* to drop and close the valve. The illustration also shows the bearing for the forward end of the shaft, with its ball and socket boxes, and the adjustable thrust bearing at *M*. *N* is the piping for pressure oiling. The steam piston is provided with a lever arm *P* for hand regulation.

The time required for opening this valve becomes less as the load diminishes; the steam pressure in back of the throttle

valve varies, therefore, periodically; and so that its mean value decreases with the load.

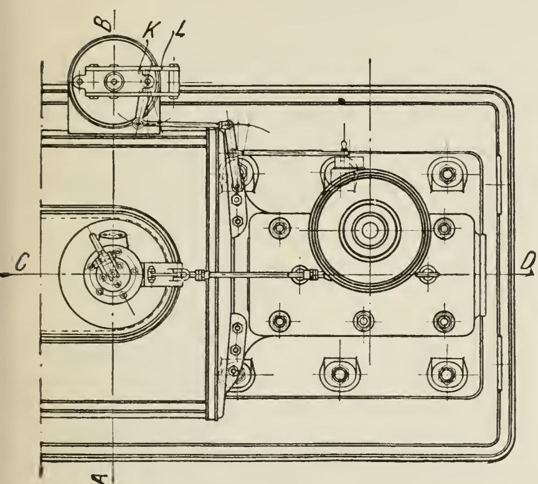


Fig. 187 represents the governor used by *Brown, Boveri & Cie.* Eccentric x serves for producing the principal oscillating motion of the governing valve g. The governor displaces the central position of the oscillations and thereby varies the time required for the intermittent steam admissions.

In Figs. 188* and 188a* are shown the throttling curves at half and at full load, from a design of *Brown, Boveri & Cie.* The number of steam admissions are, recently,

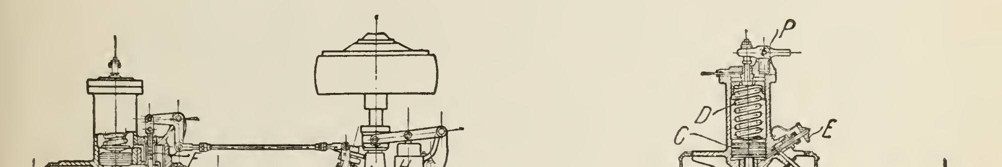
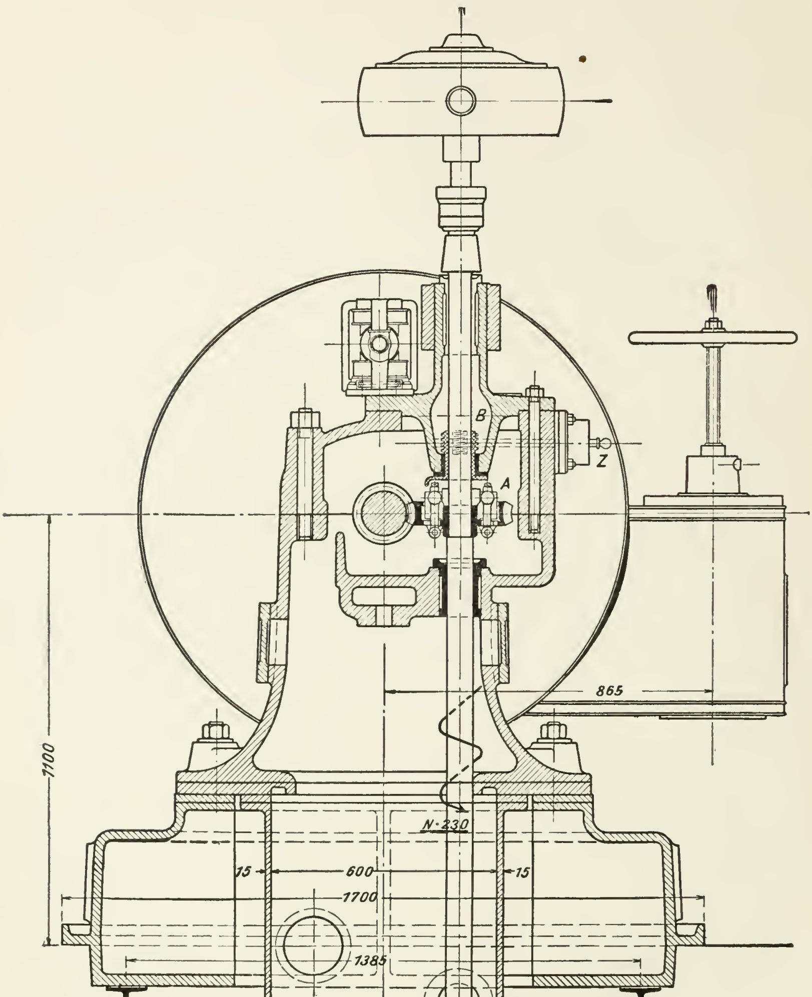


Fig. 186.

* Schweiz. Bauzeitung, 1902, Vol. I., p. 238.

150–250; the unsteadiness of the rotary motion which is brought about artificially by the pressure oscillations, cannot be very high on account of the large rotating masses of the Parsons turbine.



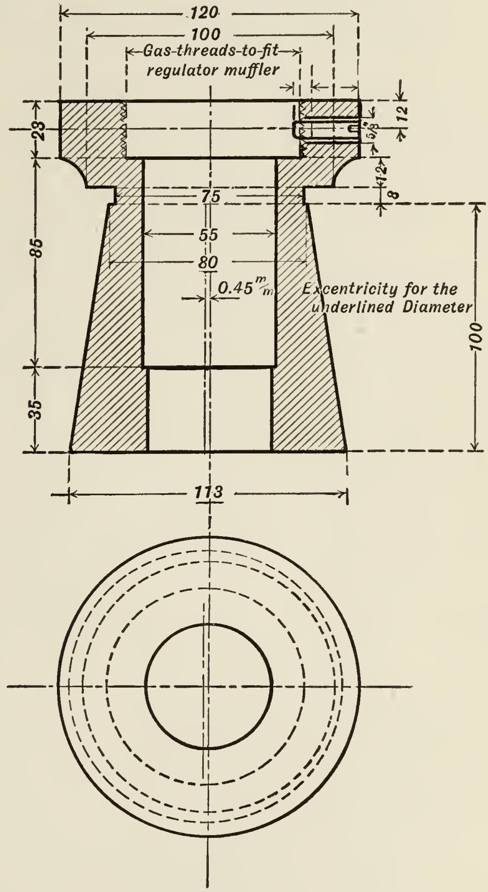
Dimensions given are in mm.

Fig. 186*a*.

Favorable reports have generally been made concerning the wear of the blades. The steam velocity seldom reaches the value of 350 m. to 450 m. (1 148.3 ft. to 1 476.4 ft.), and then only in the low pressure wheels, hence is about half as great as in the Laval turbine; the kinetic energies per unit mass are in the ratio $\frac{1}{4}$,

and this seems to be the reason for the small wear. In this turbine, as in the others, the introduction of superheated steam, by doing away with the water, should have a beneficial influence on the steam tightness.

A large number of results are on hand of experiments concerning the steam consumption of the Parsons turbine. Above all are to be mentioned the excellent investigations of *Lindley*, *Schröter* and *Weber* with the turbines at *Elberfeld*.* Table 4 contains further results made public by *Stoney*,† before the International Engineering Congress at Glasgow, 1901.



Dimensions given in mm.
Fig. 186*b*.

In the column for steam consumption per kw. hour, there has been entered the total consumption in one hour (in parentheses). As the data is not given concerning the efficiency of the dynamo, the actual power is compared to the power of the perfect turbine dynamo, in which the total available heat $\lambda_1 - \lambda_2'$, is changed without loss into electric energy. The theoretical consumption per kw. hour is therefore, in French units,

$$D_0 = \frac{637}{0.736\,(\lambda_1 - \lambda_2')} = \frac{865.5}{A\,L_0}\ \text{kg.},$$

* Zeitschr. d. Ver. deutsch. Ing., 1900, p. 829, etc.

† Chief Engineer with C. A. Parsons & Co., Newcastle-on-Tyne.

TABLE 4.

RESULTS OF STONEY'S EXPERIMENTS WITH PARSONS TURBINES.

	Location of the Turbine and Kind of Electric Current.	Pressure (Gauge).		Condenser Pressure (Absolute).		Superheat.		Power kw.	No. of Revolutions per Min.	Actual Steam Consumption per kw. Hour, D_e.		Theoretical Steam Consumption per kw. Hour, D_0.		$\eta = \frac{D_0}{D_e}$
		Kg. per Sq. Cm.	*Lb. per Sq. In.*	Kg. per Sq. Cm.	*Lb. per Sq. In.*	C.°	*F.°*			Kg.	*Lb.*	Kg.	*Lb.*	
1	Newcastle.	5.62	79.94	0.0414	0.59	. .	. .	24.7	4990	13.06	28.79	5.19	11.44	0.397
2		5.41	76.95	0.0345	0.49	. .	. .	11.8	4630	15.38	339.06	5.11	11.26	0.332
3		5.20	73.96	0.0311	0.44	. .	. .	5.15	4570	20.68	45.59	5.09	11.22	0.246
4		5.48	77.94	0.138	1.96	. .	. .	23.8	4900	15.19	33.49	6.41	14.13	0.422
5		5.55	78.94	1.036	14.7	. .	. .	19.7	4780	31.07	68.50	12.79	28.20	0.412
6	Blackpool (Alternating)	8.86	126.02	0.0691	0.98	. .	. .	52.7	5044	12.7	28.00	5.074	11.19	0.400
7		9.28	131.99	0.0518	0.74	. .	. .	. . .	4880	(145.1)	(319.89)	. . .	. .	. .
8	Blackpool (Alternating)	8.93	127.02	0.104	1.48	. .	. .	108.5	4800	12.16	26.81	5.40	11.91	0.445
9		8.93	127.02	0.0656	0.93	. .	. .	51.4	4600	13.56	29.89	5.04	11.11	0.372
10		8.93	127.02	0.0553	0.79	. .	. .	. . .	4450	(136.1)	(300.05)	. . .	. .	. .
11	West-Bromwich (Direct)	9.07	129.01	0.076	1.08	30.	54.0	123	3500	11.57	25.51	5.01	11.04	0.433
12		9.42	133 98	0.079	1.12	35.6	64.1	122	3520	10.80	23.81	4.96	10.94	0.459
13	Winrick (Direct)	7.03	99.99	0.0414	0.59	46.7	84.1	119	3640	11.02	24.29	4.75	10.47	0.431
14		6.40	91.03	0.0829	1.18	38.3	68.9	121	3685	11.48	25.31	5.40	11.91	0.470
15		6.54	93.02	0.0829	1.18	34.4	61.9	80	3500	12.88	28.39	5.41	11.93	0.420
16		6.82	97.00	0.0760	1.08	15.6	28.1	42	3200	16.33	36.00	5.40	11.91	0.331
17	Blackpool (Direct)	9.07	129.01	0.0829	1.18	32.2	58.0	226	3045	9.98	22.00	5.14	11.33	0.515
18		8.58	122.04	0.0553	0.79	33.3	59.9	232	3010	9.93	21.89	4.81	10.60	0.484
19		8.37	119.06	0.107	1.52	. .	. .	204	3000	10.98	24.21	5.52	12.17	0.503
20		9.14	130.00	0.0691	0.98	. .	. .	. . .	3010	(430.9)	(949.97)	. . .	. .	. .
21	Scarborough (Alternating)	8.86	126.02	0.112	1.59	. .	. .	529	2400	10.30	22.71	5.47	12.06	0.531
22		9.00	128.01	0.0794	1.13	. .	. .	258	2400	11.98	26.31	5.15	11.35	0.430
23		11.53	163.99	0.0656	0.93	. .	. .	. . .	2600	(670.0)	(1477.1)	. . .	. .	. .
24	Cheltenham (Alternating)	9.14	130.00	0.114	1.62	. .	. .	553	3000	9.84	21.69	5.46	12.04	0.554
25		9.14	130.00	0.117	1.66	. .	. .	278	3000	11.88	26.19	5.49	12.10	0.462
26		9.35	132.85	0.207	2.94	. .	. .	553	3000	10.70	23.59	6.12	13.49	0.572
27		9.14	130.00	0.207	2.94	. .	. .	453	3000	11.25	24.80	6.15	13.56	0.547
28		9.49	134.98	0.207	2.94	. .	. .	276	3000	13.45	29.65	6.11	13.47	0.455
29	Blackpool (Alternating)	10.26	145.92	0.100	1.42	38.9	70.0	515	2500	9.68	21.34	5.04	11.11	0.520
30		10.55	150.05	0.104	1.48	. .	. .	502	2500	10.48	23.10	5.18	11.42	0.495
31		9.49	134.98	0.0932	1.33	. .	. .	497	2500	10.89	24.01	5.26	11.60	0.483
32		9.35	133.39	0.0932	1.33	36.7	66.1	507	2500	9.57	21.08	5.08	11.20	0.531
33		10.69	152.14	0.0345	0.49	. .	. .	. . .	2500	(680.4)	(1500.1)	. . .	. .	. .
34		11.25	160.00	0.221	3.14	. .	. .	. . .	2500	(1147.6)	(2530.0)	. . .	. .	. .
35		10.97	156.03	0.038	0.54	2.8	5.0	. . .	2500	(664.5)	(1465.0)	. . .	. .	. .
36	Elberfield (3 Phase)	9.11	129.57	0.063	0.90	10.2	18.4	1190.1	1487	8.81	19.42	4.82	10.63	0.547
37		9.47	134.70	0.053	0.75	11.1	20.0	994.8	1461	9.14	20.15	4.69	10.34	0.513
38		9.76	138.81	0.054	0.77	8.0	14.4	745.3	1470	10.12	22.31	4.70	10.36	0.464
39		9.40	133.70	0.046	0.65	29.1	52.4	498.7	1473	11.42	25.18	4.54	10.01	0.398
40		9.14	129.86	0.050	0.71	17.0	30.6	246.5	1485	15.21	33.53	4.66	10.27	0.304
41		9.49	124.98	0.037	0.53	13.5	24.3	. . .	1488	(1183)	(2607.9)	. . .	. .	. .

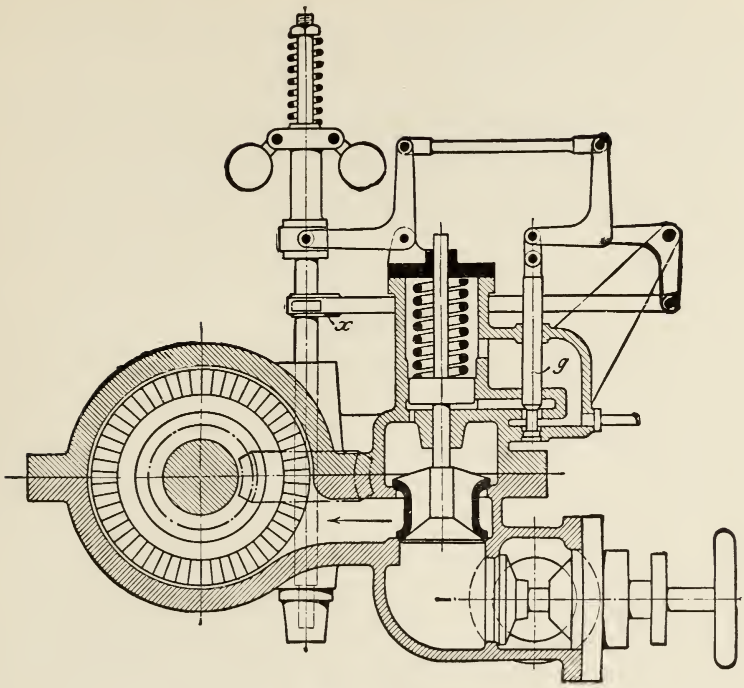


Fig. 187.

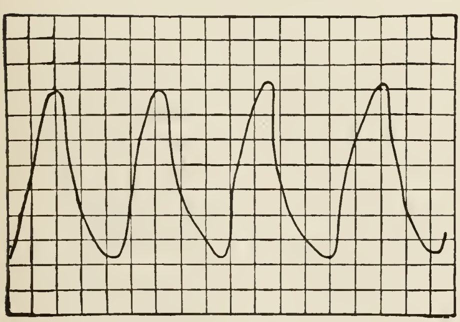


Fig. 188.

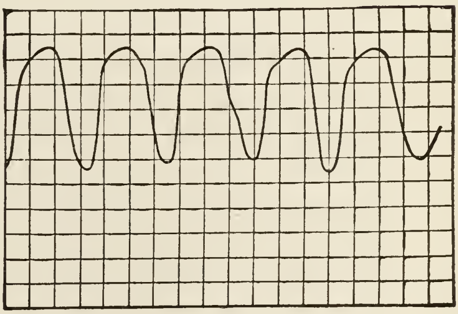


Fig. 188a.

in English units,

$$D_0 = \frac{2\,544.65}{0.7456\,(\lambda_1 - \lambda_2')} = \frac{3\,412.7}{A\,L_0}\ \text{lb.},$$

and the ratio $\eta_{el} = \dfrac{D_o}{D_e}$ represents the thermodynamic efficiency referred to the steam condition at the turbine entrance and the electric power; in these figures, therefore, the losses of the dynamo are included, which naturally change from step to step.

The Schweizerische Bauzeitung gives the following results of a second series of tests with the turbines at Elberfeld. (Table 5.)

TABLE 5.

Turbine No.	Power k.w.	Steam Temperature.		Saturated or Superheated.	Steam Consumption.						Mechanical Efficiency of Dynamo.
					Per kw. hour.		Per Electric h. p. Hour at Dynamo.		Per Effective h. p. Hour at Turbine Shaft.		
		C.°	*F.*°		Kg.	*Lb.*	Kg.	*Lb.*	Kg.	*Lb.*	
I.	1 030	18.20	359.6	Saturated	9.42	20.77	6.93	15.49	6.37	14.24	0.919
	735	183.0	361.4	"	10.12	22.31	7.43	16.61	6.80	15.20	0.915
	470	184.8	364.6	"	11.31	24.93	8.32	18.60	6.73	15.04	0.809
	1 022	208.7	407.7	Superheated	9.10	20.06	6.69	14.95	6.17	13.79	0.922
	758	211.0	411.8	"	9.64	21.25	7.09	15.85	6.47	14.46	0.912
	481	207.0	404.6	"	10.87	23.96	8.00	17.88	7.11	15.89	0.888
II.	1 042	181.0	357.8	Saturated	9.69	21.36	7.13	15.94	6.48	14.48	0.909
	506	185.0	365.0	"	11.34	25.00	8.35	18.66	6.77	15.13	0.811
	1 030	226.9	440.4	Superheated	8.96	19.75	6.59	14.73	6.06	13.55	0.920
	510	219.0	426.2	"	10.71	23.61	7.83	17.50	7.01	15.67	0.880

The efficiency of the dynamo has been calculated by the author by dividing the last two columns, and shows a conspicuous difference at half load. If we take about 86 kw. as the mean value of the losses of hysteresis, excitation, air and bearing friction, and (supported by a remark in the experiment report of *Lindley, Schröter and Weber*) 4 kw. as lost in heat in the field windings, or a total of 90 kw. at normal load, then can the experiments at Newcastle be recalculated for the effective power. Hereby we draw a comparison with a perfect machine which works with the same pressure and temperature as exists *in the chamber in back of the regulating valve*, whereby the temperature was figured from the condition in front of the valve, without considering the loss of heat by radiation. The recalculation gives the values in Table 5*a*.

TABLE 5a.

Experiments in Newcastle with a 1 000 Kw. Turbine, Referred to the Effective Power and to the Steam Condition in the Steam Chamber.

Experiment No.		II.	I.	III.	IV.	V.	VI.	VII.
							Dynamo. with excitation	Dynamo. without excitation
Electric power	kw.	1190	995	745	499	246		
Total loss in dynamo (estimated)	kw.	92	90	88	87	86	86	10
Total consumption of work . .	kw.	1282	1085	833	586	332	86	10
That is, effective power at turbine shaft	Fr. $h.p._e$	1742	1474	1132	796	451	117	13.6
The Same	*English* $h.p._e$	1718	1454	1116	785.1	444.9	115.4	13.41
Mech'l efficiency of the dynamo	%	92.8	91.7	89.4	85.2	74.2	. . .	. . .
Observed steam consumption per Fr. $h.p._e$ per hour . .	Kg.	6.02	6.17	6.66	7.15	8.36	15.8	87.0
Observed steam consumption per Eng. $h.p._e$ *per hour* . . .	*Lb.*	13.46	13.79	14.89	15.98	18.69	35.32	194.5
Theoretical steam consumption per Fr. $h.p._e$ per hour . .	Kg.	3.73	3.77	3.89	3.96	4.48	5.54	6.47
Theoretical steam consumption per Eng. $h.p._e$ *per hour* . .	*Lb.*	8.34	8.43	8.69	8.85	10.02	12.38	14.46
Thermodynamic efficiency, referred to effective power . .	%	61.9	61.0	58.5	55.3	53.5	35.0	7.5

To determine the "indicated" power, the data of the Newcastle experiments does not suffice, but it is interesting to make even an approximate investigation of this value. By "indicated" power, we mean, as previously explained, the sum of the effective powers (at the shaft), and the work of friction resistances of the turbine, understanding that the steam friction on the blades is not included. The latter is taken into account at the very beginning in the condition equations (that is, its representation in the entropy diagram). The following can be said concerning the friction between the face areas of the rotating blades and exposed drum periphery between two rotating wheels.

The steam jet leaving a guide blade close to the periphery of the casing, which is at rest, has a velocity c_x in the clearance space x (Fig. 189), whose peripheral component, c_{ux}, with the usual angles, is no doubt greater than the peripheral velocity u. In the clearance space x the steam stream causes a friction on the face area of the blades in the direction of driving, and it would be best

to add the positive work so supplied to the indicated work ; but the losses due to eddy currents may be assumed to have been considered in the condition curve. The flow in clearance space y, that is at the periphery of the drum, is analogous to the above, if we consider it as a motion relative to the drum, because the angles of the guide and rotating blades are nearly equal ; but the peripheral component c_{uy} has a direction opposite to u, exerting a retarding friction on the drum. But the corresponding work is very slight, because we are dealing with friction of smooth surfaces. The

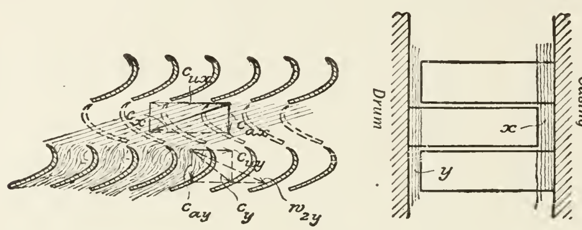


Fig. 189.

same may be true in the case of friction of the labyrinth piston, as the rubbing surfaces are small. The friction of the bearings still remains as a considerable quantity, however, and this becomes less with increasing load, as the turbine is more thoroughly heated, raising the temperature of the bearing. The drum and labyrinth friction increase with greater power on account of higher steam density, and it is therefore quite possible that the total work of friction does not vary much with the load ; and the friction running light may be taken as constant. Therefore for the Parsons turbine, we may use approximately the same designation as for a reciprocating engine,

Indicated Power = Effective Power + Power running light.

A rough approximation of the weight of the turbine at Elberfeld and the calculation of the bearing friction according to *Lasche*, makes it probable that the work of running light is somewhere near 125 h. p. With this assumption the experiments at Newcastle would give the table 5*b*.

TABLE 5b.

Experiment No.	II.	I.	III.	IV.	V.	VI.	VII.
1. Effective Power as in Table 6 French h.p.$_e$	**1 742**	**1 474**	**1 132**	**796**	**451**	**117**	**13.6**
1a. Effective Power as in Table 6 English h.p.$_e$	1 718	1 454	1 117	785.1	444.8	115.4	13.42
2. Assumed work for running light French h. p.	**125**	**125**	**125**	**125**	**125**	**125**	**125**
2a. Assumed work for running light English h.p.	123.3	123.3	123.3	123.3	123.3	123.3	123.3
3. Indicated Power . . French h.p.$_i$	**1 867**	**1 599**	**1 257**	**921**	**576**	**242**	**138.6**
3a. Indicated Power . . English h.p.$_i$	1 841	1 577	1 240	908.4	568.1	238.7	136.7
4. Observed steam consumption per h.p.$_i$ hour . . . . Kg.	**5.61**	**5.69**	**6.00**	**6.18**	**6.55**	**7.62**	**8.54**
4a. Observed steam consumption per h.p.$_i$ hour . . . . Lb.	12.54	12.72	13.41	13.81	14.64	17.03	19.09
5. Theoretical steam consumption per h.p. hour . . . . Kg.	**3.73**	**3.77**	**3.89**	**3.96**	**4.48**	**5.54**	**6.47**
5a. Theoretical steam consumption per h.p. hour . . . . Lb.	8.34	8.43	8.68	8.85	10.01	12.37	14.46
6. Thermodynamical efficiency referred to indicated power . . . . %	**66.5**	**66.3**	**64.8**	**64.1**	**68.4**	**72.7**	**75.8**
7. Available heat drop per kg. steam referred to condition in the steam chamber (after leaving valve) Calories	**170.8**	**169.2**	**163.5**	**161.0**	**142.3**	**115.0**	**98.5**
7a. Available heat drop per lb. steam referred to condition in the steam chamber (after leaving valve) B. t. u.	307.4	304.6	294.3	289.8	256.1	207.0	177.3
8. Transformed into indicated work Calories	**113.4**	**112.0**	**106.2**	**103.0**	**97.2**	**83.6**	**74.6**
8a. Transformed into indicated work B. t. u.	204.1	201.6	191.2	185.4	175.0	150.5	134.3
9. Assumed exhaust loss . . Calories	**10.8**	**10.8**	**7.0**	**5.8**	**2.1**	**0.7**	**0.4**
9a. Assumed exhaust loss . . . B t. u.	19.44	19.44	12.6	10.44	3.78	1.26	0.72
10. Actual loss of energy per kg. steam $7-(8+9)$ . . . . . Calories	**46.6**	**46.4**	**50.3**	**52.2**	**43.0**	**30.7**	**23.5**
10a. Actual loss of energy per lb. steam $7a-(8a+9a)$ . . . . . B. t. u.	83.88	83.52	90.54	93.96	77.4	55.26	42.30
11. The same in % of available drop . %	**27.3**	**27.4**	**30.7**	**32.4**	**30.2**	**26.7**	**23.9**

Of especial interest are the figures that show the gradual decrease of energy-loss per kilogram or pound of steam, with increase of load. Later (see Article 79) will be given the deductions that can be made from these results for the work of running light.

The best results so far (end of January, 1904) have been obtained by Brown, Boveri & Cie. of Baden, with their 3 000 kw. machine for the Frankfort Electric Works. The experiments were

made by the director of the works himself, under running conditions, and are given in Table 6.*

TABLE 6.

EXPERIMENTS OF THE ELECTRICAL WORKS IN FRANKFORT WITH A 3 000 KW. TURBINE.

Experiment No.		I.	II.	III.
Steam pressure at admission valve (Gauge?)	Atm.	**12.63**	**12.8**	**10.6**
Steam pressure at admission valve (Gauge)	*Lb. per sq. in.*	185.6	188.1	155.8
Temperature of superheated steam	C°.	**298**	**295**	**312**
Temperature of superheated steam	*F°.*	568.4	563 0	593.6
Vacuum in % of barometer reading	%	**93.2**	**91.8**	**90.0**
Load	Kw.	**1 945**	**2 518**	**2 995**
Steam consumption per kw. hour	Kg.	**7.20**	**7.09**	**6.70**
Steam consumption per kw. hour	*Lb.*	15.87	15.63	14.77

The number of revolutions at normal running was 1 360 per minute.

The direct current excitation was supplied by two special machines. The surface condenser was also driven by a separate machine. It was not stated whether or not the power consumed was deducted from the total power.

Finally, *Brown, Boveri & Cie.* state that with a 2 000 kw. steam turbine at only 1 440 kw. load, a steam consumption of 7.165 kg. (15.8 lb.) per kw. hour has been reached, under the following conditions: 11.9 atmospheres (174.45 lb. per sq. in.) pressure, 252° C. (488.3° F.) steam temperature, 96% vacuum, and 1 500 r. p. m. Running without load, but with excitation 1 208.4 kg. (2 664.06 lb.) of steam was consumed per hour, with 12.3 atmospheres (180.77 lb. per sq. in.) pressure, 205° C. (401°F.) steam temperature, and 96.8% vacuum. For intermediate loads, the total consumption varies directly with the load. Of especial interest is the information that the consumption of power of the independently driven surface condenser was only 20 kw., that is, $1\frac{1}{2}$%.

The Westinghouse Machine Company of Pittsburg notifies me

* Described in "Die Stadtischen Electrizitätswerke zu Frankfort a. M." by *S. Singer*, director of Works, Frankfort, 1903, p. 35.

that the 1 500 kw. turbine illustrated in Fig. 183 at 150 lb. boiler pressure and 26-inch vacuum, gave the following steam consumption :—

At full load	8.67 kg.	19.11 lb. per kw. hour,
At $\frac{3}{4}$ load	9.20 kg.	20.28 " " "
At $\frac{1}{2}$ load	10.44 kg.	23.02 " " "
At $\frac{1}{4}$ load	12.70 kg.	28.00 " " "

The experiments from which these results are probably derived have in the meantime been published in the *Electrical World*, September, 1902, by Prof. *Wm. Lispenard Robb*, and are given in Table 7, page 288.

The experiments must have been conducted during ordinary running, which explains the unavoidable variation of load. The curves shown elsewhere are somewhat more regular, but do not prevent us from giving credence to the above results. The consumption was measured by weighing the volume of feed water. It was not stated whether the steam pressure was "absolute." The values given in Experiment 5 may be referred to the minimum value of the vacuum.

From these results, especially those of *Stoney*, there is plainly shown the great influence of superheat and degree of vacuum on the steam consumption. The experiments with a 500 kw. machine at Blackpool, Table 4, Nos. 29 and 30, show an improvement of 1% for every 5.1° C. (9.2° F.) superheat, while the decrease of theoretical consumption was only $5.18 - 5.04 = 0.14$ kg. (0.309 lb.) per kw. hour $= 2.7\%$ of 5.18, that is, only $\frac{38.9}{2.7} = 14.4°$ C. (25.9° F.), a saving of 1% of the consumption. Similarly, Nos. 31 and 32 give 1% for 3.3° C. (5.9° F.), while, theoretically, 1% for each 11° C. (19.8° F.). In absolute figures, the results of Nos. 29 and 30 are as follows: with an assumed efficiency of the dynamo of 0.90, and an exhaust and bearing friction loss of 8%, the total steam friction losses per kg. of steam are 61.6 calories with saturated steam, and 58.5 calories with superheated steam. In English units this would be for 1 lb. of steam 111.12 B. t. u. for saturated, and 105.53 B. t. u. for superheated steam. The friction corresponded to 3.1 calories for 61.6 calories per kg. (5.6 B. t. u. for

TABLE 7.

RESULTS OF EXPERIMENTS WITH A 1500 KW. WESTINGHOUSE TURBINE AT HARTFORD, CONN.

	Date, 1902.	Power.			Duration of Experiment Hours.	Steam Pressure.		Vacuum.		Superheat.		Coal.		Steam Consumption.	
		Mean Kw.	Maximum Kw.	Minimum Kw.		Kg. per Sq. Cm.	*Lb. per Sq. In.*	Kg. per Sq. Cm. Abs.	*Lb. per Sq. In. Abs.*	C.°	*F.°*	Kg. per Kw. Hour.	*Lb. per Kw. Hour.*	Kg. per Kw. Hour.	*Lb. per Kw. Hour.*
1	Feb. 8 . .	1998	2185	1900	4	10.92	155.3	0.111	1.579	23.2	41.8	0.772	1.702	8.67	19.11
2	Jan. 28 .	1675	1820	1480	6	10.62	151.05	0.095	1.351	22.2	40.0	0.779	1.717	9.17	20.22
3	May 9 . .	1371	1570	1110	6	10.66	151.6	0.122	1.735	17.8	32.0	0.922	2.033	9.96	21.96
4	May 12 .	834	940	660	6	10.76	153.0	0.105	1.493	19.7	35.5	1.067	2.352	11.17	24.63
5	May 8 . .	888	980	750	6	10.72	152.5	0.145	2.062	18.2	27.8	1.117	2.463	12.04	26.54
6	May 7 . .	471	730	310	6	10.66	151.6	0.113	1.607	10.6	19.1	1.330	2.932	14.51	31.98
7	May 13 .	364	520	150	6	10.75	152.9	0.091	1.294	16.1	29.0	1.507	3.322	15.19	33.49

111.12 B. t.u. per lb.) ; that is, 5.7%. As the superheat was 38.9° C. (70° F.), *the decrease in the work of steam friction was* 1% *for each* 6.8° C. (12.2° F.) *superheat.* Further experiments must verify this drop in friction coefficients, which is very important; because slight superheating extends into the turbine for a short distance only, and the greatest part of the condition change remains in the saturated territory.

It is assumed that both experiments in Elberfeld were made under similar conditions. The comparison shows a gain of 1% for every 8° C. (14.4° F.) with turbine I. (27° C. or 48.6° F. superheat) and 1% for about 6° C. (10.8° F.) for turbine II. (46° C. or 82.8° F. superheat) ; hence considerably less than with experiments 31 and 32.

Experiments 24 and 26 with a 500 kw. machine at Cheltenham gave results bearing on the *influence due to vacuum changes.* In going from 0.207 kg. per sq. cm. (0.46 lb. per sq. in.) back pressure to 0.114 kg. per sq. cm. (0.25 lb. per sq. in.), a gain of steam consumption was obtained of $\frac{10.70 - 9.84}{10.70} = 8.95\%$, while the theoretical gain was $\frac{6.12 - 5.46}{6.12} = 12.4\%$. On decreasing the back pressure to 0.1 kg. per sq. cm. (0.22 lb. per sq. in.) the corresponding figures were 4.65% and 6.40% ; *there was, therefore, an increase of vacuum of* $\frac{4.65}{6.40} = 0.73$; that is, $\frac{3}{4}$ *of the theoretical gain was actually achieved.* But it is to be observed that with one and the same turbine the exit velocity with low vacuum increases nearly in simple ratio with the larger steam volumes ; the loss at exit, therefore, increases as the square of this ratio. If this loss at 0.114 atmospheres (0.25 lb. per sq. in.) vacuum is 5%, then at 0.207 atmospheres (0.46 lb. per sq. in.) it would equal about $\left(\frac{0.114}{0.207}\right)^2 5 = 1.4\%$, and the difference $5\% - 1.4\% = 3.6\%$ is near the value that is given as the difference between the theoretical (12.4%) and the actual (8.95%) gain. From this follows the importance of a good vacuum in the steam turbine. The experiments of *Stoney* show that with the *Parsons* Turbine this was excellently accomplished.

63. THE SCHULZ TURBINE.

The *Schulz* turbine for stationary purposes, which shall be first discussed, is a reaction turbine which balances the axial steam pressure in a manner deserving of a great deal of attention. The turbine is divided into a high pressure and a low pressure group, with the steam flow in opposite directions. Fig. 190 was taken from the German patent drawing No. 137 792 (November, 1900), and shows at E the steam entrance on the high pressure side, at F the exit from the high pressure side. Leaving F the steam passes through the governing valve G and the pipe K to the low pressure side of the turbine H, from which the steam exhausts through I to the condenser. M is a live steam pipe that could be used for running the turbine in a reversed direction, and N is a thrust bearing for taking up any remaining axial forces. The turbine may be completely balanced for a certain steam and condenser pressure and a certain number of revolutions. The balancing remains perfect, if admission and vacuum pressures change proportionately ; this does not exactly occur in practice, still the axial force is not great if the drum dimensions of A and B are correctly fixed (similarly as with the Parsons turbine where analogous conditions exist) and the use of a regulating valve would become superfluous.

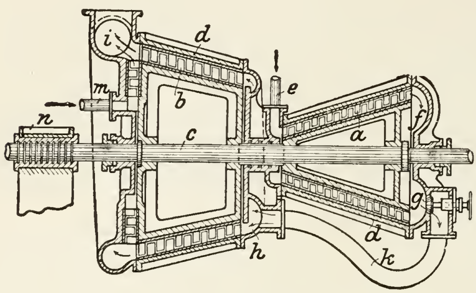
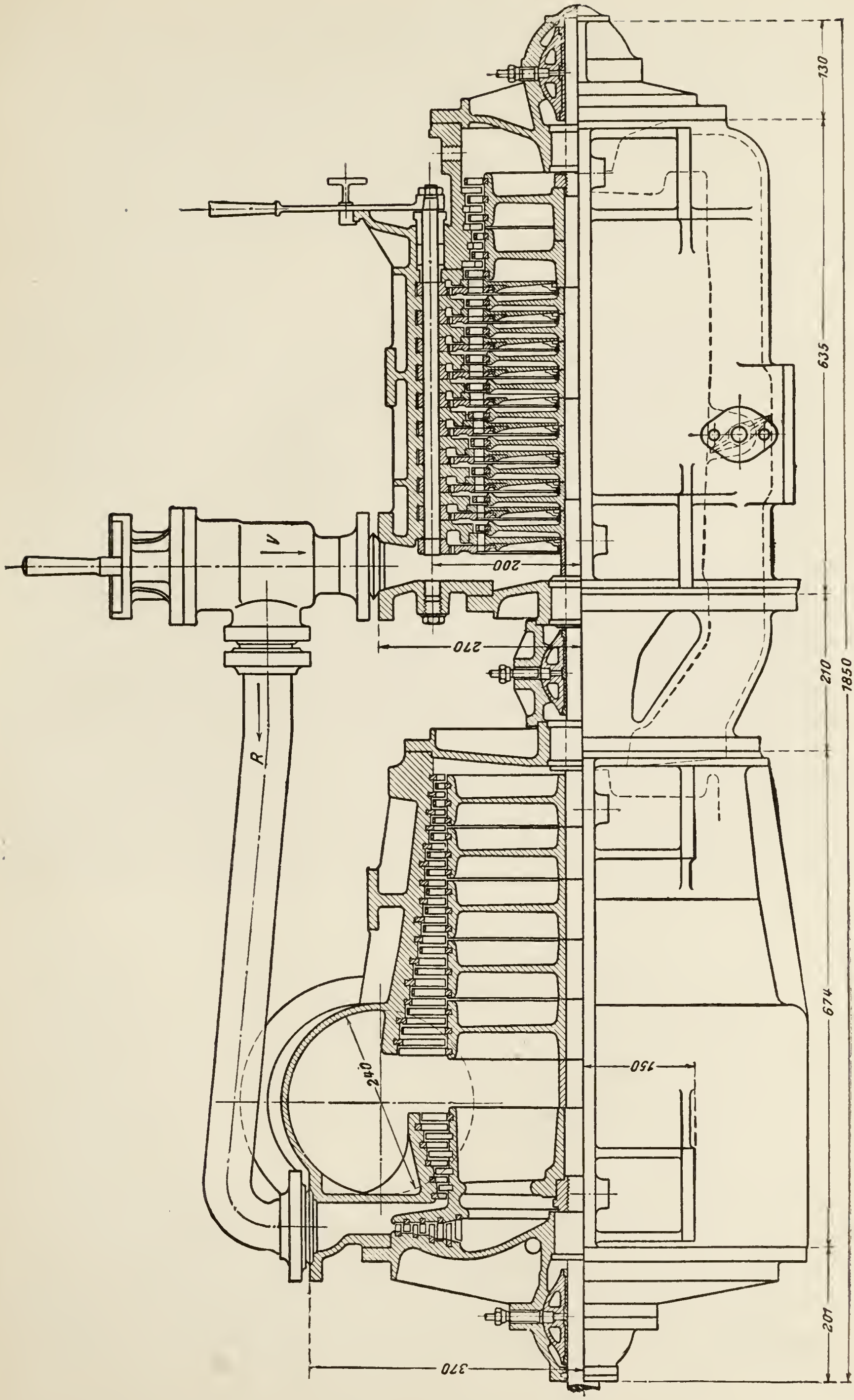


Fig. 190.

For marine purposes *Schulz* constructs the high pressure side mostly as an impulse turbine, and leaves the axial pressure of the low pressure side remain in order to balance the thrust of the propeller. An experimental turbine of this type, built by the Krupp Germainiawerft of Kiel, is shown in Fig. 191. The first ten impulse wheels are fitted with the governor (in this case regulated by hand) described on page 209. In Fig. 192 is a reproduction of a photograph of the turbine mounted on a test foundation.



Dimensions given in mm.
Fig. 191.

The long screws visible on the high pressure cover serve to set the guide wheels on their discs so that the cross-section through which the steam flows may be changed at will. The pipe R shown in Fig. 191 takes steam from the reverse valve V for running the turbine backwards; the steam, according to a method protected by patents, acts upon an axial and a radial turbine. The latter serves the same purpose as the labyrinth packing of a Parsons turbine, and balances the propeller thrust when running backwards.

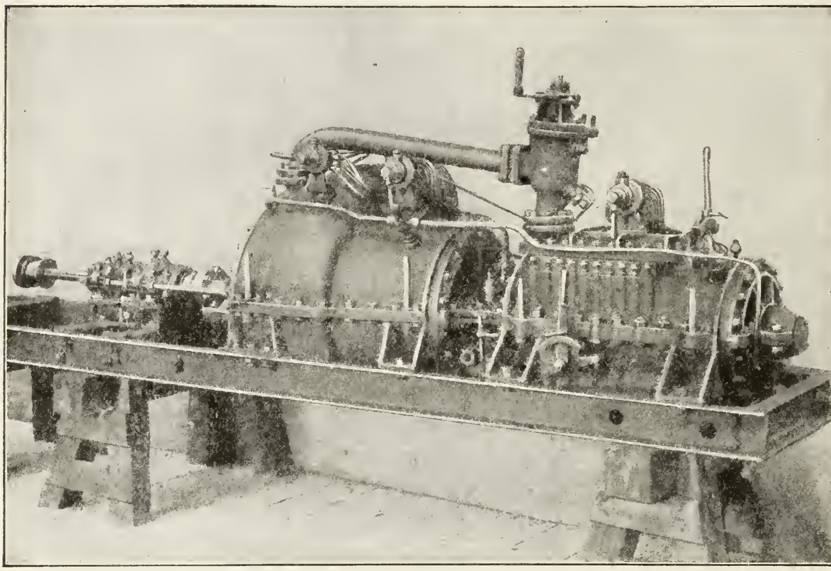


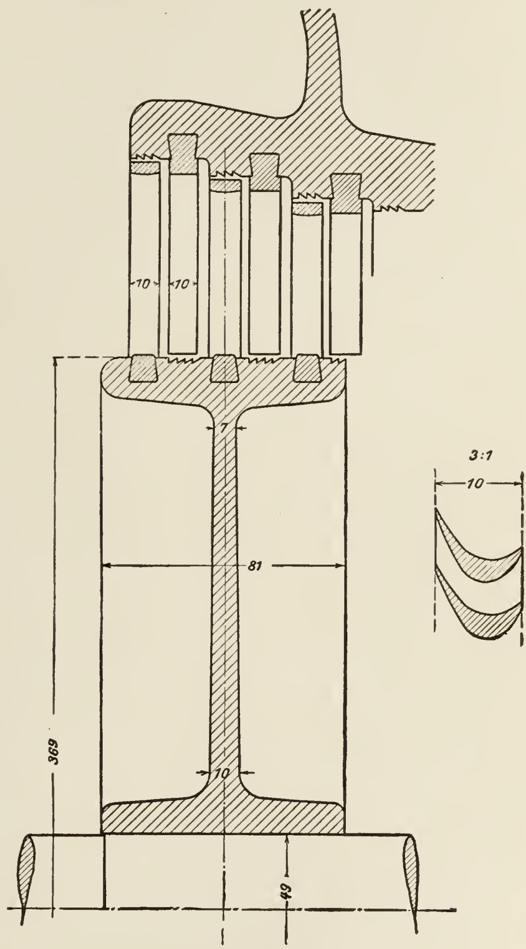
Fig. 192.

The construction of the high pressure steel wheels can be seen from Fig. 132, page 208. The blades are made of drawn delta metal, the covering ring of steel. Fig. 193 represents a working drawing of the low pressure wheel, which unites three rotating rings combined, and shows the saw-tooth profile for the diminution of steam leakage.

The designer conducted a series of experiments with an experimental turbine whose results are graphically represented in Fig. 194. The apex of the power parabola was reached in increasing the speed to 5 000 revolutions per minute. The boiler pressure remained constant; the mean pressure after the high pressure turbine was,

in the order of the curves A to F, 1.09; 1.12; 1.35; 1.60; 1.80; 1.90 kg. per sq. cm. abs. (15.5; 15.9; 19.2; 22.8; 25.6; 27.0 pounds per square inch absolute); the mean condenser pressure was 0.3 kg. per sq. cm. (4.3 pounds per square inch) absolute. Whether the indentations of the curves A and B are organic phenomena (as is the case with many hydraulic turbines) could not be ascertained from the given data.

Further interesting experiments investigating the change of pressure with the change of guide wheel cross-section are represented in Fig. 195. The number of revolutions was about 1 440. The pressure was determined by means of an indicator connected with a cock shown in Fig. 196, and also seen in Fig. 192. Curve A corresponds to normal running. In curve B the first ten steps received one-sixth opening. In C, steps 1, 5 and 10 each one-sixth opening. In D only the first step received one-sixth opening. The guide cross-section, decreased for the time being gave, naturally, a strong drop of pressure, which is clearly seen from the figure. For further calculated results, the steam condition must be known; that is, the degree of superheat at the turbine. Still, the good results of the regulation



Dimensions given in mm.
Fig. 193.

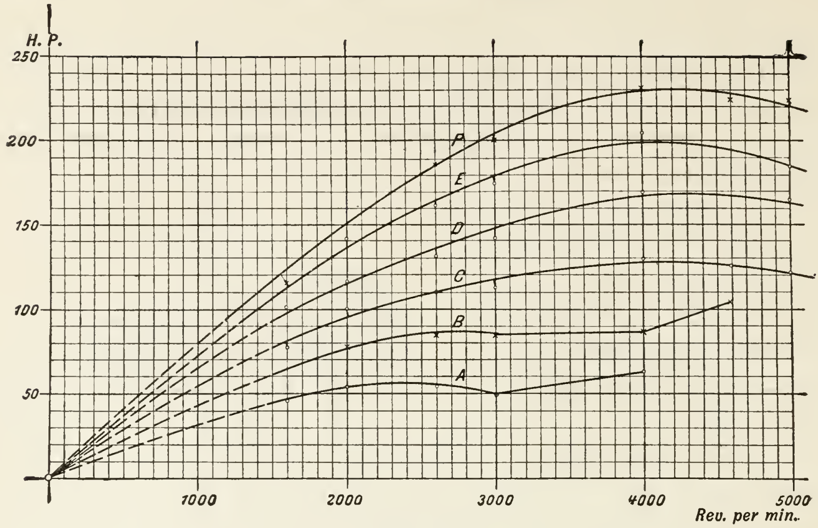


Fig. 194.

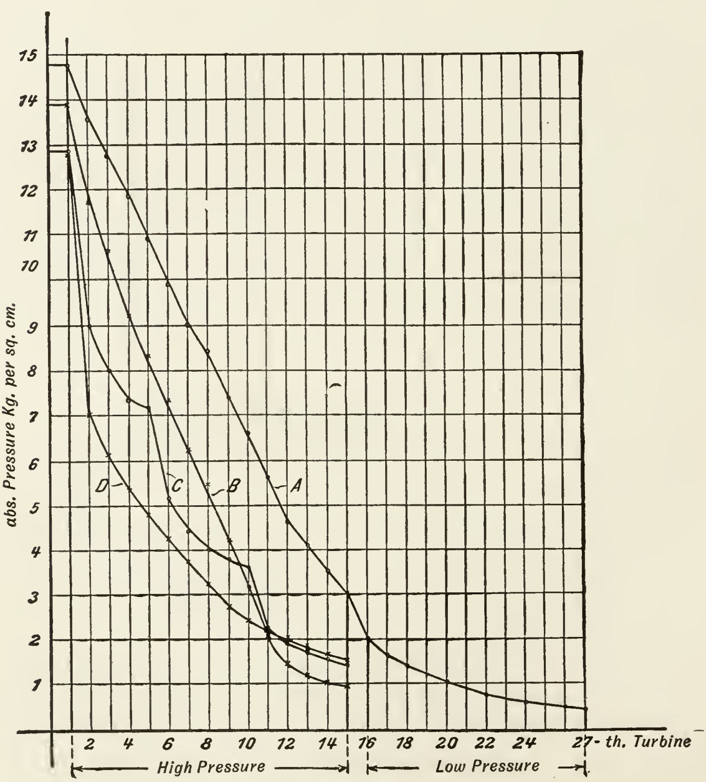


Fig. 195.

according to method B are clearly seen ; that is, working with one-sixth cross-section and full steam pressure.

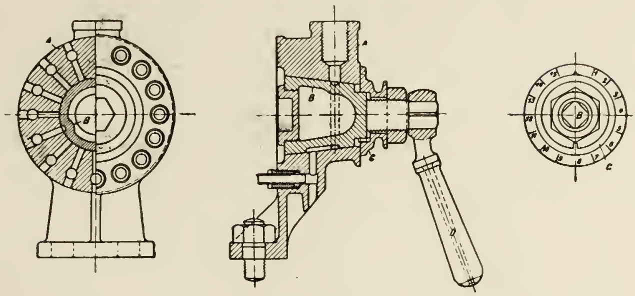


Fig. 196.

The construction of the marine type of *Schulz* turbine will be discussed later.

64. THE LINDMARK TURBINE.

*Lindmark** made use of the possibility of retransforming the flow energy of the steam into pressure, by means of conical nozzles, in order to construct a turbine with the smallest possible peripheral velocity. From curve C, Fig. 29, page 64, can be determined that with steam at 10.5 kg. per square cm. (149.4 pounds per square inch) pressure at the nozzle entrance, there is an expansion to the narrowest part of the nozzle to about 7.5 kg. per sq. cm. (106.7 pounds per square inch), and then a compression in the diverging part to nearly 9 kg. per sq. cm. (128 pounds per square inch). Hence there occurs in spite of the considerable drop of pressure, a pressure loss of only about $\frac{1}{2}$ atmosphere (7.4 pounds per square inch). The velocity at the wide end of the nozzle is so slight that the energy of flow may be neglected, and the condition of the steam may be determined by making the heat contents equal at the two extremities of the nozzle. The entire occurrence may be seen in the entropy diagram (to no certain scale) in Fig. 197. A_1 is the initial condition, A_1A_2 is the actual expansion line to the pressure at the narrowest part of the nozzle, which differs from the adiabatic line A_1A_2' by the constant increase of entropy (that is, transformation of kinetic energy into heat). The subsequent

* Formerly chief draughtsman with the Laval Company in Stockholm.

compression leads to A_3, and the actual occurrence again differs from the adiabatic compression A_2A_3' by the increase of entropy. A_3 is the point of intersection of the compression curve with the curve $\lambda_1 =$ constant and gives the final condition from its pressure and temperature. The total work of friction is represented by area $A_1B_1B_3A_3A_2A_1$, and it depends on the size of this area how much lower the final pressure p_3 is than p_1. But here also the total value does not in any way represent an actual loss; it is, however, the product of the increase of entropy B_1B_3 and the lowest absolute temperature; that is, the temperature at condenser pressure. Let this be C_1B_1; that is, equal to the area $B_1B_3C_3C_1$.

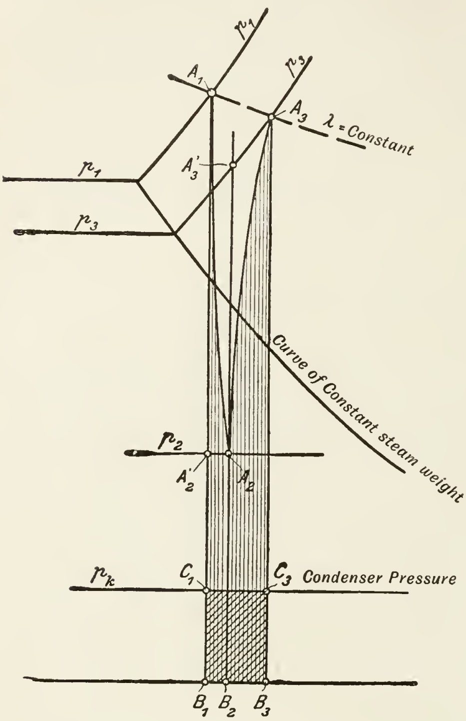


Fig. 197.

According to what has been said before, it is now probable that if the final velocity of expansion at p_3 exceeds the acoustic velocity, a steam shock occurs and the losses increase rapidly. For this reason, the transformation of velocity into pressure has a practical limit. It is entirely immaterial in what manner the steam at condition A_2 acquired its velocity. This can be the exit velocity from the rotating wheel of a turbine; if the steam is led to a nozzle it undergoes a compression, as in our experiment, and this is the scheme adopted by *Lindmark*.

Lindmark observed a part of these occurrences from his own experiments, as can be seen from his German patent No. 142 964 of Feb. 23, 1902, which was not granted until Aug. 8, 1903. He failed, however, to discover the dangerous steam shock, although he realized the increase of pressure losses with increasing expansion,

and did not intend to go below the critical pressure ratio (that is, about 0.58). The manner in which *Lindmark* developed his idea practically can be seen in Figs. 198 and 199. The first of these

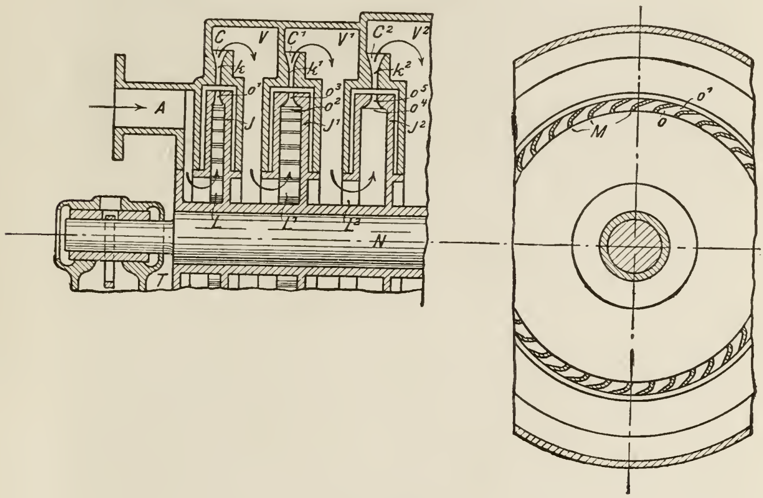


Fig. 198.

many-stage turbines (Fig. 198) worked in a certain measure with total reaction, by having the steam enter the hollow rotating wheel, and transform the entire pressure drop into velocity. A special guide wheel is unnecessary on account of the small value of the radial velocity. With relatively small peripheral velocity, a considerable exit-velocity remains, and this, as shown above, is retransformed into pressure energy by the enlarged ring channel, which must be called a "diffuser" as in hydraulic turbines. By a suitable channel the steam is led to the inside of the second wheel, and so on.

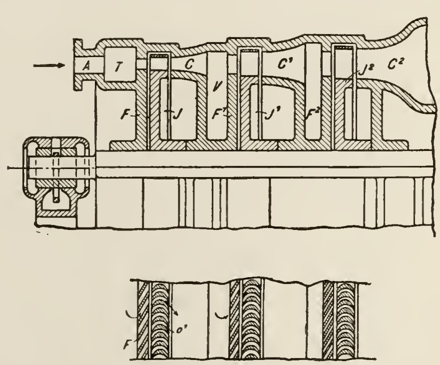


Fig. 199.

Fig. 199 shows the application of this principle to an impulse turbine with axial flow throughout. In both cases full peripheral

admission is absolutely necessary in order to avoid eddy currents at entrance to the diffuser. The inventor hopes for better results with the reaction turbine, for the following reasons: The reaction

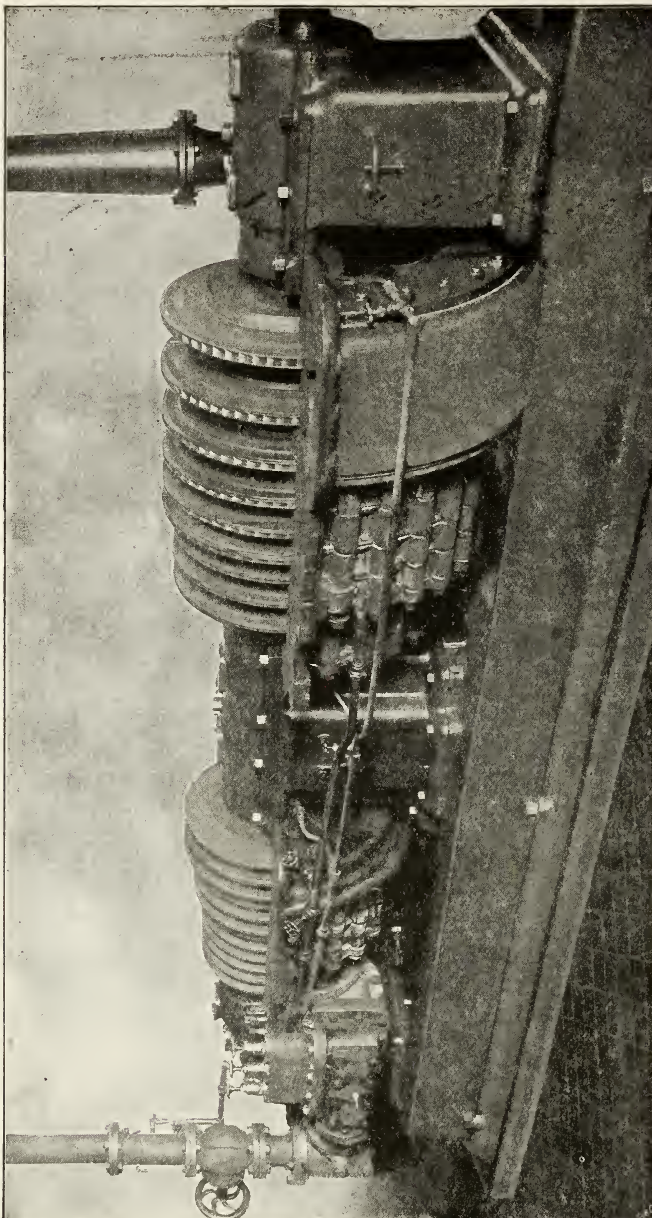


Fig. 200.

blades can be made extremely short. With pressure differences that are not excessive, the diffuser channel will also be short. High steam velocities (300–400 meters, or 984–1 312 ft.) can only occur in these short paths whose length for a single stage is confined to a few centimeters. In all the remaining parts we can work with very small velocity. The actual steam friction will be therefore very slight. The wheel friction also can be small, as there exist no idle blades, and the steam is in contact with only

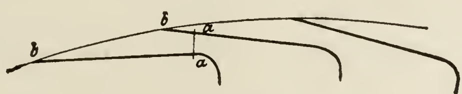


Fig. 201.

smooth surfaces. As a disadvantage of this system may be noted the necessity for full peripheral admission, which leads in large units to very narrow channels. The leakage between the wheel and its casing may be a source of disturbance, as the difference between the pressures in the clearance space and the entrance to the rotating wheel is larger than with the ordinary turbine. Still, the final verdict must be left to experience.

Fig. 200 shows the first 300 h. p. experimental design, consisting of 21 wheels with 500 to 800 mm. (19.7 to 31.5 inches) diameter, and working at 3 000 revolutions per minute. The highest flow velocity, according to reports of the inventor, was 250 to 350 meters (820 to 1 148 ft.). Fig. 201 shows a profile of the blades. The design corresponds to pure reaction, according to scheme in Fig. 198. The results of experiments with this peculiar type are awaited with great interest.

65. THE GELPKE-KUGEL TURBINE.

The turbine shown in Fig. 202 works with slight reaction and radial peripheral admission. The path of the steam, as can be seen, follows a sinuous course. Fig. 203 represents the guide blades *A* purposely made with slighter bends; *B* is the actual working rotating blades, that allow the steam to exhaust at a moderately sharp angle to the wheel periphery. In the channel *C* which leads outward are more rotating blades, that should work

with pure impulse (that is, approximately constant pressure), therefore only deviate the steam. Channel D now receiving the steam,

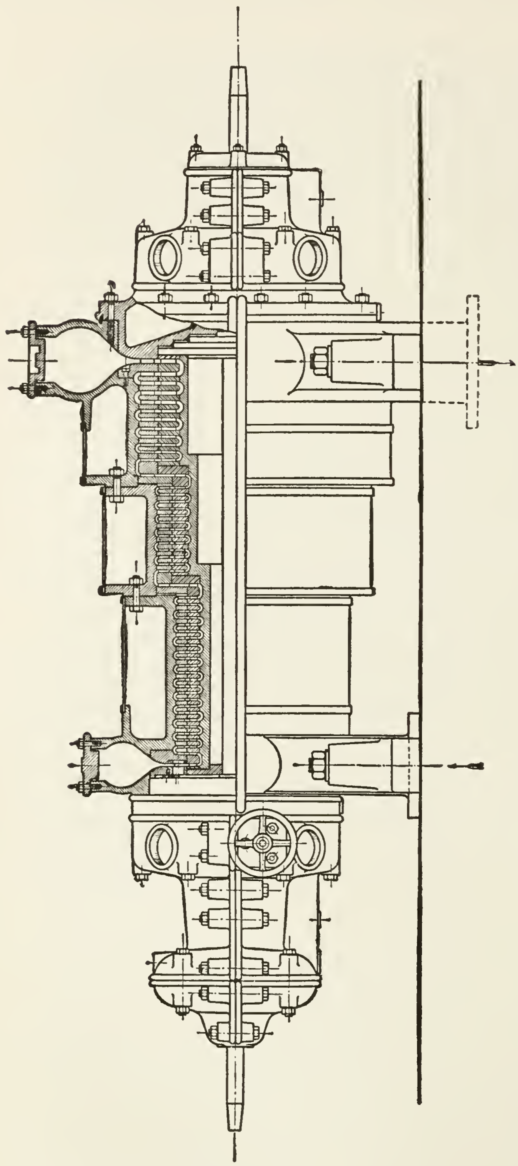


Fig. 202.

is provided with only a few blades guiding radially. By a succession of similar systems, the 25-stage turbine shown in Fig. 202 is constructed. The excess pressure on the forward end and on the free ring areas of the drums is partly balanced by pressure pistons arranged in steps (on the right hand end of the shaft), to which live steam is admitted. The main part of this excess pressure is taken up in an oil pressure thrust bearing. The blades are cut from a drawn profile and have suitable attachments for the side rings, I. In the assembled condition they are pressed together with the carrying rings L and K, on the rotating drum, or the casing, respectively.

The regulation is accomplished by turning the guide blades E of the first wheel, in the manner shown in Fig. 203, that very much suggests the usual construction of the *Francis* turbine. H is a movable ring that carries along the

guide blades E by means of the bolt G and the rod F. The high steam velocity existing during throttling at exit from these guide blades is not utilized in the rotating wheels, as a "continuity" forces the steam in the rotating blades to take as high a velocity as though we had used the usual throttling.

The constructors give as advantages of their system the fact that the blade lengths can be completely fitted to the theoretical

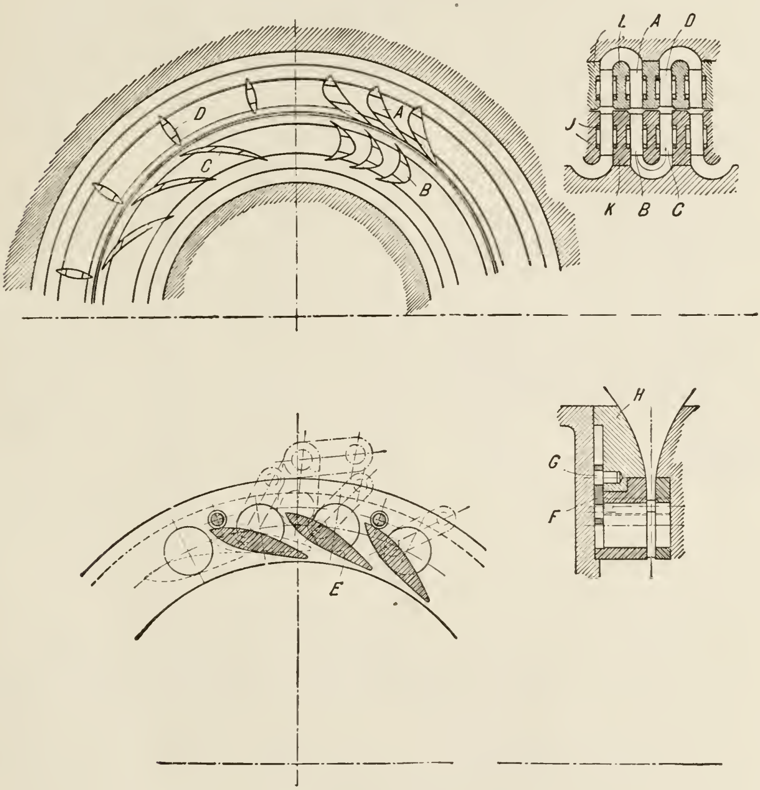


Fig. 203.

values, also that the losses in the clearance spaces are slight, because the flow takes place at right angles. The length of the steam path and the pressure of a further bearing in each guide and rotating wheel must be considered as disadvantages. Still, it is hoped that the effects of these disadvantages may be reduced to a harmless degree by the use of smaller velocities (20 to 40 meters, or 65 to 131 ft.).

In the vertical construction the very awkward divisions of the casings of other systems may be avoided, and the externally smooth drum may be inserted at the top of the casing, which is very advantageous. A turbine of 140 h. p. at 4 500 revolutions per minute has been constructed, and will soon be tested.

66. HISTORICAL REVIEW.

K. Sosnowski, in his book, "Roues et turbines à vapeur," Paris, 1897, gives some very interesting information on the fertile history of the steam turbine. The book is chiefly a compilation of old patents taken from the *Parisian Archive;* and with the permission of the author, we give the following examples.

As the oldest trace of the steam turbine, we might consider the "Eolypyle" used by the Egyptian priests, and which was described by *Heeron* of *Alexandria* about the year 120 B. C. The apparatus consisted of a hollow ball placed over a fire, and made to rotate by the reaction of a steam jet exhausting from a bent tube.

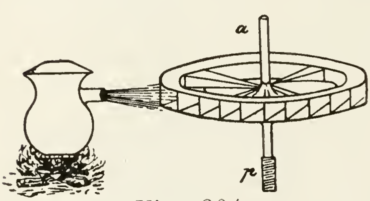


Fig. 204.

Giovanni Branca, Italian architect, suggested in 1629, the construction of a machine shown in Fig. 204. He is therefore the predecessor of *de Laval.*

Real and *Picon* constructed in 1827 the first many-stage impulse turbine (see Figs. 205 to 205*c*). The rotating wheel *g* has undoubtedly only primitive blades, the guide wheel *b* has slanting holes drilled to serve as nozzles. The enlarged sketch, Fig. 205*a*, shows how single divisions were formed by the guide discs *b*, in which the wheels *g* moved. Fig. 205 shows an assembly, in which only the steps on the shaft are shown, but which shows by counting, that it represented a 31-stage turbine.

Since *James Sadler* in 1791 described a reaction turbine of the type built by *Segner*, this principle has been numerously applied. In 1853 *Tournaire* presented to the French Academy an exceedingly clear description of a many-stage reaction turbine (Figs. 206, 206*a*). He *emphasized* that it worked chiefly because of the differences of pressure between the entrance to and exit from the blades,

which is increased by the relative velocity. The cross-section of the blade channel must be greater at entrance than at exit. He

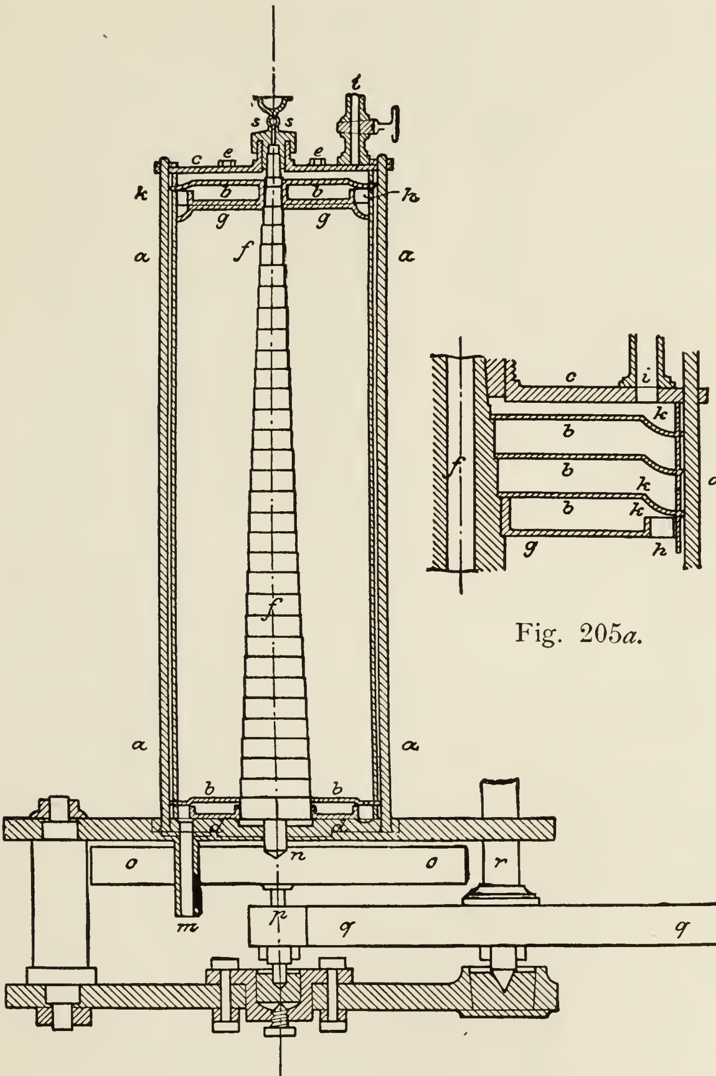


Fig. 205*a*.

Fig. 205.

recognized that the very high peripheral velocity otherwise necessary could be decreased by the application of many-stage expansion.

Perrigault and *Farcot* in 1864 took out the first patent for reversing the direction of the steam current, of course in a prac-

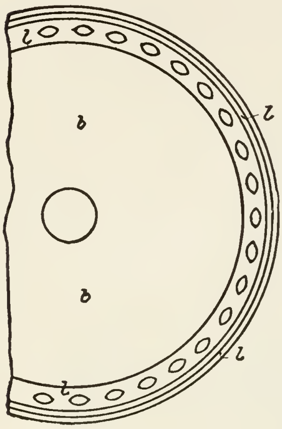


Fig. 205*b*.

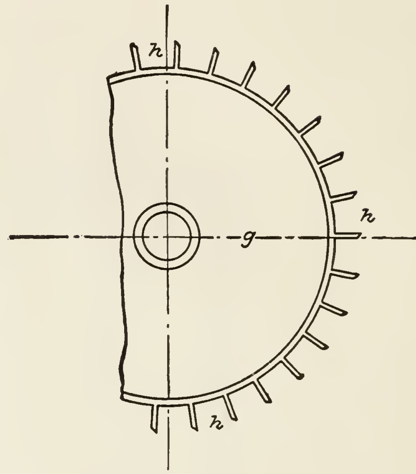


Fig. 205*c*.

tically useless form shown in Fig. 207. *Ferranti's* suggestions, shown in Fig. 208, are little better. In order to drive the steam repeatedly through the wheel, the pressure must decrease from step to step. But here is found a considerable loss in the clearance with corresponding loss in efficiency.

The turbine of *Hanssen*, 1870, is again a many-stage reaction motor with axial peripheral admission. *Cutler* in 1879 presents a relatively well-constructed radial many-stage turbine.

De Laval in 1883 made the first application of a steam turbine by applying it to his milk-separator. In 1884 *C. A. Parsons* constructed the many-stage reaction turbine with axial peripheral admission, which is kept in the Kensington Museum in London as an object

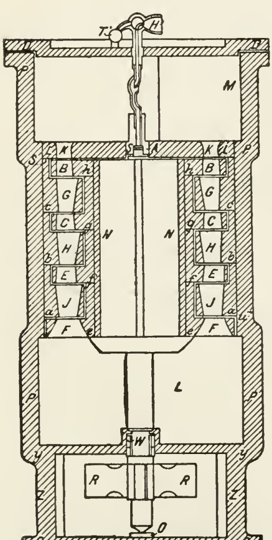


Fig. 206.

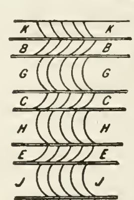


Fig. 206*a*.

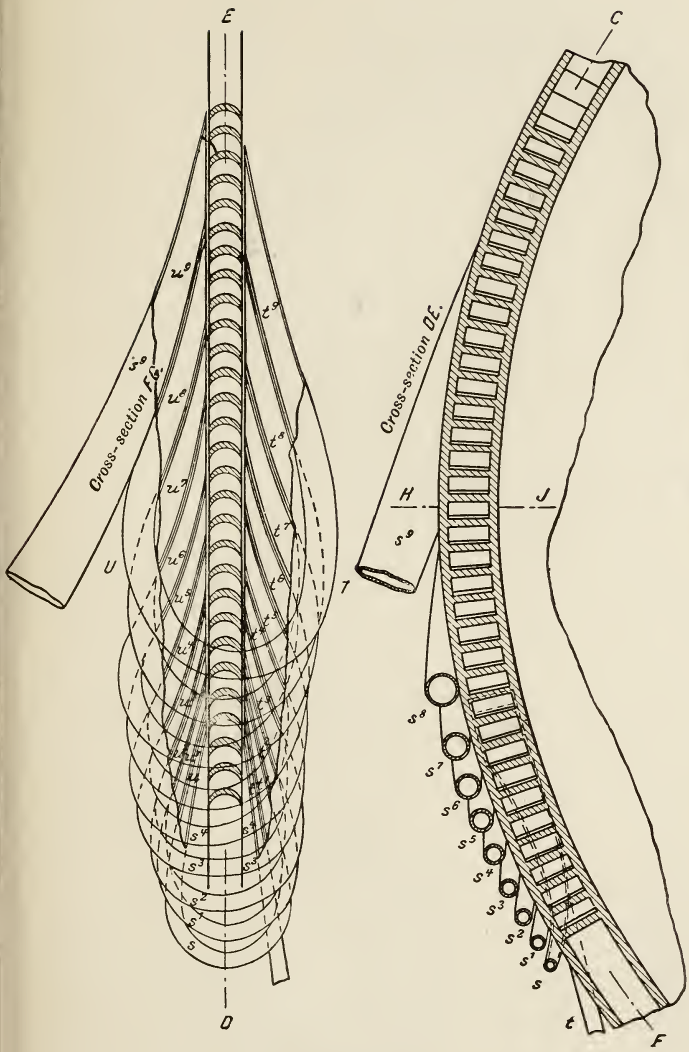


Fig. 207.

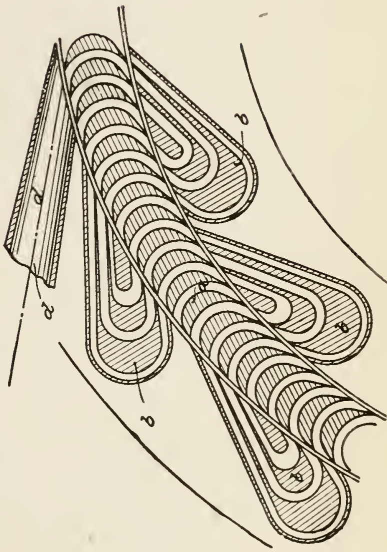


Fig. 208.

of historical interest. In 1890 followed the construction of radial peripheral admission turbines, which has been again discontinued.

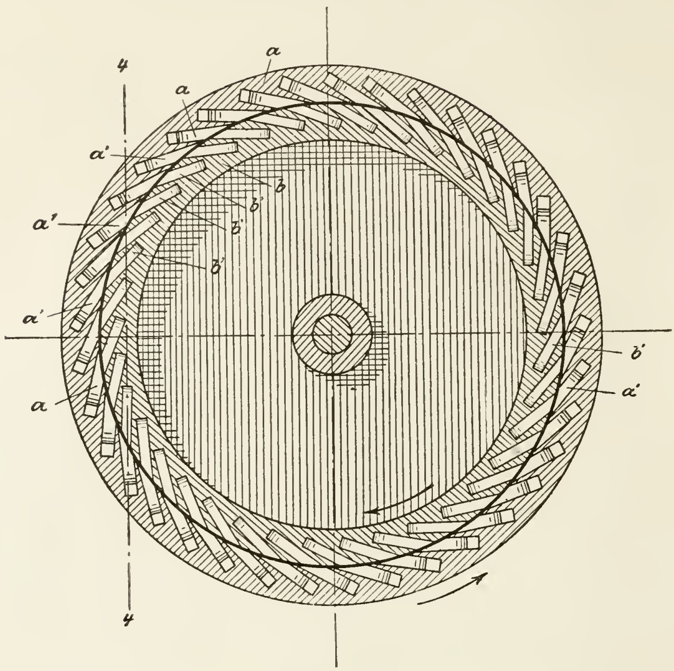


Fig. 209.

The patents of *Altham*, 1892, show design of blades similar to the *Stumpf* construction (Fig. 209), and even make use of reverse blades (Figs. 209 and 209*b*).

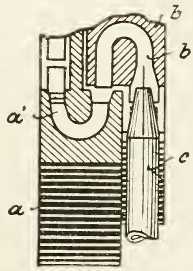


Fig. 209*a*.

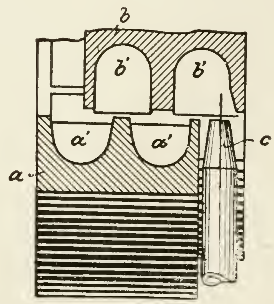


Fig. 209*b*.

The majority of the older patents showed lack of knowledge of the laws of steam flow. One idea especially led inventors on in

spite of constant failure: to *decrease the velocity of the steam by mixing it with fluids or gases.* An especially instructive experiment of this kind was made by *Escher, Wyss & Cie.*, who squirted mercury into an expanding steam jet. The experiment failed, not considering other reasons, from the fact that the finely divided mercury mixed with the condensed steam so that it was inseparably mixed.

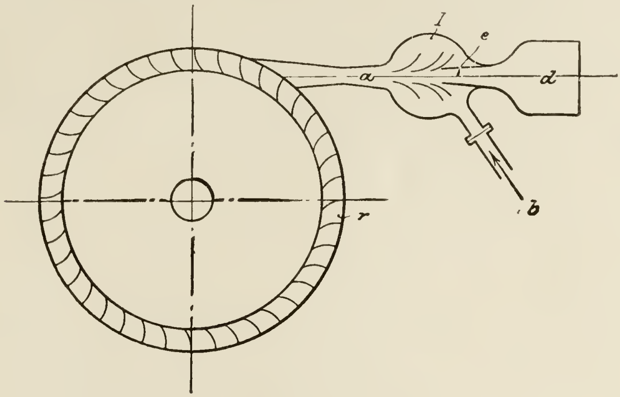


Fig. 210.

Fig. 210 shows the suggestion of *Piquet* (1894), in which a type of injector was to be used to unite the materials. The inventor recognized that the mixing took place chiefly according to the laws of inelastic bodies; and therefore, if the velocity is to be decreased considerably, there must be a loss of kinetic energy that would amount to $\frac{1}{2}$ to $\frac{3}{4}$ of the available work.*

* As patents are being taken up to the present time on this useless idea, it is well to investigate it somewhat more closely. The mixing of fluids must give, besides the loss due to shock, a poor performance in the blade channels, because the individual drops of the "rain of this mixture" must become separated from the steam mass on account of the sharp bending of its path. It therefore suffices to investigate the mixing of two similar vapors, which, according to Fig. 211, are admitted into a cylindrical mixing chamber at A through the inner nozzle F with the cross-section F_1 and the outer nozzle with the cross-section F_2. At the condition of constant flow let w_1, w_2 be the entrance velocities, p_0 the common pressure. After the eddy currents are neutralized, there exists at B a pressure p and a velocity w.

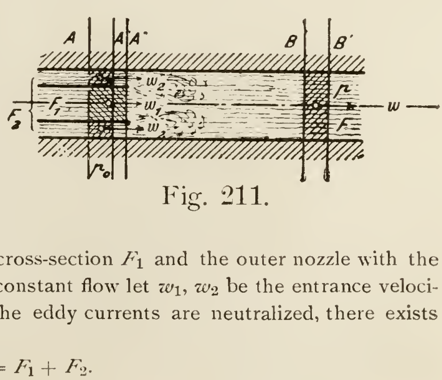


Fig. 211.

$$F = F_1 + F_2.$$

67. LATEST SUGGESTIONS.

THE FULLAGER TURBINE.*

Fullager replaces the balance pistons of *Parsons* by a few discs at the low pressure end of the turbine, which is also provided with

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Cross-section A moves in the time dt, so far as the outer tube is concerned, to A', in the inner tube to A''; cross-section B to B'. On any given element dm the force dP is exerted in the axial direction, so that

$$dm.\frac{dw}{dt} = dP \text{ or } dm.\,dw = dP.\,dt$$

and represents the equation of flow. The summation

$$\Sigma\, dm.\,dw = dt\,\Sigma\, dP \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

of all elements in the enclosed steam mass between A and B, expresses the "law of momentum," and gives for

$$\Sigma\, dP = (F_1 + F_2)p_0 - Fp = F(p_0 - p),$$

while $\Sigma\, dm\,.\,dw$ is the increase of velocity in the time element dt. The mass parts lying between B and A' or A'' respectively have equal velocities at the beginning and at the end of the occurrence; therefore between B and B' there appears a momentum of dMw, and between A and A'' in the inner tube there disappears $dM_1 w_1$, between A and A'' in the outer tube there also disappears $dM_2 w_2$. If G_1, G_2 and $G = G_1 + G_2$ stand for the weights flowing through per unit time, then

$$dM = \frac{G\,dt}{g}; \quad dM_1 = \frac{G_1\,dt}{g}; \quad dM_2 = \frac{G_2\,dt}{g};$$

and we have

$$\Sigma\, dm.\,dw = \frac{dt}{g}\left[G\,w - (G_1 w_1 + G_2 w_2)\right]$$

and equation 1 reads

$$G\,w - (G_1 w_1 + G_2 w_2) = Fg(p_0 - p) \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

Otherwise the law of energy gives the expression

$$G_1\left[\lambda_1 + A\frac{w_1^2}{2g}\right] + G_2\left[\lambda_2 + A\frac{w_2^2}{2g}\right] = G\left[\lambda + A\frac{w^2}{2g}\right] \quad . \quad . \quad . \quad . \quad (3)$$

and the "law of continuity" is

$$G\,v = Fw \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

From equations 2, 3 and 4, we can determine p, v, w of the condition at B, and convince ourselves that, in general, p is larger than p_0. We must, therefore, let the steam expand in a continued conical nozzle to the initial pressure or initial temperature in order to compare the kinetic energy before and after mixing. The calculation proves the above statement regarding the loss of energy, which can also be seen from the remark that equation 2, if we place the right side $= 0$, expresses the law of inelastic shock.

In the first edition of this book, the flow was taken as bent, also with the important assumption, $F = F_1 + F_2$; still, it is simpler and stronger to assume the mixing space cylindrical, as here done.

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* Swiss patent No. 24 039, April, 1901. As Fullager took out other patents in common with *Parsons*, we may consider these turbines as belonging to the Parsons sphere of interest.

labyrinth packing. It is identical with the *Parsons* construction in all other points (Fig. 212). The pressure of the entering live steam at A on the face area of the first drum is balanced by admitting live steam through channel a to a steam-tight space,

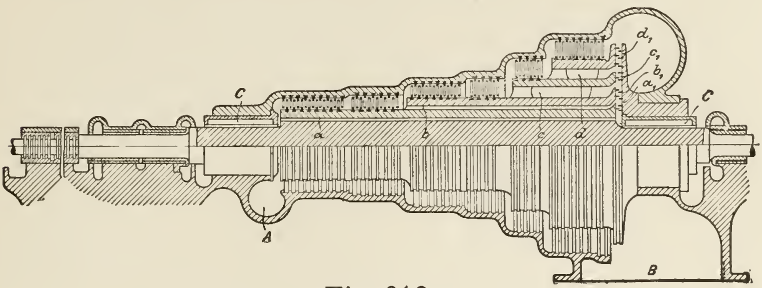

Fig. 212.

kept tight by the labyrinth ring a_1, at the right end of the turbine. The same is true for the second drum through channel b and labyrinth ring b_1, and for the others through c, c_1, and d, d_1. If there is leakage at any one place, the escaped steam can perform useful work in the next division of the turbine. These constructions greatly shorten the length of the turbine, but have the disadvantage that steam-tightness takes place in the cylindrical face surfaces at x (Fig. 212a), therefore the original position of the shaft must be absolutely maintained. Of course we could make the steam-tight contact at y, that is, allow here a very small clearance, but we must place the thrust-bearing on the right side.

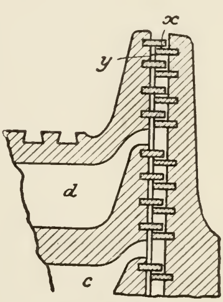

Fig. 212a.

THE SCHEME OF DOLDER.*

Dolder suggested a scheme which is similar to that of *Lindmark*, and depends on the retransformation of kinetic into potential energy. The highly superheated pressure shall, for instance, expand in a nozzle to the condenser pressure, but shall give up in the rotating wheel only a part of the flow energy. The turbine

* Schweiz. Bauzeitung, Jan., 1904.

shall therefore run slow, on the one hand, and have large exit angles on the other. The steam flowing out is deprived of some of its quantity of heat in a cooling chamber, but its kinetic energy is not destroyed. From here on it is compressed in a nozzle to the initial pressure by its own kinetic energy. In Fig. 213 (to no definite scale) is represented the entropy diagram of this occurrence, with the assumption of frictionless flow. $A_1 A_2$ is the adiabatic expansion; at condenser pressure p_2, the quantity of heat taken away must be

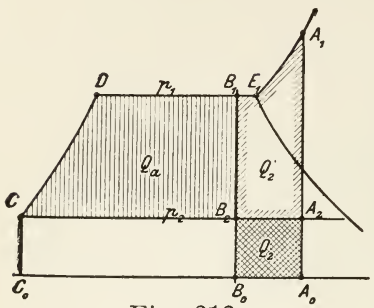

Fig. 213.

$$A_2 A_0 B_0 B_2 = Q_2,$$

so that in condition B_2 the existing kinetic energy of the steam is sufficient to compress it to p_1. To accomplish this, according to what has already been given, it is necessary that the kinetic energy in heat units must equal the area

$$C D B_1 B_2 = Q_a.$$

The original kinetic energy at the entrance to the wheel is equal to the area $C D B_1 A_1 A_2 C$, and it follows that this process has changed the area

$$B_1 A_1 A_2 B_2 B_1 = Q_i$$

into indicated work.

This theoretically interesting idea, which does away with an air pump, and if we move B_1 to E_1 would insure a good efficiency, raises an objection, that the flow in practice occurs with resistances which absorb the kinetic force, so that B_2 is moved to the left; that is, more heat must be taken away than was assumed above. Hereby the efficiency would be greatly decreased, for purely theoretical reasons. Of more importance is the question, as seen from our experiments, whether, be it in either converging or in the diverging nozzle, such considerable compressions as are here de-

manded *can by any means be reached.* The deciding objection is, finally, that the steam while its heat is being taken away will only condense locally against the cooling surfaces, and these condensed parts will lose practically all their velocity. The steam (not wet enough) remaining behind is not capable even with frictionless flow of compressing from B_2 to B_1; because the entire moisture of the steam corresponding to condition B_2 must flow along with the steam with equal velocity in microscopically fine divisions. Even if we confine ourselves to the territory of pure superheat, — that is, move B_2 to the curve of constant steam weight, in which the efficiency would become still higher, — condensed steam would form against the cooling surfaces. For these reasons, I consider this scheme practically, entirely unfeasible.

THE C. A. PARSONS DOUBLE-MOTION TURBINE.

In the English patent No. 6 142, year 1902, *Parsons* describes an impulse turbine, in which the nozzles are arranged at the circumference of a hollow wheel, and receive an equally large but opposite peripheral velocity to the rotating wheel. The effect is the same as though the nozzles were at rest, and the rotating wheel turned with double the absolute velocity, because the relative motion depends only upon the relative velocity of the nozzle and rotating wheel. We must, of course, subtract the consumption of work necessary to accelerate the steam particles to the peripheral velocity, and obtain a very little smaller exit velocity than with nozzles at rest. The advantage of this arrangement consists, therefore, in decreasing the revolutions to one-half (with diameters remaining equal), but has the disadvantage of complicating a shaft division, double construction of the dynamo, and doubling the stuffing boxes against the full pressure at the steam entrance.

According to reports in English technical journals ("Engineering," 1903), English manufacturers were going to apply this double motion to their many stage turbine, by allowing the casing to rotate in the opposite direction. The difficulty in guarding against steam leakage and of utilizing the power makes this idea appear as an experiment that has entirely failed.

Brady applied the double-motion to the few stage radial

turbines, where its impracticability may be seen from the suggested blade construction.

Here also may be mentioned an English patent of the *Siemans-Schuckert Works,* who proposed alternating current dynamos connected in parallel for the above described type of double-motion turbines, in which the division of power of the parallel working alternators automatically tends to preserve uniformity of revolutions.

THE PARALLEL CONNECTING ALTERNATORS OF PARSONS TURBINE.

According to the English patent No. 19 031, 1902, *Parsons* attempts to make the intermittent steam admission, characteristic of his turbines, synchronous with all motors in a parallel-connected group. We could derive from this, that with non-synchronous admission, difficulties may arise ; still, other observers state (for instance, the Electrical Works at Frankfort), that the steam turbine can be instantly connected in parallel with any given reciprocating engine.

THE TURBINE OF NADROWSKI.

Nadrowski uses, according to German patent No. 137 586, for regulating the admission of a radial turbine, the rotating body shown in Fig. 214, that may be moved axially and has a profile so constructed that the ratio of the cross-section F_m at the narrowest place to the end cross-section F_2 remains constant. This widening space of ring form, acting as a "nozzle," allows the steam to expand constantly to the same final pressure, and we get a "quantity" regulation. Unfortunately, this ring nozzle, applied practically, does not permit a sufficiently high peripheral velocity if we do not wish to reduce the clearance space to a fraction of a millimeter.

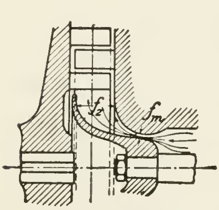

Fig. 214.

The immense number of constructions that have failed entirely, and above all, those that appear in the patent literature, cannot be entered into now, in spite of the fact that a knowledge of these

failures would undoubtedly save the beginner much useless inventive labor. All in all, it is not too emphatic a statement to say that there is hardly to be expected a greater number of misconceptions than already exist in steam turbine construction.

68. THE MARINE STEAM TURBINE.

The chief advantage which is derived from the application of the steam turbine to marine purposes, not considering the points that would apply also to turbines used on land, is the following: non-existence of vibration and economy of space. The economy of weight is also, according to the type of the turbine system, of more or less importance. Against these advantages are also disadvantages; above all, the impossibility of simply reversing, which forces us to install an especial reversing turbine that runs empty when the vessel is moving forwards. It is then allowed to run in vacuum to decrease the losses due to friction. Even if the economy of this turbine is of secondary importance, still, it should develop the full power for running backwards with the available boilers. Of course this demand will be made only in case of necessity (danger of collision), but for this it is all the more useful. The second important disadvantage is the great number of revolutions, which give the ship's propeller a too large peripheral velocity, causing the formation of hollow spaces, the so-called "cavities," and thereby favoring the creation of eddy currents. *Parsons* is to be thanked for valuable experiments with marine turbines, and his turbines are until now the only ones that have been successfully applied in ship-building. Parsons divides the turbine into several parts driving special propellers, and connects them in "series" or "parallel." The question whether one, two or three small propellers are to be placed on a shaft, and especially the determination of the most favorable propeller dimensions, belongs to ship-building. We shall confine ourselves to the following remarks of interest to the turbine constructor. The experiments of Parsons led to the general decrease of the number of revolutions; in fact, to about 500 to 1 000 per minute. It is obvious that the peripheral velocity of the turbine is correspondingly decreased at such low number of revolutions, as otherwise the drum would be too large and the blades too short. The steam velocity would advantageously be made low, and we get

as a further result (for several reasons) a considerable increase in the number of stages. If the steam friction (under otherwise equal conditions) depends only on the square of the velocity and the length of path, the economy would not decrease in spite of a longer steam path. According to reports of Parsons in English technical journals, it seems that the consumption per effective horse power is over 7 kilograms ($15\frac{1}{2}$ pounds) per hour, and would point to the fact that the splitting up of the steam by the face surfaces of the blades forms the chief part of the resistances.

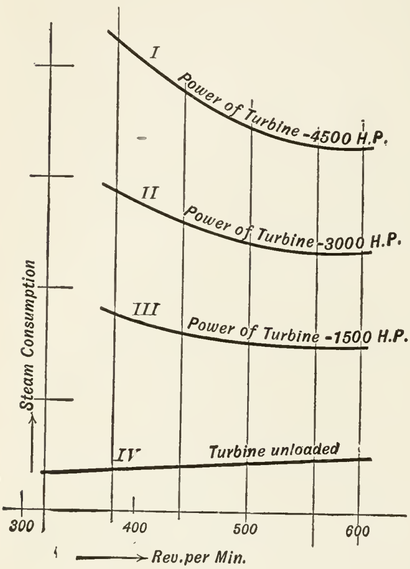

Fig. 215.

Special difficulties are brought up for the case of war vessels running at reduced speed, because here also the number of revolutions of the screw decreases; hence the steam consumption increases. *Grauert* in "Marine-Rundschau," 1904, p. 44, gives quantitative results concerning this and are reproduced in Fig. 215. With a load of 4 500 h. p. the steam consumption of the Parsons marine turbine increases 31 per cent in going from 580 to 380 revolutions. With 1 500 h. p. this difference is only 21 per cent; still, the steam consumption is in itself greater. If we estimate the quantity of steam per hour as 7.2 kilograms (15.9 pounds) per horse power at 4 500 h. p. and 580 r. p. m., then according to Fig. 215 there would be a consumption of 9.4 kilograms (20.7 pounds) per h. p. at 4 500 h. p. and 380 r. p. m.; further, 19.4 kilograms (42.7 pounds) per h. p. at 1 500 h. p. and 580 r. p. m.; and 23.3 kilograms (51.3 pounds) per h. p. at 1 500 h. p. and 380 r. p. m.

It is now of great importance to consider means of partially doing away with the increase of steam consumption which also exists with the reciprocating engine. As an example to show in which direction the inventor's energy is being exerted, we shall quote from the English patent No. 8 378,—1901, of *R. Schulz* (Fig.

216). The chief thought of the inventor is, to divide the turbine into several parts, or, what is the same thing, to construct it of several single turbines, which are capable of working either individually or in groups, and also either in series or parallel. If the number of revolutions, also the peripheral velocity, is small, the turbine is connected in series in order to get a large number of stages, and thereby obtain a good utilization of the steam ; but if the highest power is to be obtained, we use the single turbines in parallel and get satisfactory utilization of steam while the peri-

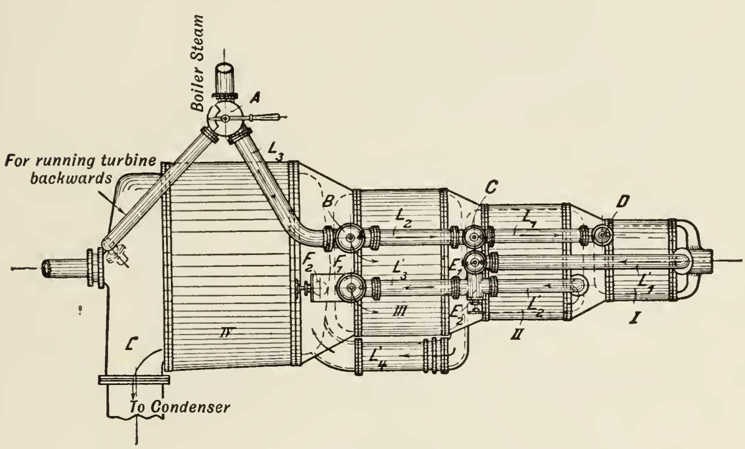

Fig. 216.

pheral velocity remains high. The patent of *Schulz* suggests four turbine divisions (I to IV) through which the steam can be led by the pipe L_1 and the valves A, B, C, D. *Pipes* L' take care of the overflow from one turbine to another ; L'' leads to the condenser. The flow in I, II, and III, is from left to right; in IV the reverse. The propeller thrust is balanced by the excess of steam pressure on the face area of the large turbine, No. IV. The reversing turbine is on the left end, and has its own steam pipe.

Schulz mentions the following turbine combinations :

1. Lowest power, smallest number of revolutions : all turbines connected in series. The steam enters at D, flows through L_1' to turbine II, through L_2' past valve E_1, to turbine III, through L_4' to turbine IV, and from there to the condenser.

2. Next higher power, larger number of revolutions: turbine I is cut off by valves D and E_1, the steam enters at C to turbine II, flows through this and then through III and IV.
3. Next higher power: turbines I and II are cut off, III and IV connected in series.
4. Next higher power: turbines III and IV in series as before; and to increase the power, turbines I and III are connected in parallel; that is, live steam enters through B in turbine III and also through D in turbine I and is led from the latter through L_1', L_3', and valve F_2 direct to turbine IV. E_1 and F_1 are valves suitably constructed to enable the entire turbine II to be cut out so that the formerly used entrance to turbine III may be used.
5. For the higher powers, turbine II, and finally I and II, are connected in parallel with III, and IV is used as a low pressure unit.

There is no doubt but that this large number of combinations is superfluous in practice, but the result was worthy of endeavor.

Nevertheless, the design of a marine turbine of this type, for the best steam utilization under the above mentioned manifold conditions, and with a resulting axial pressure that is constantly equal to the propeller thrust, forms an exceedingly interesting problem in design. It is evident that the blade angles would not suit equally well *all* these combinations. The uniformity of the power parabolas of the *Laval* turbine, shown on page 99, permits us to state that the shock that would occur by having unsuitable peripheral velocity at the entrance to the rotating wheel would cause no considerable losses. According to the curves of the *Parsons* turbine (Fig. 215), the most favorable peripheral velocity decreases only slightly with decreasing power. The experiments of *Schulz* (Fig. 194) show, on the other hand, a depression in the power curve, with small power and small steam velocity, which points to increased losses on account of the excessive peripheral velocity. We must, therefore, on the one hand, by a series of progressive trials, find the corresponding number of revolutions for a certain power; on the other hand, determine the power parabolas for the given turbine system in order to decide which mean value of the velocities has the advantage of entrance without shock, with which the blade angles must then agree.

Peculiar is the suggestion of *Stumpf*, that a double-motion turbine be constructed with peripheral admission on both sides (Fig. 217), and the power of both wheels made equal. The power of the first wheel should be transmitted by means of a solid, and that of the second by means of a hollow shaft, to a propeller. The nozzles A' and B' must follow one another at such distances that the steam from either the right or the left can exhaust freely. The objections to this idea are that if two conjugate wheels are used, the entrance to the temporarily second wheel is accompanied by shock. If the blades are correctly constructed for the one nozzle system, then they are unfavorable for the other. According to *Riedler's* previously mentioned paper, the opposite running wheels are constructed as axial turbines, each with a special blade rim ; but in this the power of the second wheel must be considerably smaller than that of the first. *Grauert* also states, that the arrangement with a hollow shaft was suggested by the German navy, but was abandoned on account of difficulties of construction. The opposite rotation of the screw has, on the other hand, as Riedler correctly mentions, the advantage that the turning motion given to the water by the first screw is neutralized by the second, and therefore gives the propeller a higher efficiency.

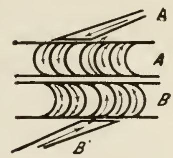

Fig. 217.

69. THE STEAM TURBINE AND THE RECIPROCATING ENGINE.

A comparison of both motor types must be made on technical and practical grounds. In the first place, the turbines of *de Laval* and *Parsons* look back upon a number of years of practical use, during which time they in general successfully stood the test. The Parsons turbine especially has come widely into use in recent years, and it is stated that there is over one-half million horse-power in actual use or in course of construction. Among the machines are a large number of 3 000 to 5 000 h. p. units, and even one of 10 000 h. p. The mistrust generally felt by the man in practice toward a new innovation seems to be disappearing ; in its place there is an increasing tendency in favor of this convenient source of power.

There may be named the following objections that might naturally arise and partly still exist : the high number of revolutions with the accompanying vibrations, never entirely avoidable, and the consequent danger of running hot ; the delicate and highly accurate work of installing certain parts ; the rapid wear on parts required to run steam tight ; the high material stresses ; and the requirement of exceedingly careful and intelligent attendance. There was doubt as to how long the high economy found in the new machines would be maintained, because the wear of the bearings, the blade tightness, and incorrect installation would increase the losses due to leakage, and decrease the efficiency.

Practical experience has already proved how groundless were part of these objections, and the advantages of the steam turbine are slowly becoming fully recognized. They are so obvious that they need not be explained in detail, and are here merely mentioned. The smaller number of moving parts, less weight, less space needed, easier taking apart and repairing, no necessity of internal lubrication, excellent regulation, disappearance of stresses due to unsymmetrical heating, suitability for highest superheating, quick starting, uniform speed, ease of running in parallel, etc.

To compare the use of both motors practically, we shall recalculate the above given consumption figures of the h.p.$_i$ hour of a turbine, with the figures of a *reciprocation engine of the same power*. For the *Parsons* turbine we have, according to the experiments of *Stoney*, *Schröter*, *Westinghouse*, and the Electrical Works in Frankfort, the following table :

Power . . . . Kw.	24.7	52.7	108.5	123.0	119.0	226	507	1190	1998	2995
Superheat . . . . . C.°	. . .	. .	. . .	30.0	46.7	32.2	36.7	10.2	23.2	126.6
Superheat . . . . . F.°	. . .	. .	. . .	54.0	84.1	59.0	66.1	18.4	41.8	236.9
Assumed Efficiency of the Dynamo	0.85	0.87	0.87	0.87	0.87	0.92	0.92	0.92	0.93	0.93
Assumed Efficiency of the Reciprocating Engine of equal power	0.85	0.87	0.87	0.87	0.87	0.92	0.92	0.94	0.94	0.94
Steam Consumption of the Turbine per Kw. Hour . Kg.	13.06	12.7	12.16	11.57	11.02	9.98	9.57	8.81	8.67	6.70
Steam Consumption of the Turbine per Kw. Hour . Lb.	28.7	27.9	26.8	25.5	24.2	22.0	21.1	19.4	19.1	14.7
Steam Consumption of the Reciprocating Engine in h. p.$_i$ per Hour . . . Kg.	6.96	7.07	6.77	6.44	6.12	6.21	5.95	5.60	5.57	4.31
Reciprocating Engine in h p.$_i$ per hour . . . Lb.	15.5	15.8	15.1	14.4	13.7	13.9	13.3	12.5	12.4	9.6

So far as it concerns impulse turbines, we refer to the data given in Articles 56 to 61.

The figures given above permit us to say *that the steam turbine working with moderate superheat has exceeded the compound reciprocating engine in steam economy.* All signs point to the fact that with high superheating, this comparison will not change.*

It is different with the triple expansion reciprocating engine. This shows with smaller powers, up to 1 000 kw., such a small steam consumption that its efficiency, taking also into consideration the consumption of oil, space required, etc., may be said to place it far in advance of the turbine. With very large powers, on the other hand, these relations are reversed. The best results obtained by *Brown, Boveri & Cie.*, in Frankfort, with about 3 000 kw., or according to our assumptions about 4 660 indicated h. p., gave for a reciprocating engine of equal power, without power required for driving the air pump, 4.31 kg. per h. p.$_i$ hour (9.6 lb. per English h. p. hour); by adding 1.5% for the air pump, 4.37 kg. per h. p.$_i$ hour (9.7 lb. per English h. p.$_i$ hour) or (feed water assumed at 0° C. = 32° F.) about 3 160 calories per h. p.$_i$ hour (12 714 B. t. u. per English h. p.$_i$ hour). On the other hand, the triple expansion steam engine of the Berliner Electric Works,† at 12.3 atmospheres (180.8 lb. per sq. in.) boiler pressure, 314° C. (597.2° F.) temperature of superheat, and 2 550 h. p.$_i$, a consumption of 40.5 kg. (8.93 lb.) or about 2 930 calories per h. p.$_i$ hour (11 789 B. t. u. per English h. p.$_i$ hour). With hardly more than one-half the power, *the triple expansion steam engine has therefore still an advantage of about 8% of the steam consumption.*

The question also arises as to the difference of oil consumption; no exact data is at hand, but we may say that the steam turbine is from 5 to 10% more economical. If we take into consideration

* Lately, according to the *Ze tschrift des Veriens deutscher Ingenieure*, 1903, p. 725, *Professor Schröter* with a 250 h. p. compound engine of *Van den Kerkhove* has attained a consumption of 5.28 kg. per h. p.$_i$ per hour (11.8 lb. per English h. p.$_i$ per hour) of saturated steam; with superheated steam at 304.6° C. (580.3° F.) a consumption of 4.31 kg. (9.6 lb.); or, a consumption of heat of 3 490 and 3 108 calories per h. p.$_i$ hour (14 042.1 and 12 505.2 B. t. u. per English h. p.$_i$ hour) respectively. For this machine the above statement does not hold, and we must wait to see how long this low consumption figure, not hitherto reached, will stand, and also whether it can be reached with other reciprocating engine types.

† Zeitschr. d. Ver. deutsch. Ing., 1902, p. 187.

the care, and with horizontal machines the space required and the cost of foundation, the greater economy will be found on the side of the steam turbine.

In very numerous installations, the machine is not constantly at full load, so there is an important question to be answered

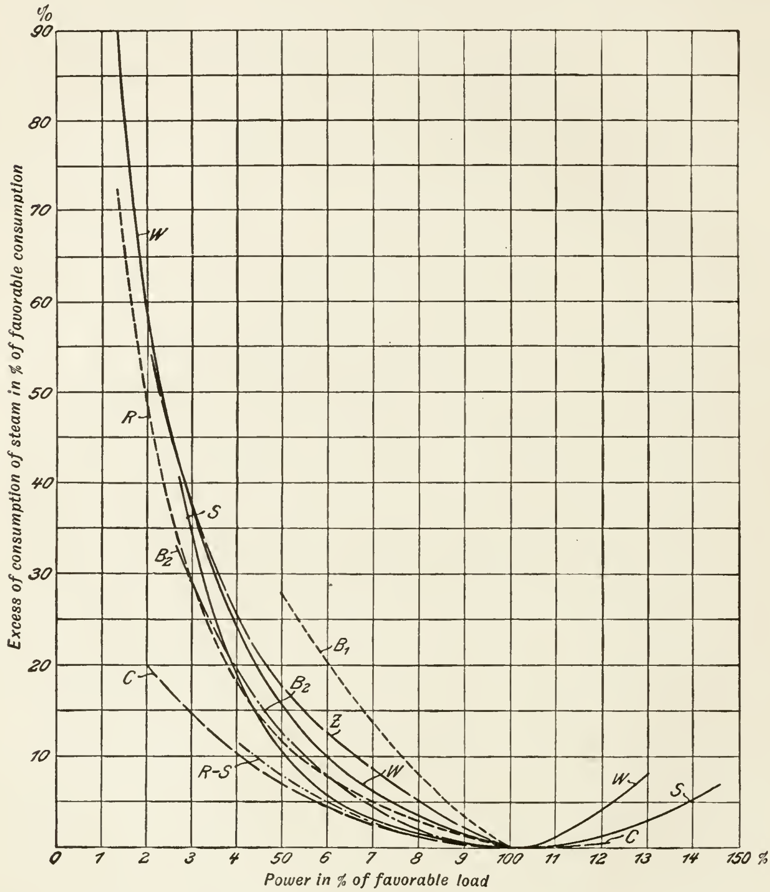

Fig. 218.

how much the steam consumption changes with the reciprocating engine and the steam turbine at over or under load. The relation is best seen by comparing, not the absolute values, but the changes in per cent. As the idea of normal load is not sufficiently definite, there is given in the graphical representation, Fig. 218, the most favorable steam consumption (per kw. hour); and that load at

which it occurs is taken at unity. As abscissas we take the ratio of the actual power to the just defined "normal load;" as ordinates, the increase of steam consumption per kw. hour from the lowest value, in per cent, of the latter. The comparison in this form is entirely free from objections, as there exists no general agreement in regard to the value of the overload that can be demanded of a steam motor; on the other hand, it may represent the correct standard for the steam turbine, because it is forced, on account of economy, to work so that the normal load is reached at full admission pressure at entrance to the first guide wheel. The overload must be taken care of by an overload valve of any type. Fortunately, experiments are at hand with a Westinghouse-Parsons turbine, that works with an overload valve. The British Thomson-Houston Co. also give results of experiments where the turbine was loaded throughout the entire limits of favorable steam consumption.

In the few stage impulse turbine with only a few wheels, live steam is admitted to the first wheel at overload through opening nozzles or guide blades. As the guide apparatus leading to the second wheel retains its cross-section, a back pressure occurs in the first chamber, and therefore a less favorable working than at normal load, which holds good for all velocities and blade angles. Still, the gain is greater than if steam were introduced into a lower stage.

The following may be said in explanation of Fig. 218:

Curve C refers to a Curtis turbine of 500 kw. rated power, according to "Engineering," 1904, I., p. 182.

Curve R–S refers to a Riedler-Stumpf turbine in Moabit, according to p. 193 of the source referred to.

Curve S represents the normal change of steam consumption of a triple expansion steam engine, according to reports of Sulzer-Winterthur Bros.

Curve R represents a Rateau turbine, according to experiments on p. 266–267.

Curve Z represents the Zölly turbine, according to experiments on p. 242–243.

Curve W stands for the consumption of a Westinghouse turbine of 1250 kw. rated power, according to "Power," 1904, p. 130.

Curve B_1 refers to a Brown-Boveri-Parsons turbine of 400 kw. power, of old design, according to Zeitschr. d. Ver. deutsch. Ing., 1904, p. 120.

Curve B_2 refers to the same, but of a newer and larger design, according to the firm's reports.

The figure permits the determination that the two first named, between 70 and 100% of normal load, are identical with the steam engine, and are outdone at smaller powers. The remainder, with the exception of very small powers, are less favorable. The overload with the Westinghouse turbine is very much less favorable than with the reciprocating engine; but with the Curtis, again, very much more favorable. The advantage over the reciprocating engine therefore, and also the character of the steam consumption curve at various powers, is doubtful.

The choice between turbine and reciprocating engine depends, therefore, considerably on the degree of reliance which the purchaser places in the safety of operation of the two motor types.

V.

A FEW SPECIAL PROBLEMS OF STEAM TURBINE THEORY AND CONSTRUCTION.

70. DISTRIBUTION OF PRESSURE IN A CROSS-SECTION OF AN EXPANDING GAS OR STEAM JET.

It was stated in Article 16 that taking the pressure, density and velocity in a cross-section at constant value, is only a first approximation. The difference between this approximation and the true value will be considerable in sharp bends, and it is of practical importance to get quantitative results concerning this. With certain simple assumptions, it is possible, in fact, to give integrals for the general hydrodynamic laws of flow for *elastic* fluids, as shall here be given.

We shall assume that with frictionless flow of an elastic fluid, referred to a fixed plane, there are parallel stream courses. If x, y are rectilinear coördinates in this plane, u, v the velocity components parallel to x and y respectively, p the pressure, μ the mass per unit volume, then the well-known *Euler's* formulæ of flow are, if the mass-forces are neglected,

$$\left.\begin{aligned} \mu \frac{du}{dt} &= -\frac{\delta p}{\delta x} \\ \mu \frac{dv}{dt} &= -\frac{\delta p}{\delta y} \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

The law of continuity for *normal condition*, that is, steady flow, may be expressed as

$$\frac{\delta \mu u}{\delta x} + \frac{\delta \mu v}{\delta y} = 0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

Multiplying equation 1 by $u\,dt$ or $v\,dt$, respectively, we also get after addition and determined integration, an equation for steady flow,

$$\tfrac{1}{2}\,(u^2+v^2)+\int\frac{dp}{\mu}=\text{constant} \quad . \quad . \quad . \quad . \quad (3)$$

In this formula u^2+v^2 is the square of the resultant velocity. The flow occurs without "rotation" of the fluid particles; there exists, therefore, the so-called velocity potential; that is, a function $\phi(xy)$ of the property that is,

$$u=\frac{\delta\phi}{\delta x}, \qquad v=\frac{\delta\phi}{\delta y} \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

The relation between pressure and specific mass shall be expressed by the equation,

$$p=\alpha^2\mu \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

or, if v' represents the specific volumes (in the meaning as used until now), $\mu=\frac{\gamma}{g}=\frac{1}{v'g}$, also,

$$p\,v'=\frac{\alpha^2}{g} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5a)$$

It shall be assumed, for simplicity, that the change of condition of gases occurs isothermally, because any other assumption would lead to insurmountable difficulties. With vapors, the adiabatic law would be a closer assumption. The solution of equation 5*a* gives

$$\alpha=\sqrt{g\,p\,v'} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5b)$$

and shows that α is the acoustic velocity of the isothermal change of condition. Equations 2 to 5 now permit the elimination of u, v, p, μ, and allow ϕ to be determined. To accomplish this, take p from equation 5 and place it in equation 3, giving

$$\frac{1}{2}\left[\left(\frac{\delta\phi}{\delta x}\right)^2+\left(\frac{\delta\phi}{\delta y}\right)^2\right]+\int\alpha^2\frac{d\mu}{\mu}=\frac{1}{2}\left[\left(\frac{\delta\phi}{\delta x}\right)^2+\left(\frac{\delta\phi}{\delta y}\right)^2\right]+\alpha^2\ \text{nat. log}\ \mu$$
$$=\text{constant} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3a)$$

But equation 2 reads

$$\mu\frac{\delta u}{\delta x}+\mu\frac{\delta v}{\delta y}+u\frac{\delta\mu}{\delta x}+v\frac{\delta\mu}{\delta y}=0,$$

or, by dividing by μ, and allowing

$$\frac{u}{\mu}\frac{\delta \mu}{\delta x} = u\frac{\delta \text{ nat. log } \mu}{\delta x}$$

then

$$\frac{\delta^2 \phi}{\delta x^2} + \frac{\delta^2 \phi}{\delta y^2} + \frac{\delta \phi}{\delta x}\frac{\delta \text{ nat. log } \mu}{\delta x} + \frac{\delta \phi}{\delta y}\frac{\delta \text{ nat. log } \mu}{\delta y} = 0 \quad . \quad . \quad . \quad (2a)$$

Solve equation $3a$ in terms of nat. log μ, and place the partial derivatives according to x and y in equation $2a$. This gives

$$\frac{\delta^2 \phi}{\delta x^2} + \frac{\delta^2 \phi}{\delta y^2} - \frac{1}{\alpha^2}\left[\left(\frac{\delta \phi}{\delta x}\right)^2 \frac{\delta^2 \phi}{\delta x^2} + 2\frac{\delta \phi}{\delta x}\frac{\delta \phi}{\delta y}\frac{\delta^2 \phi}{\delta x \, \delta y} + \left(\frac{\delta \phi}{\delta y}\right)^2 \frac{\delta^2 \phi}{\delta y^2}\right] = 0. \quad (6)$$

The value α^2, referred to the above dimensions, may, as can readily be proved, be eliminated by taking

$$\phi = \alpha \psi \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6a)$$

If now the derivatives with respect to x and y be denoted by the subscripts 1 and 2, the differential equation reads:

$$\psi_{11} + \psi_{22} - [\psi_1^2 \, \psi_{11} + 2 \, \psi_1 \, \psi_2 \, \psi_{12} + \psi_2^2 \, \psi_{22}] = 0 \quad . \quad . \quad (6b)$$

Professor A. Hirsch of Zürich has taken the trouble to derive methods for the integration of this quite complicated equation, and found, among others, the following results:

Let n be any positive whole number > 1, p and q two independent parameters, their functions x, y as well as the solution of ψ, are to be represented in common. With the designations

$$N = \frac{n\,(n-1)}{2}$$

$$N_k = \frac{N\,(N-1)\ \ (N-2) \cdots (N-k+1)}{1 \cdot 2 \cdot 3 \ \cdots k}$$

$$t = p^2 + q^2$$

we write the function *Nth* degree, as

$$F\,(t) = \sum_{k=0}^{N} (-1)^k N_k \frac{n\,!}{(n+k)\,!}\frac{t^k}{2^k}$$

and its derivative,

$$\frac{dF(t)}{dt} = F'(t).$$

Further, the functions P_n and Q_n are defined by the equation

$$(p + q\,i)^n = P_n + i\,Q_n,$$

in which i represents -1; then the solution of the equation 6b can be expressed as follows:

$$x = n\,[a\,P_{n-2} + b\,Q_{n-2}]\,F(t) + 2p\,[a\,P_n + b\,Q_n]\,F'(t)$$

$$y = n\,[-a\,Q_{n-1} + b\,P_{n-1}]\,F(t) + 2\,q\,[a\,P_n + b\,Q_n]\,F'(t)$$

$$\psi = [a\,P_n + b\,Q_n]\,[(n-1)\,F(t) + 2\,t\,F'(t)],$$

in which a, b represent arbitrary constants. $\Omega =$ constant, is the equation of the orthogonal trajectories of the function of ψ; that is, the stream lines of our problem can be represented in general, and

$$\Omega = [-\,a\,Q_n + b\,P_n]\,[-\,n\,(n-1)\,F(t) + 2\,t\,F'(t)]\,e^{-\frac{t}{2}}.$$

If the above solutions are designated more exactly, by reason of their connection to the number n, as x_n, y_n, ψ_n, then two solutions belonging to m and n can be superimposed, so that

$$x = x_m + x_n$$

$$y = y_m + y_n$$

$$\psi = \psi_m + \psi_n$$

and similarly for any number of solutions.

The simplest form is obtained for $n = 2$ and $b = 0$, and this can be solved in a manner originally tried by the author, as follows:

Place for trial,

$$\psi = U + V \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

in which U is a function of x only, and V of y only. The derivatives of U with respect to x, and of V with respect to y, are designated by accents, and we have

$$\psi_1 = U', \qquad \psi_2 = V', \qquad \psi_{11} = U'', \qquad \psi_{12} = 0, \qquad \psi_{22} = V'',$$

and after placing these in equation $6b$, we have

$$U'' (U'^2 - 1) + V'' (V'^2 - 1) = 0,$$

which designation can only stand for all values of x and y when both expressions are constant, and equal but opposite in sign; that is,

$$U'' (U'^2 - 1) = a, \qquad V'' (V'^2 - 1) = -a \quad . \quad . \quad (8)$$

The integration* can be performed if, for instance, the first equation is multiplied by $2\,U'$ and written as follows:

$$(U'^2 - 1) \frac{d}{dx} (U'^2) = 2\,a\,U' \quad . \quad . \quad . \quad . \quad . \quad . \quad (9)$$

or

$$(U'^2 - 1) \frac{d}{dx} (U'^2 - 1) = 2\,a \frac{dU}{dx},$$

from which, by immediate integration, we have

$$\frac{1}{2} (U'^2 - 1)^2 = 2\,a\,U \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (10)$$

The constants can be omitted, as also ψ need only to be given correctly so far as a constant.

Let U' be designated by ξ, then equation 9 can be written in the form

$$(\xi^2 - 1)\ d\xi = a\ dx$$

and integrated,

$$\left(\frac{\xi^3}{3} - \xi\right) = a\,x \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (10a)$$

From equations 10 and $10a$ we now have a parameter representation of U as a function of x,

$$\left.\begin{aligned} x &= \frac{1}{3\,a} (\xi^2 - 3)\,\xi \\ U &= \frac{1}{4a} (\xi^2 - 1)^2 \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (11)$$

---

* I am thankful to Professor Hirsch for the correction of mistake originally made here by myself.

in which is possible a solution of the above equation with respect to ξ, placing it in the formula for U; but it is not to be recommended. In like manner, we obtain (by transposing $+a$ with $-a$) when $V=\eta$,

$$\left.\begin{aligned} y &= -\frac{1}{3a}(\eta^2-3)\eta \\ V &= -\frac{1}{4a}(\eta^2-1)^2 \end{aligned}\right\} \quad \ldots \ldots \quad (12)$$

and here is also

$$\phi = \alpha\psi = \alpha(U+V) \quad \ldots \ldots \quad (13)$$

The next problem is the finding of the stream lines as the orthogonal trajectories to the curves of constant potential.

The tangents to the angle of slope of a curve of constant potential $\phi(xy) =$ constant, is

$$\tan\tau = -\frac{\dfrac{\delta\phi}{\delta x}}{\dfrac{\delta\phi}{\delta y}}$$

The tangents at the same point of the stream line have an angle of slope τ', for which

$$\tan\tau' = \frac{dy_1}{dx_1}$$

in which x, y_1 are the coördinates of the stream line, and the condition of their being at right angles to one another, demands

$$\tan\tau \cdot \tan\tau' = -1 \quad \ldots \ldots \quad (14)$$

We are now dealing with mean functions, and for brevity, let

$$\left.\begin{aligned} x &= f(\xi), & y &= g(\eta) \\ U &= F(\xi), & V &= G(\eta) \end{aligned}\right\} \quad \ldots \quad (14a)$$

We have next

$$\tan\tau = -\frac{\dfrac{\delta U}{\delta x}}{\dfrac{\delta V}{\delta y}}$$

and

$$\frac{\delta U}{\delta x} = \frac{\delta U}{\delta\xi}\frac{\delta\xi}{\delta x} = \frac{\dfrac{dU}{d\xi}}{\dfrac{dx}{d\xi}} = \frac{F'}{f'}, \text{ likewise } \frac{\delta V}{\delta y} = \frac{G'}{g'}.$$

As tan τ is expressed in ξ and η, it would be well to choose for tan τ', and hence also for the stream curve, the same variables. We assume that also for the latter, x_1, y_1 can be replaced as in formulæ 11 and 12 by ξ η, and then we write

$$dy_1 = \frac{\delta f}{d\xi}\, d\xi = f'\, d\,\xi,\ \ dy_1 = g'\, d\eta.$$

Placing all this in equation 14, we get

$$\frac{f'^2}{F'}\, d\xi - \frac{g'^2}{G'}\, d\eta = 0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (14b)$$

The integration of this equation is possible after inserting the function values from equation 11, and now gives the *equation of the "stream lines group"* in the form

$$\xi^2 + \eta^2 - 2 \text{ nat. log } \xi\eta = \text{constant} \quad . \quad . \quad . \quad (15)$$

The velocity at a certain point of a stream line designated by ξ and η is obtained by differentiating ϕ as a mean function of x, y

$$\left.\begin{aligned} u &= \frac{\delta\phi}{\delta x} = \frac{\delta\phi}{\delta\xi}\frac{\delta\xi}{\delta x} = \frac{\dfrac{\delta\phi}{\delta\xi}}{\dfrac{\delta x}{\delta\xi}} = a\,\xi \\ v &= \frac{\delta\phi}{\delta y} = \frac{\delta\phi}{\delta\eta}\frac{\delta\eta}{\delta y} = \frac{\dfrac{\delta\phi}{\delta\eta}}{\dfrac{\delta y}{\delta\eta}} = -\,a\,\eta \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad (16)$$

Herewith are obtained the resulting velocity and pressure as per equation $3a$.

The particular solution, so derived, of the general differential equation for ϕ, is but little available because of the mean values of the variables ξ and η. But very simple formulæ are obtained if we confine ourselves to the small values of ξ and η. If, for instance, 0.1 be the upper limit, then the sum of the first two terms of equation 15 is always smaller than 0.02; the third term, on the other hand, is always larger than 9.20. We therefore make a negligible error if we neglect, within the given limits, $\xi^2 + \eta^2$ as compared with the logarithm, and equation 15 becomes

$$-2 \text{ nat. log } \xi \eta = \text{constant}$$

or

$$\xi \eta = \text{constant} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (15a)$$

For the same reasons, ξ^2 and η^2 as compared to 3, may be omitted in equations 11 and 12, so that

$$x = -\frac{\xi}{a} \qquad y = +\frac{\eta}{a} \quad . \quad . \quad . \quad . \quad . \quad . \quad (17)$$

which values, when substituted in equation 15a, express the equation of stream lines for the coördinate axes x, y. Then

$$x\,y = \text{constant} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (17a)$$

that is, *the stream lines are equilateral hyperbolas.* In this simplification is also

$$U = \frac{1}{4a}(1 - a^2x^2)^2, \qquad V = -\frac{1}{4a}(1 - a^2y^2)^2 \quad . \quad . \quad (18)$$

and the velocity potential,

$$\phi = \frac{a}{4a}\left[(1 - a^2x^2)^2 - (1 - a^2y^2)^2\right] \quad . \quad . \quad . \quad (19)$$

or, nearly, as ax and ay are small, while they are of the same order as ξ and η,

$$\phi = \frac{1}{2}\alpha\, a\,(y^2 - x^2) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (19a)$$

The velocities are now

$$u = \frac{\delta \phi}{\delta x} = -\alpha\, a\, x, \qquad v = \frac{\delta \phi}{\delta y} = \alpha\, a\, y \quad . \quad . \quad (20)$$

The pressures in any point are determined from equation 3a, which, with $p = \alpha^2 \mu$, takes the form

$$\frac{1}{2}(u^2 + v^2) + \alpha^2 \text{ nat. log} \frac{p}{\alpha^2} = \text{constant.}$$

Combining $-\alpha^2$ nat. log α^2 with the constants, and designating the *pressure at the coördinate origin* as p_0, in which $u = 0$, $v = 0$, we obtain

$$\text{nat. log} \frac{p}{p_0} = -\frac{1}{2\alpha^2}(u^2 + v^2) = -\frac{1}{2}\alpha^2 (x^2 + y^2) \quad . \quad . \quad (21)$$

or when $r^2 = x^2 + y^2$

$$p = p_0 e^{-\frac{1}{2}\alpha^2 r^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (21a)$$

that is:

Pressure and velocity depend only on the distance the chosen stream point is from the coördinate origin.

We have now finally reached a velocity-potential, with the above approximations, which correspond to a flow without compression; that is, to the assumption μ = a constant. This also holds true, even when μ varies slightly, as can be seen by inserting numerical values in the exact equation 15. We shall assume the constants of this equation collectively equal to 6.52; that is, calculating the values of ξ, η, $3\,a\,x$, $3\,a\,y$ lying on the stream line, from formulæ 15, 11 and 12, the following table is derived:

$\xi =$	1.0	0.7	0.4	0.3	0.2
$\eta =$	0.0635	0.0702	0.1046	0.1350	0.2
$3\,a\,x =$	− 2	− 1.757	− 1.136	− 0.873	− 0.592
$3\,a\,y =$	0.1896	0.2102	0.3125	0.4025	0.592

If, on the other hand, xy = constant, then the values of $3\,a\,y$, for instance, must be, as in the above order,

0.1752 0.1994 0.3083 0.4015 0.592.

The difference, therefore, for a graphical representation of the flow, is negligible. As the stream lines are symmetrical when referred to a line passing through the origin at an angle of 45°, then the second branch of the curve is determined by the above values. Beyond the limits $\xi = 1$ and $\eta = 1$, the equations do not give any continuation of the stream lines, and it must next be investigated whether or not the flow beyond these limits can remain free from rotation.*

In order to consider a concrete example, we shall assume superheated steam of 440° C. (824° F.) absolute temperature, with the approximately correct condition equation:

* For our problem, this is immaterial, as we can imagine the condition at the stream orifice as artificially produced.

In French units

$$pv = 47\,T$$

p in kg. per sq. meter, v in cubic meters, T absolute temperature C°.

In English units

$$pv = 0.6\,T$$

p in lb. per sq. in., v in cubic feet, T absolute temperature F°. We get $\alpha = \sqrt{gvp} = 450$ meters (1476 feet) per second. Let the arbitrary constant $\alpha = 10$, and as the limits of steam flow, let

$$xy = 4$$

and take cm. as the unit of length. Fig. 219 represents the stream limits (in this case a *channel with right-angled profile*); the stream

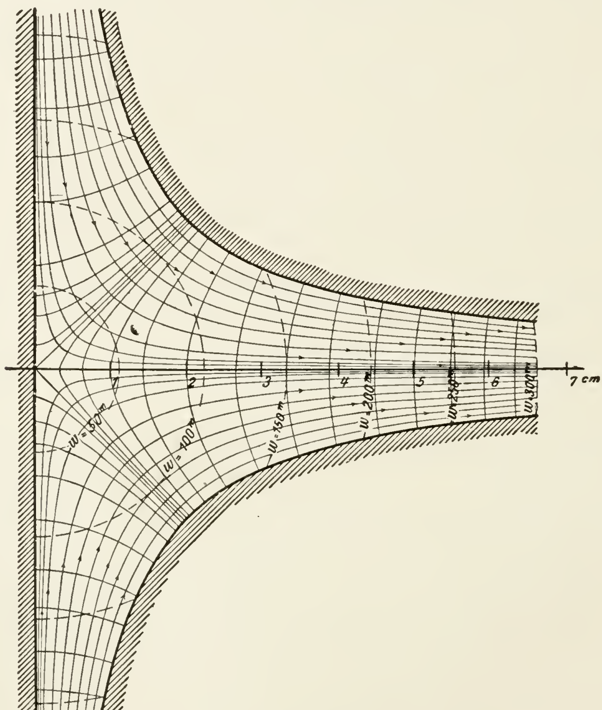

Fig. 219.

lines; also the lines ϕ = constant, that is the stream cross-section; and finally, the curves of constant velocity and constant pressure respectively. The inserted figures give the velocities in meters

per second. Our formulæ give a flow towards the coördinate origin; but as the sign of ϕ may be altered without making any other change, and again give a solution of the problem, the steam direction is shown reversed, thus making the problem easier to imagine, and *the figure can now be said to be an illustration of the orifice of a nozzle.* The main conclusion of this investigation may be said to be the proof, that *the pressures and velocities of the steam jets, so soon as the limits of small bends of the stream courses are reached, neutralize each other very quickly, even when the velocity has reached hundreds of meters.*

Let p_r be the pressure at the rim for the point x, y, and p_m the pressure at the center of the stream for the same abscissa x. Formula 21 gives

$$\text{nat. log}\,\frac{p_m}{p_0} = -\frac{1}{2}a^2x^2;\ \text{nat. log}\,\frac{p_r}{p_0} = -\frac{1}{2}a^2(x^2+y^2)$$

or

$$\text{nat. log}\,\frac{p_m}{p_r} = \frac{1}{2}a^2y^2.$$

Placing $p_m = p_r + \Delta p$ in which Δp is intentionally a small quantity, the logarithm may be developed, and there results

$$\frac{\Delta p}{p_r} = \frac{1}{2}a^2y^2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (22)$$

Let for our problem $x = 6$ cm., $y = \frac{4}{6}$ cm. $= 0.0066$ m., then

$$\frac{\Delta p}{p_r} = 0.0022.$$

If the pressure in the stream center were 5 atmospheres, then the pressure at the stream rim would be only about 0.01 atmosphere less. We would, therefore, look in vain, even in a conically diverging nozzle, for pressure differences between the center and rim of the stream, if we did not have extraordinarily delicate measuring apparatus. This is even less, because at the orifice of a nozzle the flow is not interfered with by a plane, and therefore the steam particles at the middle are not forced to follow a course so sharply bent.

70a. PRESSURE DISTRIBUTION IN A TURBINE BLADE CHANNEL.

If we cut from a steam stream the portion included between two stream lines, as is shown in Fig. 220, there results a channel that greatly resembles the channel between two turbine blades. The derived formulæ can be used without change, and give an easily seen representation of the pressure distribution. A line passing through the coördinate origin at 45° cuts the direction of flow at right angles; r_1 represents the inner distance along this line and r_2 the outer distance. With small blade depths, we may introduce in the differential of p, according to equation 21*a*, Article 70,

$$dp = -p_0 e^{-\frac{a^2 r^2}{2}} \frac{a^2}{2} d(r^2) = -\frac{a^2}{2} p\, d(r^2).$$

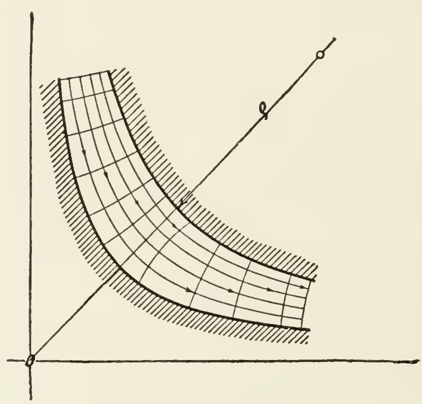

Fig. 220.

Introducing for pressure p a constant mean value p_m, we obtain as an approximate value of the pressure differences between the inner and outer blade limits

$$p_2 - p_1 = -\frac{a^2}{2} p_m (r_2^2 - r_1^2) \quad . \quad . \quad . \quad . \quad (23)$$

This formula can also be derived directly if we observe that below the 45° line, no tangential acceleration is present. An element bounded by two stream planes and two infinitely close normal planes must therefore be in equilibrium under the influence of the pressures and deflecting force $dm \frac{w^2}{\rho}$ in which ρ is the radius of

curvature of the course. This leads, if we use equation 8 of Article 31 with

$$\sigma_r = \sigma_t = -p, \quad \text{further } y = \text{constant}; \quad x^2\omega^2 = w^2$$

to

$$-\rho \frac{dp}{dx} + \mu w^2 = 0,$$

or

$$\frac{dp}{dr} = -\frac{\mu w^2}{\rho} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (24)$$

For the equilateral hyperbola, however, we find $\rho = r$, and equation 20 gives

$$w^2 = u^2 + v^2 = \alpha^2 a^2 r^2 \quad . \quad . \quad . \quad . \quad . \quad (25)$$

and finally

$$p = \alpha^2 \mu \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (26)$$

and from equations 24 to 26 either equation 21a or equation 23 may be derived.

For an actual turbine blade channel, for instance in a Laval turbine, the ratios, as was explained on page 97, are considerably different, because there cannot be considered in the above discussion either the circumstance that the stream in the blade channel flows in straight lines with equal pressure throughout, or the very great friction resistances. The flow in the blade channel is an unusually complicated occurrence, all the more so as there is added to the friction the splitting up against the blade edges, and we must leave to experiment the determination of blade forms that give favorable steam performance.

71. DEFLECTION OF A HORIZONTAL DISC OF VARIABLE THICKNESS, DUE TO ITS OWN WEIGHT.

The application of horizontal turbine wheels, as for example occurs in the Curtis turbine of sizes up to 5 meters (16.4 ft.) in diameter, must present to the designer the question of the deflection of the wheel due to its own weight, as this deflection might very easily reach the amount of clearance between the individual guide and rotating wheels. This deflection can be calculated in a comparatively simple manner.

Assume a symmetrical disc of variable thickness at rest with its axis vertical, as shown in the wheel in Fig. 102, page 161. The

thickness of the disc, y, at a distance x from the axis, let us here call h. The deflection downward will be considered as positive, and is z at the distance x. An outer rim will not be considered; the hub is relatively small in diameter, therefore x_1 is small in comparison with x_2. The profile of the disc corresponds to the equation

$$h\,x^{\alpha} = c, \quad \text{or} \quad h = c\,x^{-\alpha} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

and let α, that is, also the slope of the tangent at the profile line towards the middle plane of the wheel, be so small that we can place the cosine of the angle of slope $=1$ in the conditions of equi-

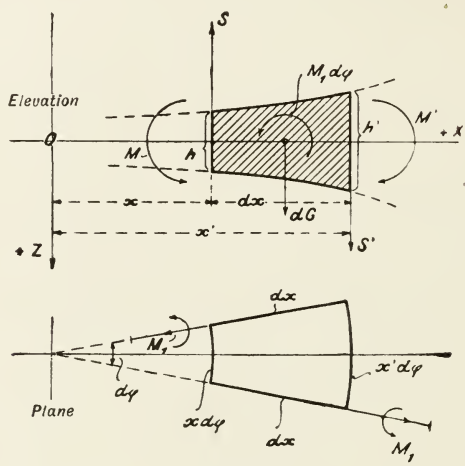

Fig. 221.

librium of stresses in a disc element. With the absences of rim forces there are present in any one cross-section at right angles to the middle plane of the disc, only bending and shearing stresses. The former we may, as with a uniformly thick disc, place proportional to the distance of the surface element in question from the middle plane, and let σ_x be the absolute value of the bending stresses in the *outermost fiber* of a section at right angles to the radius; σ_y be the same in a meridian section.

The bending moment M which is exerted upon the face area $x\,d\phi\,h$ of the disc element represented in Fig. 221 in the direction of the arrow, has the value: Resisting Moment × Bending Stress at the outer fiber; that is,

$$M = \frac{1}{6}(x\,d\phi)\,h^2\,\sigma_x \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (2)$$

The same for the opposite face surface is

$$M' = \frac{1}{6}(x'd\phi)\,h'^2\,\sigma_x'.$$

The Moment at the side areas, $dx\,h$, is

$$M_1 = \frac{1}{6}\,dx\,h^2\,\sigma_y \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (3)$$

in the sense of the "axis" as shown in plan view of Fig. 221.

Besides this there is a shearing force exerted in each face area; and in $x\,d\phi\,h$ the force

$$S = x\,d\phi\,h\,\tau_m \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (4)$$

in which τ_m represents the *mean value of the shearing stresses.**

In like manner,

$$S' = x'\,d\phi\,h'\,\tau'_m.$$

In the side areas the shearing force is, for reasons of symmetry, = 0. Finally there acts in the center of gravity of the element, vertically *downward,* due to its own weight, the force

$$dG = x\,d\phi\,dx\,h\,\gamma \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (5)$$

in which γ represents the specific weight.

The forces given must be in equilibrium with each other; the sum of the moments must therefore disappear, for instance, for an axis passing through the center of gravity at right angles to the plane XOZ. Combining the moments M_1 gives the turning moment $M_1\,d\phi$ about this axis, and the first condition of equilibrium is therefore,

$$M' - M - M_1\,d\phi + S\,dx = 0 \quad . \quad . \quad . \quad . \quad . \quad . \qquad (6)$$

or after substituting the individual values, as

$$M' - M = \frac{dM}{dx}\,dx,$$

---

* This line of reasoning corresponds substantially to the methods used by all authorities, as, for instance, *Grashof;* but is much simpler than the method of the latter. The degree of approximation to the exact solution may be *just as close* as that of the ordinary theory of bending of *de Saint-Vénant.*

we have,

$$\frac{d\,(x\,h^2\,\sigma_x)}{dx} - h^2\,\sigma_y + 6\,x\,h\,\tau_m = 0 \quad . \quad . \quad . \quad (7)$$

The second condition of equilibrium we shall not refer to one element, but to an entire disc, limited by a vertical cylinder of radius x. The total weight of the same is

$$G_x = \int_{x_1}^{x} 2\,\pi\,x\,dx\,h\,\gamma \quad . \quad . \quad . \quad . \quad . \quad (8)$$

The reaction P exerted vertically upwards through the center is equal to the weight of the entire disc; therefore,

$$P = \int_{x_1}^{x_2} 2\,\pi\,x\,dx\,h\,\gamma \quad . \quad . \quad . \quad . \quad . \quad . \quad (9)$$

Vertically downwards we have finally the total shearing force $2\,\pi\,x\,h\,\tau_m$. Equilibrium demands that

$$G_x + 2\,\pi\,x\,h\,\tau_m = P \quad . \quad . \quad . \quad . \quad . \quad . \quad (10)$$

From this we calculate

$$x\,h\,\tau_m = \frac{1}{2\,\pi}\Big[P - G_x\Big] = \Big[\int_{x_1}^{x_2} x\,dx\,h\,\gamma - \int_{x_1}^{x} x\,dx\,h\,\gamma\Big]$$

or, also

$$x\,h\,\tau_m = \int_0^{x_2} x\,dx\,h\,\gamma - \int_0^{x} x\,dx\,h\,\gamma = \frac{P_0}{2\,\pi} - \frac{\gamma\,h\,x^2}{2-\alpha} \quad (11)$$

if we represent

$$P_0 = \int_0^{x_2} 2\,\pi\,x\,dx\,h\,\gamma = \frac{2\,\pi\,\gamma\,h_2\,x_2^{\,2}}{2-\alpha} \quad . \quad . \quad . \quad (12)$$

as the "ideal" weight of the disc, imagined carried out to the axis, in which it is assumed that $\alpha < 2$, and the disc of uniform thickness, $\alpha = 0$, as is later shown, must be neglected. By inserting $x\,h\,\tau_m$ from equation 11 in equation 7, the shearing stresses are eliminated, and we have

$$\frac{d\,(x\,h^2\,\sigma_x)}{dx} - h^2\,\sigma_y = -\frac{6\,P_0}{2\,\pi} + \frac{6\,\gamma\,h\,x^2}{2-\alpha} \quad . \quad . \quad . \quad (13)$$

The elongation of a disc element on the tension side of the deflection shown in Fig. 222 at a distance $\frac{h}{2}$ in the radial direction is

$$\epsilon_x = \frac{\left(\rho + \frac{h}{2}\right) d\delta - \rho\, d\delta}{\rho\, d\delta} = \frac{h}{2\rho} \quad . \quad . \quad . \quad . \quad (14)$$

and in the direction of y, that is, measured circumferentially,

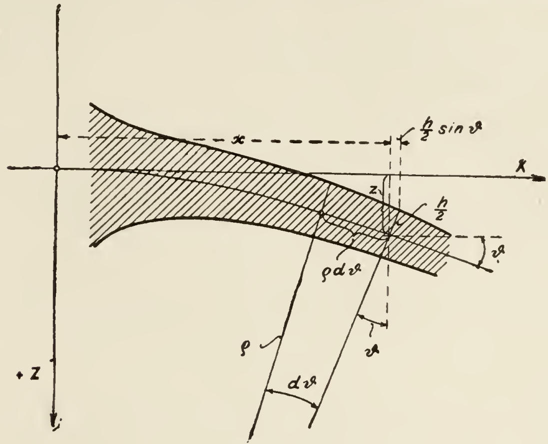

Fig. 222.

$$\epsilon_y = \frac{2\,\pi\left(x + \frac{h}{2}\sin\,\delta\right) - 2\,\pi\,x}{2\,\pi\,x} = \frac{h}{2\,\pi}\sin\,\delta \quad . \quad . \quad . \quad (14a)$$

or, with the allowable approximation,

$$\frac{1}{\rho} = \frac{d^2z}{dx^2} = z''\,;\ \sin\,\delta = \infty \tan\,\delta = \frac{dz}{dx} = z',$$

$$\epsilon_x = \frac{h}{2}z'', \qquad \epsilon_x = \frac{h\,z'}{2\,\pi} \quad . \quad . \quad . \quad . \quad (14b)$$

From this we get, as in Article 39,

$$\left.\begin{aligned} \sigma_x &= \frac{E}{1 - \nu^2}\left(z'' + \nu\frac{z'}{x}\right)\frac{h}{2} \\ \sigma_y &= \frac{E}{1 - \nu^2}\left(\nu\, z'' + \frac{z'}{x}\right)\frac{h}{2} \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad (15)$$

and the differential equation 13 reads

$$\frac{d}{dx}\left[h^3(x z'' + \nu z')\right] - h^3\left(\nu z'' + \frac{z'}{x}\right) = \frac{12(1-\nu^2)\gamma h x^2}{E(2-\alpha)}$$
$$- \frac{6(1-\nu^2)P_0}{\pi E} \quad \ldots \ldots \ldots \quad (16)$$

or, taking equation 1 into consideration,

$$z''' + (1 - 3\alpha)\frac{z''}{x} - (1 + 3\alpha\nu)\frac{z'}{x^2} = a_1 x^{n_1} - a_2 x^{n_2} \quad . \quad (17)$$

and the notation

$$\left.\begin{array}{ll} n_1 = 2\alpha + 1 & n_2 = 3\alpha - 1 \\ a_1 = \dfrac{12(1-\nu^2)\gamma}{(2-\alpha)E c^2} & a_2 = \dfrac{6(1-\nu^2)P_0}{\pi E c^3} \end{array}\right\} \quad . \quad (18)$$

For solving this equation, we place

$$z = u + b_1 x^{k_1}$$

and designate the right side of equation 17 as $f(x)$; we then get

$$f(u) + b_1[k_1(k_1-1)(k_1-2) + (1-3\alpha)k_1(k_1-1)$$
$$- (1 + 3\alpha\nu)k_1]x^{k_1-3} = a_1 x^{n_1} - a_2 x^{n_2}$$

If

$$k_1 = n_1 + 3$$

then x^{n_1} disappears, and b_1 may be determined from the equation,

$$(n_1 + 3)[(n_1 + 2)(n_1 + 2 - 3\alpha) - (1 + 3\alpha\nu)]b_1 = a_1 \,. \quad (19)$$

In like manner the second term on the right may be set aside by substituting

$$u = v + b_2 x^{n_2+3}$$

from which b_2 may be calculated from the equation

$$(n_2 + 3)[(n_2 + 2)(n_2 + 2 - 3\alpha) - (1 + 3\alpha\nu)]b_2 = -a_2 \quad (19a)$$

The remaining equation

$$f(v) = 0 \quad \ldots \ldots \ldots \quad (17a)$$

is integrated by the addition of $v = b_0 x^\lambda$ in which λ must satisfy the equation

$$\lambda^3 - (2 + 3\,\alpha)\,\lambda^2 + 3\,\alpha\,(1 - \nu)\,\lambda = 0.$$

The three roots are

$$\left.\begin{matrix}\lambda \\ \lambda'\end{matrix}\right\} = \left(1 + \frac{3\,\alpha}{2}\right) \pm \sqrt{\left(1 + \frac{3\,\alpha}{2}\right)^2 - 3\,\alpha\,(1 - \nu)}\,; \quad \lambda'' = 0 \quad . \ (20)$$

from which the complete integral of equation 17 is

$$z = b_0\,x^\lambda + b_0'\,x^{\lambda'} + b_0''\,x^0 + b_1\,x^{n_1+3} + b_2\,x^{n_2+3}.$$

For $x = 0$ we require $z = 0$, and this gives $b_0'' = 0$; likewise for $x = 0$, z' must also $= 0$, which is only possible when $b_0' = 0$. $(\lambda' - 1)$ is always a negative real quantity, as can easily be seen, if b_0' is not $= 0$; then with $x = 0$, the values of z' would be infinitely great.

The corresponding solution of the problem is, therefore,

$$z = b_0\,x^\lambda + b_1\,x^{n_1+3} + b_2\,x^{n_2+3} \quad . \quad . \quad . \quad . \quad (21)$$

The still arbitrary constant b_0 is determined by the rim conditions, that for $x = x_2 = r$, the bending stress σ_x disappears; that is,

$$\left(z'' + \nu\frac{z'}{x}\right)_{x=r} = 0\,. \quad . \quad . \quad . \quad . \quad (21a)$$

The disappearance of the shearing stresses is satisfied when P_0 is made equal to the "ideal" wheel weight. Completing the solution, we have

$$b_0 = -\frac{1}{\lambda\,(\lambda - 1 + \nu)}\Big[(n_1 + 3)\,(n_1 + 2 + \nu)\,b_1 r^{n_1+3-\lambda}$$

$$+ (n_2 + 3)\,(n_2 + 2 + \nu)\,b_2 r^{n_2+3-\lambda}\Big], \quad . \quad . \quad . \quad . \quad (22)$$

whereby the problem is completely solved. The stresses themselves we obtain by substituting the derived values from equation 21 in equation 15.

The equations are inconvenient, but at least do not demand tiresome trials. When the wheel diameter reaches several meters, the deflection can be measured in millimeters, and the calculation should not be omitted.

For convenience, we shall again give the *steps in their proper order* for this calculation. Equation 1 is obtained from the design of the wheel. We calculate P_0 from equation 12; n_1, n_2, a_1, a_2 from equation 18; b_1, b_2, from equations 19 and 19a; λ from equation 20; b_0 from equation 22; and obtain the deflection from equation 21.

For the *disc of uniform thickness*, the integration is to be performed separately, and we obtain, with $\alpha = 0$, h = constant = h_0,

$$z = \frac{a_1 x^4}{32} - \frac{a_2 x^2}{4}(\text{nat. log } x - 1) + \frac{a_3 x^2}{4} \quad . \quad . \quad (23)$$

in which

$$a_1 = \frac{6(1-\nu^2)\gamma}{E h_0^2}, \qquad a_2 = \frac{6(1-\nu^2)P}{\pi E h_0^3} \quad . \quad . \quad (24)$$

which formulæ have already been derived by *Grashof.*

For the determination of a_3 condition 21a again serves; and we have

$$a_3 = -\frac{3+\nu}{4(1+\nu)}a_1 r^2 + \left[\text{nat. log } r + \frac{1-\nu}{2(1+\nu)}\right]a^2 \quad . \quad (25)$$

and finally the deflection at the rim,

$$(z)_{x=r} = \frac{3(1-\nu)(7+3\nu)}{16}\frac{\gamma r^4}{E h_0^2} = 1.037\frac{\gamma r^4}{E h_0^2} \quad . \quad (26)$$

By a numerical example we can prove that by increasing the thickness of the disc towards the shaft according to equation 1, which is demanded by the stresses due to centrifugal force, the deflection due to its own weight is considerably lessened. The effect of a thickened rim may also be calculated; still, the presentation of this calculation would lead us too far.

71a. STRAIGHTENING-OUT OF A VERTICALLY ROTATING DISC BY ITS OWN CENTRIFUGAL FORCES.

With discs of considerable dimensions, the danger may arise that the disc is straightened out more or less by its own centrifugal forces, therefore under circumstances might draw up and scrape along its top side. The value of this straightening out can be approximately determined at least for a *disc of uniform thickness.*

In Fig. 223 is shown a plan and elevation of a disc element of the same shape as before. To the forces due to its own weight, dG, S, S', M, M', M_1, there is added on account of centrifugal force, $dF = \mu\,(x\,d\phi\,h\,dx)\,\omega^2 x$, in which μ is the specific mass; the radial force $R = x\,d\phi\,h\,\sigma_r$ exerted on the face area $x\,d\phi\,h$; its opposing force, $R' = x'\,d\phi\,h\,\sigma_r'$; and the tangential force $T = dx\,h\,\sigma_t$

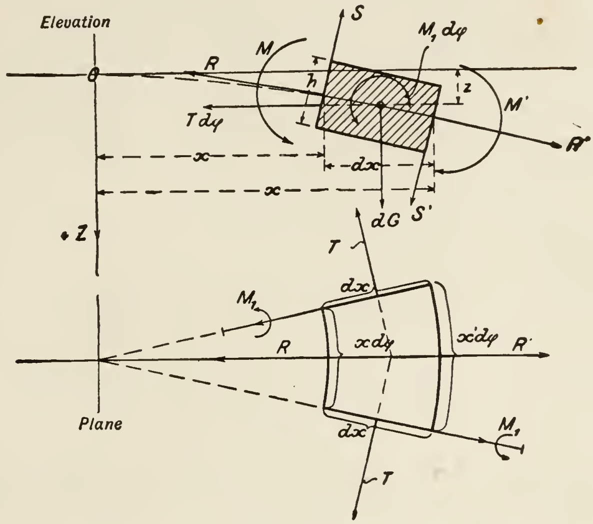

Fig. 223.

on the side areas, $dx\,h$. σ_r, σ_t represent the radial and tangential stresses, distributed *uniformly* over the entire cross-section, while σ_x, σ_y are used in the sense as before. The moments M_1 give as before $M_1\,d\phi$ which is inserted in the figure. The forces T can also be combined into a resultant $T\,d\phi$, which is exerted radially inward. The equilibrium of these four systems again demands the disappearance of the moments about any one axis, and the disappearance of the sum of the force components in any one direction. The first condition, referred to an axis through the center of gravity at right angles to $X\,O\,Z$, gives as before,

$$\frac{d\,(x\,\sigma_x)}{dx} - \sigma_y + \frac{6\,x}{h}\tau_m = 0 \quad . \quad . \quad . \quad . \quad . \quad (27)$$

We shall also take the sum of the components in the direction of the tangent to the elastic curve of the meridian section. The slope of this tangent is so small that curve, sine, and tangent may

be interchanged, $= \frac{dz}{dx} = z'$, and the cosine may be $= 1$. Resolving dG into tangential and normal components, the first component will $= dG\,z'$, and the condition of equilibrium is

$$R' - R - Td\phi + dF + dG\,z' = 0 \quad . \quad . \quad . \quad (28)$$

or

$$\frac{d\,(x\,\sigma_r)}{dx} - \sigma_t + \mu\,\omega^2\,x^2 + \gamma\,x\frac{dz}{dx} = 0 \quad . \quad . \quad (29)$$

The third condition we again refer to the vertical forces acting on a disc element cut from vertical cylinder of radius x. The sum of the vertical components of R is

$$2\,\pi\,x\,h\,\sigma_r\,z'$$

and we get

$$2\,\pi\,x\,h\,\sigma_r\,z' + 2\,\pi\,x\,h\,\tau_m + \gamma\,\pi\,x^2\,h - P_0 = 0,$$

from which

$$x\,h\,\tau_m = \frac{P_0}{2\,\pi} - \frac{\gamma\,h\,x^2}{2} - x\,h\,\sigma_r\,z'.$$

Introducing this in equation 27, we have

$$\frac{d\,(x\,\sigma_x)}{dx} - \sigma_y - \frac{6\,x\sigma_r}{h}z' - \frac{3\,\gamma}{h}x^2 + \frac{3\,P_0}{\pi\,h^2} = 0 \quad . \quad . \quad (30)$$

In equations 29 and 30 we substitute for σ_x σ_y σ_r σ_t expression 15, Article 71, and expression 12, Article 39, respectively, in order to determine the unknown z and ξ as function of x. We get around the difficulty of this calculation by the assumption that the stresses σ_r and σ_t for a first approximation have the same value, as though the forces of gravity were absent. We have then for ξ, equation 38, Article 39,

$$\xi = a\,x^3 + b_1\,x + \frac{b_2}{x} \text{ with } a = -\frac{(1 - \nu^2)\mu\,\omega^2}{8\,E},$$

in which $b_2 = 0$ for the entire disc, so that with $x = 0$, $\xi = 0$. At the rim of the wheel, $\sigma_r = 0$; that is, according to equation 12, Article 39,

$$\left(\nu\frac{\xi}{x} + \frac{d\xi}{dx}\right)_{x=r} = 0,$$

and from this follows

$$b_1 = -\frac{(3+\nu)\,a r^2}{1+\nu}$$

and finally,

$$\sigma_r = a'\,(r^2 - x^2) \text{ with } a' = \frac{(3+\nu)\,\mu\,\omega^2}{8} \quad . \quad . \quad (31)$$

which value, with equation 15, we introduce in equation 30, with the further abbreviation,

$$a'' = \frac{6\,(1-r^2)}{E\,h^3} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (32)$$

and get the differential equation

$$x\,z''' + z'' - \frac{z'}{x} + a''\left[\frac{P_0}{\pi} - 2a'\,h\,(r^2 - x^2)\,x\,z' - h\,\gamma\,x^2\right] = 0 \quad (33)$$

The integration can be performed without difficulty by summation of series, but necessitates awkward calculations, if we wish to derive a numerical result. Therefore an approximate method is introduced by substituting, for the derivative of z' expressed in the brackets, a simple function of x. As z constantly increases from the center, we shall assume the simplest form,

$$z = a_0\,x^2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (34)$$

with the unknown but constant a_0, whereby it is to be remarked that actually, z increases more quickly than the square of x. Therefore the influence of centrifugal force expressed in the term $2\,a'h\,(r^2 - x^2)\,x\,z'$, has been somewhat too highly estimated.

Inserting, therefore,

$$z' = 2\,a_0\,x$$

in equation 33, and using the notations

$$A_0 = \frac{a''\,P_0}{\pi}, \quad A_1 = a''\,h\,(\gamma + 4\,a_0\,a'\,r^2), \quad A_2 = 4\,h\,a_0\,a'\,a''. \quad (35)$$

it takes the form

$$z''' + \frac{z''}{x} - \frac{z'}{x^2} = -\frac{A_0}{x} + A_1\,x - A_2\,x^3 \quad . \quad . \quad . \quad (36)$$

in which the left side may also be written as

$$z''' + \frac{d}{dx}\left(\frac{z'}{x}\right);$$

integrating, we have

$$z'' + \frac{z'}{x} = - A_0 \text{ nat. log } x + \frac{1}{2} A_1 x^2 - \frac{1}{4} A_2 x^4 + A_3$$

The left side is $= \frac{1}{x}\frac{d}{dx}(x z')$; we can therefore, after multiplying by x, again integrate, and finally get

$$z = - A_0 \frac{x^2}{4} (\text{nat. log } x - 1) + \frac{1}{32} A_1 x^4 - \frac{1}{144} A_2 x^6 + \frac{1}{4} A_3 x^2 \quad (37)$$

The last two integration constants are $= 0$, because for $x = 0$, must $z = 0$, as also $z' = 0$. The still arbitrary A_3 follows from the condition that for $x = x_2 = r$ it must be the bending stresses, that is,

$$\left(z'' + \nu \frac{z'}{x}\right)_{x=r} = 0.$$

This gives

$$A_3 = \frac{1}{1+\nu} \left\{ A_0 \left[(1 + \nu) \text{ nat. log } x + \frac{1 - \nu}{2}\right] - \frac{1}{4} A_1 \ (3 + \nu) r^2 \right.$$
$$\left. + \frac{1}{12} A_2 (5 + \nu) r^4 \right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (38)$$

and finally we have the deflection of the rim for $x = r$, if the value of the constants A_0 to A_3 is inserted,

$$(z)_{x=r} = \frac{3}{16} (1 - \nu) \ (7 + 3 \nu) \frac{\gamma r^4}{E h^2}$$
$$- \frac{1}{96} (3 + \nu) \ (1 - \nu) \ (17 + 5 \nu) \frac{\mu \omega^2}{E h^2} a_0 r^6 \quad . \quad (39)$$

If $\omega = 0$, we then have

$$z_0 = \frac{3}{16} (1 - \nu) \ (7 + 3 \nu) \frac{\gamma r^4}{E h^2} \quad . \quad . \quad . \quad . \quad (40)$$

which agrees with equation 26 of the previous article.

If we let

$$\beta = \frac{1}{96}(3+\nu)(1-\nu)(17+5\nu)\frac{\mu\,\omega^2 r^4}{E\,h^2} \quad . \quad . \quad . \quad (41)$$

the effective deflection would then be

$$z_r = z_0 - \beta\, a_0\, r^2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (42)$$

The still unknown value a_0 must in order to satisfy the assumption, equation 34, be so calculated that

$$z_r = a_0\, r^2.$$

We therefore have

$$z_r = z_0 - \beta\, z_r\,,$$

and from this finally,

$$z_r = \frac{z_0}{1+\beta} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (43)$$

The centrifugal force, as can easily be proved, exerts a considerable influence on the deflection. For instance, z_r will become one-half of z_0 when $\beta = 1$, and this means with a disc of 4 meters (13 feet) diameter, 3 centimeters (1.18 inches) thick, (when $\mu = 7.8 \times 10^{-6}$, $E = 2 \times 10^6$, $\nu = 0.3$) an angular velocity $\omega = 56.9$, therefore rotating, $n = 543$ per minute. But if we increase the revolutions threefold, that is, to 1630, than $\beta = 9$, hence the *deflection is only one-tenth of that at a position of rest.*

The presence of a thickened rim and variable thickness of disc would influence the ratio of both of these deflections the less, the higher the velocity. We could calculate the deflection for this case also by means of the approximate method just used, and still the above simple example justifies the statement, *that for the high number of revolutions usual in turbine construction, the deflection of a horizontal, rotating disc due to its own weight is almost entirely neutralized, while in motion, by the centrifugal force.* In general, therefore, the clearance between the guide and rotating wheels must be made at least equal to the value of this deflection. But we could also make *the meridian line of the wheel a flat arc, concave upwards, according to equation* 21, so that its own weight would bend the middle area to a horizontal plane, and the centrifugal forces would only call forth an expansion in this horizontal

plane. This doubtless demands more skill in the shop; and, by taking into account the possible vibration of the wheel, we are led not to make the clearance too small.

71b. THE STRESSES IN DISC WHEELS DUE TO UNEQUAL HEATING.

It has lately been observed that with single-stage turbines, on account of the heat radiation of the nozzle rings, the rim of the disc wheel has a much higher temperature (up to 100° C. or 180° F. difference) than the disc body, which radiates heat to the colder casing. This is the reason wheel constructions are found, by which, for example, the crown is divided by radial saw-tooth sections into many independent segments, so that it can expand freely. Then also, the internal stresses in the material have a chance to balance themselves. Much more dangerous stresses might be imagined when accidents occur while running. It might be possible, for instance, with unskilled attendance, for cooling water to back into the turbine casing (while shutting down), and cool the rim of the wheel while the disc remains hot.

The investigation is greatly simplified by observing that the stresses which are caused by unequal heating may be united with the centrifugal force stresses, according to the principle of "superimposition," that is, they can be calculated as if the wheel were at rest.

An especially obvious case occurs if a disc of uniform thickness h is investigated, whose rim, of cross-section f, is suddenly cooled $t°$ below the constant initial temperature. As the rim tries to contract, it puts a radial pressure on the disc, and is itself put under stress, exactly like a shrunk ring. Let us assume for simplicity that the disc is whole (without bore, or the hub so strong that the disc may be considered whole) ; then, by specializing formula 38 in Article 39, or by immediate consideration, we can prove that the (pressure) stress σ is equally large in all directions throughout the disc, and that the linear compressive forces are

$$\xi = \frac{1-\nu}{E} \sigma x \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

By now neglecting the difference between x_2 and x_3 and placing both $= r$, we have at the disc rim

$$\xi_r = \frac{1-\nu}{E} \sigma r \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

The radius of the ring, whose value was originally r, is decreased by cooling through $t°$ C. (or F.°) to

$$\Delta r = r \epsilon t \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

where ϵ is the coefficient of heat expansion. But the disc presses the ring with a force σ over the breadth h radially apart, by which an enlargement of

$$\xi_r' = \frac{h \sigma r^2}{E f} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

is brought about, and $\xi_r + \xi'_r$ must $= \Delta r$, or by introducing the values in equation 2, 3, and 4, we have

$$\frac{1-\nu}{E} \sigma r + \frac{h \sigma r^2}{E f} = r \epsilon t\,;$$

from which may be calculated

$$\sigma = \frac{E \epsilon t}{(1-\nu) + \dfrac{h r}{f}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

The stress σ_1 in the ring, we get approximately from the load due to σ on the inner circumference of the ring, by the so-called boiler formula,

$$\sigma_1 = \frac{r h \sigma}{f} = \frac{E \epsilon t}{(1-\nu) \dfrac{f}{r h} + 1} \quad . \quad . \quad . \quad . \quad . \quad (6)$$

Formulæ 5 and 6 are peculiar in that the stresses depend only upon the difference of temperature, t, and the product $r h$, but not individually upon the value of the radius. *A disc twice as large but half as thick experiences therefore equal stresses with equally strong and equally heated rim.*

There is no difficulty in calculating a constant distribution of

temperature, if we assume a suitably simple law of distribution. The fundamental formulæ 8 and 9 in Article 39 hold good; still, ϵ_r and ϵ_t denote, after subtracting the expansion due to heat, only the resulting elastic specific expansion, which may be calculated as follows: let t be the constant excess of temperature beyond the initial temperature in all parts of the circumference of radius x, and varying with x. Let the radius after heating be $x+\xi$. A ring of radius x would be expanded by heat alone,

$$\xi' = \epsilon x t.$$

Only the excess $\xi''=\xi-\xi'$ forms the elastic deformation; therefore the tangential expansion is

$$\epsilon_t = \frac{\xi''}{x} = \frac{\xi}{x} - \epsilon t \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

In like manner the distortion of the point at the distance dx is $\xi^* = \xi + \frac{d\xi}{dx} \cdot dx$, and the total expansion of the element dx is $\frac{d\xi}{dx} \cdot dx$. Heat alone gives the part $\epsilon\, t\, dx$; as the elastic elongation in the radial direction, we have, therefore,

$$\epsilon_r = \frac{\frac{d\xi}{dx} dx - \epsilon\, t\, dx}{dx} = \frac{d\xi}{dx} - \epsilon\, t \quad . \quad . \quad . \quad . \quad . \quad (8)$$

The equation of stresses, formula 12, Article 39, therefore reads

$$\left.\begin{aligned} \sigma_r &= \frac{E}{1-\nu^2}\left[\nu\left(\frac{\xi}{x} - \epsilon\, t\right) + \frac{d\xi}{dx} - \epsilon\, t\right] \\ \sigma_t &= \frac{E}{1-\nu^2}\left[\left(\frac{\xi}{x} - \epsilon\, t\right) + \nu\left(\frac{d\epsilon}{dx} - \epsilon\, t\right)\right] \end{aligned}\right\} \quad . \quad . \quad . \quad (9)$$

and the fundamental equation 13 becomes

$$\frac{d^2\xi}{dx^2} + \left(\frac{dl\, n\, y}{dx} + \frac{1}{x}\right)\frac{d\xi}{dx} + \left(\frac{\nu}{x}\frac{dl\, n\, y}{dx} - \frac{1}{x^2}\right)\xi$$
$$- (1+\nu)\,\epsilon\frac{dt}{dx} - (1+\nu)\,\epsilon\, t\frac{dl\, n\, y}{dx} + Ax = 0 \quad . \quad . \quad (10)$$

The equation is easily integrated, if we again let $y = cx^a$, and, for the temperature, the law

$$t = Bx^n \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (11)$$

or form a sum of exponential terms. We find

$$\xi = ax^3 + bx^{n+1} + b_1 x^{\psi_1} + b_2 x^{\psi_2} \quad . \quad . \quad . \quad (12)$$

in which α, ψ_1, ψ_2 are defined by equations 18 and 20, Article 39, while

$$b = \frac{(1+\nu)\,\epsilon\,(\alpha + n)\,B}{n(n+1) + (1+\alpha)(n+1) + (\alpha\nu - 1)} \quad . \quad . \quad (13)$$

For the determination of b_1 and b_2, the rim conditions serve, as in Article 39, taking into account the temperature differences. From equation 10 we can easily prove the statement, that the solutions for the disc at rest, but heated, cannot be interchanged for the rotating disc with $t = 0$. This, however, may be said, that the differential equation as well as the rim conditions in ξ and its derivatives, are linear.

72. CRITICAL VELOCITY OF A CONSTANT AND UNIFORMLY LOADED SHAFT WITH VARIABLE DIAMETER.

In the general equation 4, Article 49, m_1 in this case stands for the sum of the masses of the wheels per unit length m_1'; and the masses of the shaft itself, $\mu\pi r^2$; and the above mentioned equation may be written with the insertion of $J = \frac{\pi}{4} r^4$;

$$\frac{\pi}{4} r^4 E \frac{d^4 y}{dx^4} = (m_1' + \mu\pi r^2)\,\omega^2 (y + e) \quad . \quad . \quad . \quad (1)$$

in which r, according to the assumption, is to be variable. In order not to make the calculation too difficult, we shall assume a shaft supported at the two ends and enlarged towards the middle, whose radius decreases towards the shaft's ends according to the law,

$$r^4 = r_0^4 \left(1 - \beta^2 \frac{x^2}{l^2}\right) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

The coördinate origin is again in the middle of the distance between bearings. Besides this, it is assumed that either m_1' so changes that the sum $m_1' + \mu \pi r^2$ has over all a constant value, or $\mu \pi r^2$ has a mean value substituted in the calculation, so that the sum m_1 remains unchanged from cross-section to cross-section. If we introduce the new variable

$$z = \beta \frac{x}{l}$$

then equation 1 appears in the form

$$\frac{\pi}{4}(1 - z^2) E \frac{\beta^4}{l^4} \frac{d^4 y}{dz^4} = m_1 \omega^2 (y + e)$$

or with the notation

$$\alpha = \frac{4 m_1 \omega^2 l^4}{\pi E r_0^4 \beta^4} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

and under the supposition that e remains constant :

$$(1 - z^2) \frac{d^4 y}{dz^4} = \alpha \, (y + e) \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

The limiting values of z, which correspond to $x = 0$ and $x = l$, are 0 and β, and in this intermediate space, the above formula is integrated by a converging series of the form

$$y = a_0 + a_2 z^2 + a_4 z^4 + a_6 z^6 + \cdots \quad . \quad . \quad . \quad . \quad (5)$$

The uneven powers disappear on account of the symmetry. If we introduce this series in the differential equation, then a_0, a_2 are first arbitrary, while the remaining values may be represented by

$$\left.\begin{aligned}
a_4 &= \frac{\alpha (a_0 + e)}{1 \cdot 2 \cdot 3 \cdot 4} \\
a_6 &= \frac{\alpha (a_0 + e)}{3 \cdot 4 \cdot 5 \cdot 6} + \frac{\alpha a_2}{3 \cdot 4 \cdot 5 \cdot 6} \\
a_8 &= \frac{\alpha}{5 \cdot 6 \cdot 7 \cdot 8}\left[\left(1 + \frac{\alpha}{1 \cdot 2 \cdot 3 \cdot 4}\right)(a_0 + e) + a_2\right] \\
&. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad .
\end{aligned}\right\} \quad (6)$$

As each coefficient in $(a_0 + e)$ and a_2 is linear, y may be written in the form

$$y = (a_0 + e) R_0 + a_2 R_2,$$

in which R_0 and R_2 are the power series of z. The constants a_0 and a_2 are now determined from the condition, that for $x = l$, that is $z = \beta$, y, as well as the bending moment, that is $\frac{d^2y}{dx^2}$, must disappear. If we denote the second derivatives of the series R_0, R_2 according to z, as R_0'', R_2'', and denote the value of the expression for $z = \beta$ by the subscript β, there result the condition equations,

$$a_0 (R_0)_\beta + a_2 (R_2)_\beta = - e (R_0)_\beta$$

$$a_0 (R_0'')_\beta + a_2 (R_2'')_\beta = - e (R_0'')_\beta$$

From these a_0 and a_2 may in general be calculated as actual finite values. Only in the case when the determinants

$$D = \begin{vmatrix} (R_0)_\beta & (R_2)_\beta \\ (R_0'')_\beta & (R_2'')_\beta \end{vmatrix} = (R_0)_\beta (R_2'')_\beta - (R_0'')_\beta (R_2)_\beta \quad . \quad . \quad (7)$$

disappear, will a_0 and a_2, hence also the deflection y, be infinitely large. The critical velocity may therefore be determined from the equation

$$D = 0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

For this purpose it is necessary to introduce the values of $a_4\ a_6 \ldots$ in the series R, and solve equation 8 for the values of ω^2 occurring in α. This process is very inconvenient in spite of the not altogether poor convergence of the series, and therefore an approximate value of ω_k shall be derived by affecting all the terms in the series R with a higher power, such as z^6 or β^6 respectively. This calculation leads to the equation

$$1 - \frac{1}{6} \alpha \beta^4 - \frac{1}{45} \alpha \beta^6 = 0 \quad . \quad . \quad . \quad . \quad . \quad . \quad (9)$$

or, after inserting the values of α, we finally get the critical velocity,

$$\omega_k = \sqrt{\frac{3\pi}{2}\frac{r_0^4\,\epsilon}{m_1\,l^4}\frac{1}{1+\frac{2}{15}\beta^2}} = 3.464\sqrt{\frac{J_0 E}{M\,l^3}\frac{1}{1+\frac{2}{15}\beta^2}}. \quad (10)$$

in which $J_0 = \frac{\pi}{4} r_0^4$ is the moment of inertia of the area of the mean cross-section of the shaft, and M is the total mass of the discs and shaft. If, further, r_1 is the radius of the shaft in the bearing, then follows from equation 2,

$$\beta^2 = 1 - \frac{r_1^4}{r_0^4} \quad . \ . \ . \ . \ . \ . \ . \ . \ . \quad (11)$$

The critical velocity, therefore, varies but little as compared to a uniform shaft.

An especially simple and still exact solution can be had for the *special case*, in which the load is proportional to the *square of the radius of the shaft, and this radius proportional to the deflection;* that is, for the law

$$J\,E\frac{d^4y}{dx^4} = \frac{\pi}{4} r^4\,E\frac{d^4y}{dx^4} = m_1'\,r^2\,\omega^2\,y,$$

or

$$\frac{d^4y}{dx^4} = \frac{4\,m_1'\,\omega^2}{\pi\,E\,a} \quad . \ . \ . \ . \ . \ . \ . \ . \ . \quad (12)$$

with

$$r^2 = a\,y \quad . \ . \ . \ . \ . \ . \ . \ . \ . \quad (13)$$

We shall here take no account of an eccentricity (e), and the critical velocity is again determined from the condition, that the shaft remains in neutral equilibrium under the influences of the centrifugal forces and the elastic resisting forces. The general integration of equation 12 gives, for a shaft supported at both ends, of length $2\,l$,

$$a\,y = r^2 = \frac{m_1'\,\omega^2\,l^4}{6\,\pi\,E}\left[\left(\frac{x}{l}\right)^4 - 6\left(\frac{x}{l}\right)^2 + 5\right] \quad . \ . \ . \quad (14)$$

If we now prescribe the radius r, for instance at the shaft center, for $x = 0$, that is, place $r = r_0$, the angular velocity must assume a certain "critical" value, so that equation 14 will hold good. We have therefore,

$$r_0^2 = \frac{5}{6\pi} \frac{m_1' \omega_k^2 l^4}{E} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (15)$$

and

$$\omega_k = r_0 \sqrt{\frac{6\pi E}{5 m_1' l^4}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (16)$$

73. SYMPATHETIC VIBRATION OF THE FOUNDATION; HARMLESSNESS OF "RESONANCE."

The shaft, which is never free from vibration, transmits to the foundation of the turbine a periodically varying force, by which the former is set into sympathetic vibration. The foundation may be considered as a rigid mass, resting on an elastic bed; and there is cause to fear that under certain circumstances, the number of revolutions of the turbine may correspond to the natural number of vibrations of the foundation, and that when other vibrations take place, a dangerous "resonance" could occur. It is of practical interest to determine that this resonance is not dangerous to the

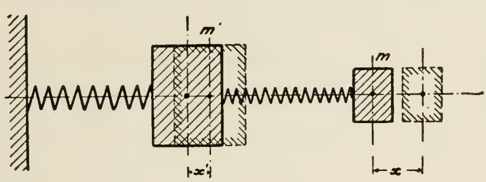

Fig. 224.

turbine, and could in no way lead to extraordinarily high vibrations, for the reason that the turbine shaft is not a rigid body, but is in itself elastic. On the other hand, the sympathetic vibrations gain considerable because the critical velocity of the shaft is decreased or increased. The simplest proof of the correctness of the above statement can be had from the consideration of "an elastic double pendulum," as, for instance, the connection of the mass m with the mass m' by means of a spring, the former being connected by another spring to a first point, as shown in Fig. 224.*

* The mass m may represent the shaft with its flexibility, and m' the elastic foundation. Let the periodic force

$$P = a \cos \omega t$$

act on the mass m, which will cause a vibration in the horizontal direction. The deflections of m and m' from the position in which the springs are neither in compression

For the shaft of the turbine we may consider for simplicity that the foundation will only yield vertically, and consider the de-

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or extension are x and x', and the forces acting at that time are X and X', proportional to the distortion; that is,

$$X = \alpha (x - x'),$$
$$X' = \alpha' x'.$$

The equations of motion of masses are

$$m \frac{d^2x}{dt^2} = -X + P,$$
$$m' \frac{d^2x'}{dt^2} = -X' + X.$$

If we assume that there also exists a so-called "dampening," even though very small, in the form of friction, which is proportional to the velocity, then there remains after a sufficient length of time, only a vibration synchronous with P; that is, it is the solution of both equations as given by the formulæ $x = A \cos \omega t$ and $x' = A' \cos \omega t$, in which

$$A = \frac{\beta' a}{\alpha^2 - \beta\beta'}; \qquad A' = -\frac{\alpha a}{\alpha^2 - \beta\beta'}$$

with $\beta = m\omega^2 - \alpha; \qquad \beta' = m'\omega^2 - (\alpha + \alpha').$

The values for critical oscillation, that is, infinitely large values of A and A', are therefore obtained when

$$\alpha^2 - \beta\beta' = 0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

If m would swing alone, by considering m' fixed, we would derive the critical value from the equation

$$m\omega^2 - \alpha = 0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

If the force P were to act on the mass m' in the absence of m, we would have, similarly, to let

$$m'\omega^2 - \alpha' = 0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

Neither the one nor the other condition permits a satisfaction of equation 1.

There exist, therefore, critical values of vibrations for the elastic double pendulum, but these do not agree with the values that hold good for a single pendulum.

The mass m' remains uninfluenced if either the elastic resistance, that is α', or m' itself is infinitely large. If the ratios of these limits approach each other, then ω_k^2 will differ very little from the value $\frac{\alpha}{m}$ derived from equation 2. We can insert this value approximately in the expression for β' in equation 1, and get

$$\alpha^2 - (m\omega_k^2 - \alpha)\left(m'\frac{\alpha}{m} - \alpha - \alpha'\right) = 0$$

from which the corrected

$$\omega_k^2 = \frac{\alpha}{m}\left[1 + \frac{1}{\frac{m'}{m} - \frac{\alpha'}{\alpha} - 1}\right];$$

hence it may be said that the critical number of revolutions is increased by the sympathetic vibrations of the "foundation" (m'), if the mass is of the same size as compared to m; therefore the elastic reaction (α') is small as compared to that of the "shaft" (α) because the term standing next to unity becomes positive. The critical number of vibrations is decreased if the opposite conditions arise.

flection of a point in the shaft as given by the coördinates x in the direction of the shaft, y at right angles horizontally, and z at right angles vertically. Using again the notation of article 49, but placing the eccentricity $e=0$, the movement of the shaft may be represented by the equations

$$J\,E\frac{\delta^4 y}{\delta x^4}=-m_1\frac{\delta^2 y}{\delta t^2}$$

$$J\,E\frac{\delta^4 z}{\delta x^4}=-m_1\frac{\delta^2 z}{\delta t^2}$$

in which on the right side there appears the *d'Alembert* forces of inertia as a "load" on the shaft. The rotation around the axis through the center of gravity of an element cut by two planes at right angles to the axis, occurs with constant angular velocity, because we wish to assume an equilibrium of the turning moments. The two equations are therefore sufficient; from them y and z are to be so determined for a shaft of length $2l$ supported at both ends, that for $x=l$ we must have $y=0$ while $z=\zeta$. Here, ζ is the instantaneous deflection of the periodic vibration of the foundation, which is maintained by the elastic reaction, $\alpha\,\zeta$, of the bed; and by the shearing force of the shaft (exerted at its end cross-section) acting on the mass m' of the foundation. On this we construct the corresponding laws of motion, and in addition observe that with $x=l$ the bending moment disappears for the shaft supported at both ends. For the simplest case of a symmetrical deflection of the shaft, and a sinuous vibration of the foundation, we get the solution

$$y=[a'\,(e^{kx}+e^{-kx})+b'\cos k\,x]\sin\omega\,t$$

$$z=[a\,(e^{kx}+e^{-kx})+b\cos k\,x]\sin(\omega\,t+\epsilon),$$

in which ϵ is a value dependent upon the initial conditions. For the constants we get, as the eccentricity is made $=0$, finite values only at the critical number of revolutions, and only on the one hand, for the vertical vibrations, when

$$\text{hyp. tan}\,(k\,l)-\tan\,(k\,l)=2\beta \quad . \quad . \quad . \quad . \quad (1)$$

in which

$$k^4 = \frac{m_1 \omega^2}{J E}; \qquad \beta = \frac{m' \omega^2 - \alpha}{J E k^3};$$

on the other hand, for the horizontal vibrations, when

$$\cos k l = 0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

In the case here assumed, the foundation yielding in only one direction, we get two series of critical numbers of revolutions; one for the vertical deflections of the shaft, the other for the horizontal deflections. The synchronousness of rotation with the sympathetic vibration of the foundation, that is, $m' \omega^2 - \alpha = 0$, *gives in itself no critical number of revolutions.*

If we assume the foundation as uniformly elastic in all directions, equation 1 still holds good, as can be proved; and further, we obtain the interesting fact, that *with resonance the shaft rotates as though it were entirely free, neglecting the force of gravity* (as, for instance, with vertical machines).

74. CONDITIONS FOR STABILITY OF EQUILIBRIUM BEYOND THE CRITICAL VELOCITY.

Let us consider a single disc having no lateral oscillations. In Fig. 225, S is the center of gravity of the disc, W is the point of application of the load bending the shaft, D the projection of the geometrical rotating axis. We shall neglect the presence of other rotating masses on the shaft. If we have relative equilibrium, S would lie on the connecting line $O\,W$ at a distance ρ_0, which

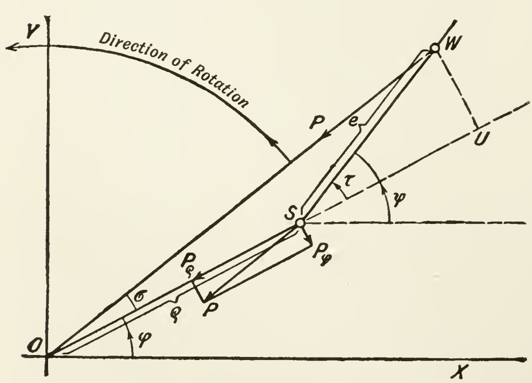

Fig. 225.

may be determined by letting the centrifugal force $m \rho_0 \omega_0^2$ equal the elastic resisting force, $\alpha (e + \rho_0)$, as before, and get

$$\rho_0 = \frac{\alpha e}{m \omega_0^2 - \alpha} \quad \ldots \ldots \ldots \quad (1)$$

The angle ϕ is determined, so long as equilibrium exists, by the uniform velocity ω_0, and is designated by ϕ_0, so that $\phi_0 = \omega_0 t$, in which t stands for time. The remaining forces acting upon the disc are kept in equilibrium ; and above all, we may imagine that the driving steam force imparts a pure moment, that is balanced by a corresponding torsion of the shaft, and which is transmitted to the working machine, which we shall assume as without mass. In order to test the stability of the dynamical equilibrium, we must increase by infinitely small functions of the time, and by deriving the equation of motion, the parameter, through which the motion under normal conditions is represented ; that is, ρ_0, ϕ_0 and the angle of OS and SW (which originally $= 0$). Fig. 225 represents a position of such a changed condition of motion, in which

$$\left.\begin{aligned} \rho &= \rho_0 + z \\ \phi &= \phi_0 + \epsilon \\ \psi &= \phi + \tau \end{aligned}\right\} \quad \ldots \ldots \ldots \quad (2)$$

where z, ϵ, τ represent infinitely small values.

For the equations of motion, ρ, ϕ and ψ are to be taken as variables,* and we must first investigate the motion of the center of gravity. The former follows as if the disc mass were placed as the center of gravity, and all forces applied there. The elastic force P, Fig. 225, is $= \alpha \overline{WO}$; but as $\overline{WO}$ is the distance of the resultants of $\overline{WU}$ and $\overline{UO}$, in which $\overline{WU}$ is perpendicular to $\overline{UO}$, so can this force be taken as the resultant of the forces $P_\phi = \alpha \overline{WU}$ and $P_\rho = \alpha \overline{OU}$ with their corresponding directions. Because of the small values of τ and σ

* It follows from the choice of the variables, specified by the nature of the problem, that we can advantageously make use of the so-called general *Lagrange* differential equations, which, in fact, give without trouble the elementary formulæ derived below.

$$\left.\begin{aligned} P_\phi &= a\,e\,\tau \\ P_\rho &= a\,(\rho + e) \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

which forces may be transposed to the center of gravity.

To find the change of ρ we shall observe the relative motion of the center of gravity in a radial slot (without weight), rotating with the radius vector. In order to accomplish this we must add the supplementary forces of relative motion, of which only the "centrifugal force" $m\,\rho\left(\frac{d\phi}{dt}\right)^2$ enters into consideration for the motion referred to, and we get

$$m\,\frac{d^2\rho}{dt^2} = m\,\rho\left(\frac{d\phi}{dt}\right)^2 - a\,(e + \rho) \quad . \quad . \quad . \quad . \quad . \quad (4)$$

To what remains we shall apply the law of the absolute motion of the center of gravity about 0 [that is, we say that the derivative of the "impulse moment" (moment of momentum) with respect to the time, is equal to the moment of the external forces], and get

$$\frac{d}{dt}\left(m\,\rho^2\,\frac{d\phi}{dt}\right) = -P_\phi\,\rho = -a\,e\,\tau\,\rho \quad . \quad . \quad . \quad . \quad (5)$$

For the motion about the center of gravity, the moment force $= a\,\overline{WO}\,e\sin(\tau - \sigma)$, or after slight transformation $= a\,e\,\tau\,\rho$; if then Θ denotes the mass moment of inertia of the disc about S, then is

$$\Theta\,\frac{d^2\psi}{dt^2} = a\,e\,\tau\,\rho \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

In the equations 4, 5, and 6 the values in equation 2 must be placed; develop them according to z, ϵ, and τ, and strike out all higher powers than the first. If we then insert the critical velocity

$$\omega_k^{\,2} = \frac{a}{m}$$

and substitute the notation

$$\delta = 1 - \frac{\omega_k^2}{\omega_0^{\,2}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

so that ρ is represented in the form

$$\rho_0 = \frac{1-\delta}{\delta} e \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (8)$$

we get for z, ϵ, and τ the linear equations

$$\left.\begin{aligned} \frac{d^2z}{dt^2} &= \delta\,\omega_0^{\,2}\, z + 2\rho_0\,\omega_0\,\frac{d\epsilon}{dt} \\ 2\,\omega_0\,\frac{dz}{dt} + \rho_0\,\frac{d^2\epsilon}{dt^2} &= -(1-\delta)\,\omega_0^{\,2}\, e\,\tau \\ \frac{d^2\epsilon}{dt^2} + \frac{d^2\tau}{dt^2} &= \frac{(1-\delta)^2}{\delta}\,\omega_0^{\,2}\,\frac{m\,e^2}{\Theta}\,\tau \end{aligned}\right\} \quad . \quad . \quad . \qquad (9)$$

The solution follows through the known relations

$$z = a\, e_0^{\lambda t} \qquad \epsilon = b\, e_0^{\lambda t} \qquad \tau = c\, e_0^{\lambda t},$$

in which (in order to distinguish it from e) e_0 is the base of the natural logarithms. This substitution gives for λ, after abbreviating with λ^2, the biquadratic equation,

$$\lambda^4 + 2\,B\,\omega_0^{\,2}\,\lambda^2 + C\,\omega_0^{\,4} = 0 \quad . \quad . \quad . \quad . \quad . \quad . \qquad (10)$$

in which

$$\left.\begin{aligned} B &= 2 - \delta - \frac{1}{2}\frac{(1-\delta)^2}{\delta}\,\nu^2 \\ C &= \delta^2 - \frac{(4-\delta)(1-\delta)^2}{\delta}\,\nu^2 \end{aligned}\right\} \begin{aligned} &\nu^2 = \frac{m\,e^2}{\Theta} = \frac{e^2}{q^2} \quad . \quad . \quad . \qquad (11) \\ &q = \text{radius of gyration.} \end{aligned}$$

The equilibrium is stable when the values z, ϵ, τ remain small for the entire duration of the motion; therefore λ, when real, may not become positive; when complex, the real part must be negative. This demands * that

$$B > 0 \qquad C > 0 \qquad B^2 - C > 0 \quad . \quad . \quad . \quad . \quad . \qquad (12)$$

For small values of δ, we may approximately replace the conditions by the condition that

* See Routh, Dynamik II, § 289.

$$\delta^3 > 4\frac{e^2}{q^2}. \qquad (13)$$

If the ratio of the radius of gyration to the eccentricity e is very large, then ν^2 would have a very small value, and *stability will occur at very small excess over the critical velocity.* This is the occurrence in practically all cases. But if the moment of inertia is negligibly small, $\Theta = 0$; then the equilibrium is unstable, and the simultaneous $e = 0$. The value of the moment of inertia is therefore of vital importance, and must be taken into account by making experiments.*

The stability of a uniformly loaded shaft can also be investigated in this manner. We can, for instance, to simplify the calculation, assume that the eccentricity varies according to a sine function, so that

$$e = e_m \sin kx,$$

and that the coördinate origin lies in the same end of the shaft, whereby $k = \frac{\pi}{l}$ and l is the length of shaft. The centers of gravity of all discs might lie in a plane; the masses m_1 per unit length are unchangeable. It is necessary to take into account the shaft's deflection, not only in the plane of the center of gravity, but also in that at right angles thereto. The solution of the general equations of motion can be accomplished for the case where we consider such an oscillation about the position of equilibrium that the shaft will only deflect, but does not experience any rotation; and for the assumption that the moment of inertia Θ_1 of the disc referred to unit length, comply with the law $\Theta_1 = \Theta_m \sin^2 kx$. If we place, as before,

$$\delta = 1 - \left(\frac{\omega_k}{\omega_0}\right)^2 \text{ and } \nu = \frac{m_1 e^2}{\Theta_m};$$

then, in these expressed values, exactly the same conditions of stability hold good as for a simple disc of mass m_1 and moment of inertia Θ_m. The calculation is nevertheless too cumbersome to repeat here.

* Sanford A. Moss, in a thesis for his doctor's degree at Cornell University (printed in May, 1903), after the publication of the above investigation, thought to have simplified the calculation by assuming e infinitely small. The final formulæ derived by Moss are therefore mathematically not correct.

75. GYROSCOPIC ACTION OF THE MARINE TURBINE.

The revolving turbine masses constitute a high rotation which imposed upon these masses a forced motion during the pitching of the vessel, or while making a sharp turn, by which reaction forces are created in the bearings, whose values it may be well to investigate for reasons of safety.

A purely vertical swinging places upon the turbine shaft only the demands of the forces of inertia, which can easily be found as the product of the mass of the wheels and the maximum acceleration. The value of the velocity reached by these oscillations is entirely without influence. It is quite different with a rotating oscillation, as is caused by the ship's pitching. In the highest and the lowest position of the shaft, the shaft is bent *in a vertical plane* by the forces of inertia; on account of the varying slope of the ship's hull, there occur also deflections *in a horizontal plane* peculiar to the high rotation. We shall concern ourselves only with the latter, and imagine the turbine masses represented in Fig. 226 by a rotating disc S about the X axis. The axis horizontal at the time $t = 0$, has turned after the time dt, through the angle $d\phi$ which is designated by the angular velocity ϵ while turning about the Y axis. The rotation about the actual turbine shaft occurs with the constant angular velocity ω. It is necessary to observe that the occurring bending moment, as a matter of fact, turns about the Z axis. To accomplish this, we must further observe that the velocity components in the direction of X change exceedingly little

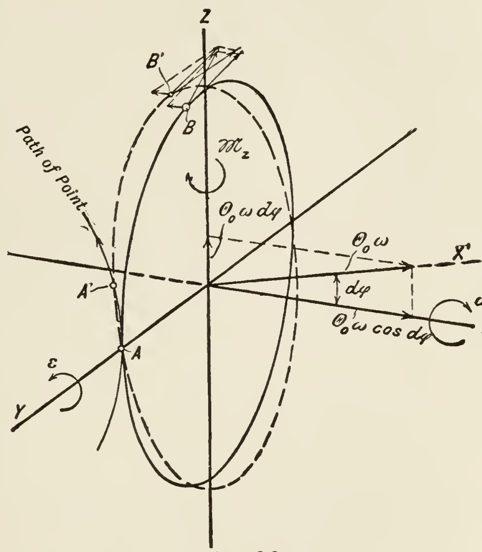

Fig. 226.

in the territory of the point of resolution lying at B, therefore only very small accelerating forces in the horizontal direction need act on this point. On the other hand, the mass point at A is forcibly drawn away from its direction of motion by the varying slope of the wheel disc, and describes a path concave to the positive direction of X. An accelerating force is now necessary in the negative direction of X, whose sum, with the opposite force-components, gives a rotating moment about the Z axis in the direction shown in the figure, which must be carried over to the outside; that is, by means of the bearing pressure on the shaft.

For numerical calculation, the law of Mechanics serves, that the increase of the moments of the value of momentum (the "impulse moment") per unit time (that is, the derivative of the impulse moment with respect to the time) is equal to the external force-moments turning about the given axis. At the time $t = 0$ the impulse moment for the Z axis $= 0$, for the X axis $= \Theta_0 \omega$, in which Θ_0 is the mass moment of inertia for this axis. After the time dt the impulse moment for the X' axis at a slope $d\phi$ is still $\Theta_0 \omega$; and we resolve the same into the components $\Theta_0 \omega \cos d\phi$ and $\Theta_0 \omega \sin d\phi$. The latter constitutes the increase of impulse moment for the Z axis during the time dt, and therefore we may write the equation

$$\frac{\Theta_0 \omega \sin d\phi - \text{Nothing}}{dt} = \Theta_0 \omega \frac{d\phi}{dt}$$

and we obtain, with $\epsilon = \dfrac{d\phi}{dt}$, the equation

$$\mathfrak{M}_z = \Theta_0 \, \omega \, \epsilon \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

in which $\mathfrak{M}_z$ is what *Stodola* calls the (Kreiselmoment) rotary moment. The angular velocity ϵ is during the time T an entire (to and fro) oscillation of the ship's hull, and the amplitude ϕ_0 is determined by the angle of slope from the formula

$$\epsilon = \frac{2 \pi \phi_0}{T} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

When the ship, during manœuvre, executes a sharp turn, then

ϵ is to be taken as the angular velocity about the Z axis, and the centrifugal moment turns about the T axis. With a many-stage turbine, with closely placed individual wheels, Fig. 227, there ensues a remarkable stress on the shaft due to the sum of the $\mathfrak{M}_z$. The individual moments $\mathfrak{M}_{1z}$ call forth in the bearings two equally large forces opposite in direction, whose moment is equal to the sum of $\mathfrak{M}_z$. In going from the bearing positions the sum of $\mathfrak{M}_z$ increases as uniformly as the moments of the bearing pressures, so that the shaft, no matter how long it may be, is only brought under stress by the shearing force P.

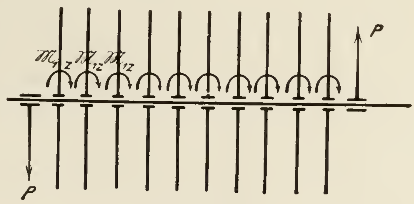

Fig. 227.

76. CRITICAL VELOCITY OF THE SECOND DEGREE, CAUSED BY THE DEFLECTION OF A UNIFORM SHAFT, DUE TO ITS OWN WEIGHT.

A shaft supported horizontally at the points A_1 and B_1, Fig. 228, is deflected while in a position of rest under the influence of its own weight. If the shaft is rotated very slowly, this form remains unchanged, because the compressed upper fiber A_2B_2 has

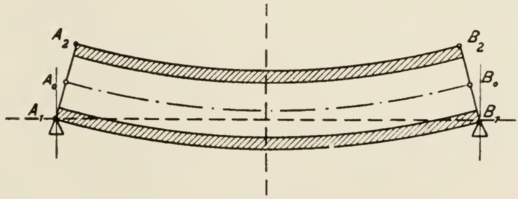

Fig. 228.

time to follow the bending stresses and to become elongated, so that after one-half revolution it has assumed the length A_1B_1. But if the rotation velocity becomes greater, mass inertia enters into the problem, and, as a consideration will show, with the result that the deflection increases up to a critical velocity, then again

decreases. The fiber A_2B_2 begins to elongate in the highest position, and a part of its accumulated stress energy is utilized for the acceleration of its mass particles in a horizontal direction. In the middle position A_0B_0 these particles possess the maximum velocity symmetrically distributed in the vertical plane through the middle of the rod, therefore also the maximum kinetic energy, which acts upon the fiber as a strong tension during the next quarter-turn, as is demanded by the pure bending stresses. But when the time of revolutions reaches the value corresponding to the simple longitudinal oscillations of the fiber, then the so-called "*resonance*" occurs; that is, the impulse increases during each period, and the critical number of revolutions has been reached. The longitudinal oscilla-

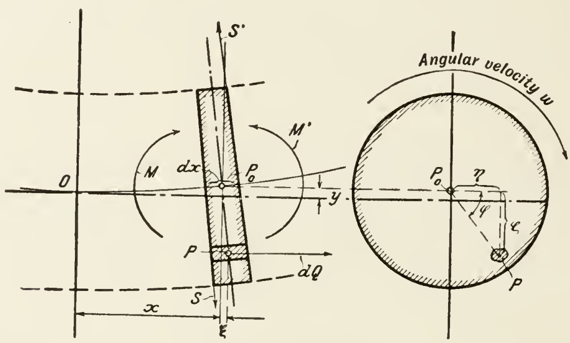

Fig. 229.

tions of a fiber depend only upon the length and material of the shaft, and it follows, à priori, *that the critical number of revolutions is independent of the diameter of the shaft.* If this velocity is exceeded, the fiber is forced into a too high oscillation, so that it has "no time" to expand sufficiently, and the shaft, for this reason, straightens out more and more.

The value of the critical number of revolutions is found by the following abbreviated calculation. Let dm in Fig. 229 be a mass element at a point P of an infinitely thin disc cut out of the shaft, that is bounded by two planes at right angles to the elastic curve. The distance of the center of gravity of the disc from the coördinate origin is x; the very small assumed ordinate of the elastic curve $= y$. The length of dm is determined by the coördinates η ζ of the side view; its distance from the plane at right angles to

shaft's axis at the coördinate origin, is $x + \xi$. As the slope of the elastic curve at the point P_0 is given by $\frac{dy}{dx}$, we get

$$\xi = \zeta \frac{dx}{dy} \quad \ldots \ldots \ldots \quad (1)$$

Counting the angle of rotation from the horizontal, then

$$\zeta = \rho \sin \phi = \rho \sin \omega t \quad \ldots \ldots \quad (2)$$

and ξ varies also as $\sin \omega t$. Consider now the inertia forces in the direction of the X axis, which are obtained from the product of dm and the negative horizontal acceleration of the mass particles; that is, by

$$dQ = -dm \frac{d^2(x+\xi)}{dt^2} = -dm \frac{d^2\xi}{dt^2} = +dm\, \omega^2 \rho \sin \omega t . \frac{dy}{dx}$$

$$= dm\, \omega^2 \frac{dy}{dx} \zeta \quad \ldots \ldots \ldots \quad (3)$$

If the inertia forces are applied to each mass element, then equilibrium must exist between their sums and the external forces. On the infinitely thin disc under consideration, there act as external forces the shearing forces S' and S, the bending moments M and M', and the force of gravity $\gamma_1\, dx$ (γ_1 denoting the weight per unit length). The components of the forces of inertia at right angles to the axis are in equilibrium (they strain the shaft radially), and the horizontal components exert the moment

$$d\mathfrak{M} = \int dQ\, \zeta = \int dm\, \omega^2 \frac{dy}{dx} \zeta^2 = \omega^2 \mu J \frac{dy}{dx} . dx \quad . \quad . \quad (4)$$

in which

μ is the specific mass $= \frac{\gamma}{g}$

J is the cross-section moment of inertia of the rod.

Equilibrium demands that

$$M' - M + S' \frac{dx}{2} + S \frac{dx}{2} + d\mathfrak{M} = 0 \quad . \quad . \quad . \quad (5)$$

$$S' - S - \gamma_1\, dx = 0 \quad \dots\dots\dots \quad (6)$$

or, what is the same thing,

$$\frac{dM}{dx} + \frac{d\mathfrak{M}}{dx} + S = 0\,; \qquad \frac{dS}{dx} = \gamma_1 \quad \dots\dots \quad (7)$$

If we differentiate the first of the above equations with respect to x, doing away with $\frac{dS}{dx}$, and using the bending equation that always holds good,

$$M = J\,E\,\frac{d^2y}{dx^2} \quad \dots\dots\dots \quad (8)$$

and we get

$$\frac{d^4y}{dx^4} + \frac{\mu\,\omega^2}{E}\,\frac{d^2y}{dx^2} + \frac{\gamma_1}{J\,E} = 0 \quad \dots\dots \quad (9)$$

as the differential equation of the problem.

Letting

$$\lambda = \omega\sqrt{\frac{\mu}{E}}\,; \qquad C = \frac{\gamma_1}{\mu\,\omega^2 J} \quad \dots\dots \quad (10)$$

the solution reads

$$y = -\frac{C\,x^2}{2} + C_1\,x + C_2 - \frac{A}{\lambda^2}\cos\lambda\,x - \frac{B}{\lambda^2}\sin\lambda\,x\,. \quad (11)$$

In the above, A, B, C_1, and C_2 are arbitrary constants determined by the limit conditions.

For

$$\left.\begin{aligned} &x = 0 \text{ is } y = 0 \text{ and } \frac{dy}{dx} = 0 \\ &x = l \text{ is } M = 0, \text{ that is, } \frac{d^2y}{dx^2} = 0 \end{aligned}\right\} \quad \dots\dots \quad (12)$$

and y must be a symmetrical function of x. From this follows $B = 0$, $C_1 = 0$

$$\left.\begin{aligned} C_2 &= \frac{A}{\lambda^2} \\ A &= \frac{C}{\cos kl} \end{aligned}\right\} \quad \dots\dots\dots\dots \quad (13)$$

so that

$$y = \frac{C}{\lambda^2 \cos(\lambda l)}(1 - \cos \lambda x) - \frac{C}{2} x^2 \quad . \quad . \quad . \quad . \quad (14)$$

Infinitely large values of y, that is, critical velocities, occur when $\cos(\lambda l) = 0$, that is

$$\lambda l = \frac{\pi}{2}; \quad 3\frac{\pi}{2}; \quad 5\frac{\pi}{2}.$$

The smallest value of the critical velocity is

$$\omega_k = \frac{\pi}{2l}\sqrt{\frac{E}{\mu}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (15)$$

and, as has been said, independent of the shaft diameter; *and fortunately so high, that only with long shafts can it come into consideration.*

We can easily prove that the duration of the free longitudinal oscillations of a rod of length $2l$ corresponds with the duration of one revolution at the velocity ω_k.*

The critical velocity is much lower with a *shaft loaded* by closely spaced discs, in which the discs greatly increase the inertia resistances of the shaft compared to the oscillations about a vertical line at right angles to the turning axis. Equation 4 would be here expressed:

$$d\mathfrak{M} = \omega^2 \mu \, dx \, (J + J') \frac{dy}{dx} \quad . \quad . \quad . \quad . \quad . \quad (16)$$

in which $\mu J'$ is the mass moment of inertia of the discs referred to a shaft of unit length for an axis at right angles to the shaft. Sim-

* The differential equation of the longitudinal oscillation of a straight prismatic rod is

$$\mu \frac{\delta^2 \xi}{\delta t^2} = E \frac{\delta^2 \xi}{\delta x^2},$$

in which ξ stands for the elongation of the rod. If we let $\xi = a \cos \omega t$, in which a depends only upon x, we get for the rod of length $2l$ held fast in the middle and having its end areas without stress

$$\xi = a \sin \lambda x \cos \omega t$$

and

$$\lambda l = \omega l \sqrt{\frac{\mu}{E}} = \frac{\pi}{2} \text{ as above.}$$

ilarly for γ_1 we place $\gamma_1+\gamma_1'$ in which γ_1' is the weight of the discs per unit length of axis. The method of integration does not change, and gives for the freely supported shaft of length $2\,l$ the critical velocity

$$\omega_k=\frac{\pi}{2l}\sqrt{\frac{E}{\mu\left(1+\frac{J'}{J}\right)}} \quad . \quad . \quad . \quad . \quad . \quad . \quad (17)$$

The effect of the disc is therefore the same as if the specific mass of the shaft material were increased in the ratio $\frac{(J+J')}{J}$.

This velocity also lies in general very much above the critical velocity. It therefore follows that the critical number of revolutions of the second degree need not be considered in practical problems, and that the designer need not search for its presence because under certain circumstances the shaft behaved unsatisfactorily.

77. TRANSMISSION OF HEAT THROUGH THE CASING AND SHAFT OF A MANY-STAGE TURBINE.

The casing and the shaft or drums of a many-stage turbine form a good conductor of heat between the admission and condenser spaces, or an intermediate receiver. The transmission of heat and the drop in temperature caused thereby form a loss of whose value the designer should be aware.

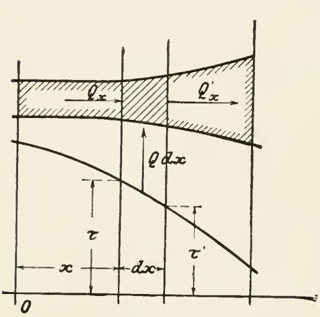

Fig. 230.

We shall take under consideration a turbine of Parsons construction, and make the assumption *that the temperature of the walls is identical at every place with the temperature of the steam flowing past that point.* This assumption is practically true with saturated steam, because the coefficient of conductivity between such steam and iron is very large; and here, where the flow velocity consists of hundreds of meters, we can neglect the temperature drop all the more because the transmitted heat is small. We shall imagine the casing and drum

stretched out in a straight rod of given variable cross-section (Fig. 230) whose surface temperature, τ, is a function of the distance x calculated from the known distribution; that is,

$$\tau = \phi(x) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

The temperature τ', at the distance $x + dx$ (taken algebraically), determines the temperature drop

$$\frac{\tau' - \tau}{dx} = \frac{d\tau}{dx}$$

at that place, and the quantity of heat that flows through the cross-section F at a distance x in unit time is

$$Q_x = -\lambda F \frac{d\tau}{dx} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

in which λ is the specific conductivity. Through the cross-section F' at distance $x + dx$ there passes the heat

$$Q_x' = -\lambda F' \frac{d\tau'}{dx} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

The difference $Q'_x - Q_x$ is furnished by the outside surface of the steam, and is designated as $Q\,dx$, in which Q is *the heat imparted to the casing by the steam per unit length in unit time.* The heat transmitted to the outside by radiation may as a first approximation be calculated independently. We have therefore

$$Q\,dx = Q_x' - Q_x = -\lambda\left[F'\frac{d\tau'}{dx} - F\frac{d\tau}{dx}\right],$$

which expression may also be written in the form

$$Q = -\lambda \frac{d}{dx}\left[F\frac{d\tau}{dx}\right] \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

The differential quotient can easily be found graphically, and permits the determination of the change of heat between wall and

steam at every point of the casing. In general, Q is always positive; that is, *heat is taken from the steam during the entire course of its flow. The total heat Q_0 taken up by the walls per unit time* is especially simple to calculate. It is, namely,

$$Q_0 = \int Q\,dx + \left(Q_x\right)_{x=0}$$

integrated over the entire length of the "rod."

The first part is taken up during the flow; the second comes through conduction from the steam chamber. We get

$$Q_0 = -\lambda\left[F_2\left(\frac{d\tau}{dx}\right)_{x=l} - F_1\left(\frac{d\tau}{dx}\right)_{x=0}\right] + \left(Q_x\right)_{x=0} \quad . \quad (5)$$

in which F_1 is the exit cross-section and F_2 the final cross-section of the "rod" represented.

But if we insert the value of equation 2, we get simply

$$Q_0 = -\lambda\, F_2\left(\frac{d\tau}{dx}\right)_{x=l} \quad . \quad . \quad . \quad . \quad . \quad . \quad (5a)$$

which expression might have been written without any calculation.

The pressure p decreases often linearly with x. In this case we would introduce the derivative of the pressure with respect to coördinate x into the formula by the notation

$$\frac{d\tau}{dx} = \frac{d\tau}{dp}\frac{dp}{dx} \quad \text{and} \quad \frac{dp}{dx} = \frac{p_2 - p_1}{l}$$

which will lead to the expression

$$Q_0 = \lambda\,\frac{p_1 - p_2}{l}\,F_2\left(\frac{d\tau}{dx}\right)_{x=l} \quad . \quad . \quad . \quad . \quad . \quad (5b)$$

in which p is expressed in atmospheres, even if so understood in $\dfrac{d\tau}{dp}$.

The derivatives are approximately determined from the steam tables. For instance, if $p_2 = 0.1$ kg. per sq. cm., $\tau = 45.58$; for

$p_2' = 0.2$; $\tau' = 59.76$, so that $\frac{d\tau}{dp} = \frac{\Delta\tau}{\Delta p} = 141.8$. Let $l = 2$ meters, and $\lambda = 50$ heat units per sq. meter hour, with $p_1 = 10$ atmospheres and $F_2 = 0.25$ sq. meters which corresponds to quite a large turbine. Formula *5b* gives

$$Q_0 = 50 \, \frac{10 - 0.1}{2} \, 0.25 \cdot 141 \cdot 8 = \text{about } 8\,800 \text{ heat unit hours.}$$

With good lagging, the amount of heat conducted and radiated to the outside would be equally large. As this example refers to a turbine of over 1 000 h.p., we may take the heat thus lost at full load as a very small correction. At no load this value would be of considerably more importance, but the loss of heat on account of the decrease of temperature on all sides will be considerably less.

For the case of superheated steam, the calculations must be made more exactly; that is, taking into consideration the coefficient of surface conductivity, which will lead to a complicated differential equation of the second order.

78. THE DIFFERENTIAL EQUATION FOR PRESSURE DISTRIBUTION IN A MANY-STAGE AXIAL REACTION TURBINE.

While the design of a new turbine causes little trouble* as soon as the fundamental principles are understood, we may say, inversely, that the question of the performance of this turbine with considerably different load, presents a problem whose solution is scarcely reliable. In the latter case, the peripheral velocity, the angles, and the cross-sections are given, from which the absolute values and the distribution of pressure are to be found. It is possible to get

* This statement has been falsely taken up by the critics, as though the author did not recognize the considerable difficulties which the *constructive* design presented in all parts, when safety of running is considered. From the general sense we see that we were only concerned with the determination of the power, that is, the work of the steam, and the fundamental turbine dimensions depending upon it. In fact, for practical success, as repeatedly demonstrated by experience, we are more concerned with the greatest care in construction than with any secret "wrinkle," and obviously no blunder should be made against the old-established rules of machine construction.

mathematically, in certain simple cases, a comparative comprehension of these occurrences which is perhaps worth stating.

In Fig. 231 is represented a velocity diagram of a given wheel. Let $c_2{}^*$ be the velocity with which the steam leaves the preceding

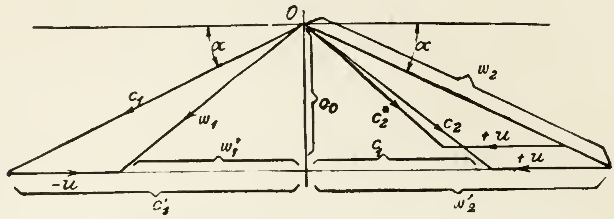

Fig. 231.

rotating wheel. The fundamental equations for the guide and rotating wheels can be written, according to formula 3*c*, Article 14*a*, in the form

$$\frac{c_1{}^2 - c_2{}^{*2}}{2g} = -\int_p^{p'} v\ dp - R_1,$$

$$\frac{w_2{}^2 - w_1{}^2}{2g} = -\int_{p'}^{p''} v\ dp - R_2.$$

Adding we get

$$\frac{c_1{}^2 - c_2{}^{*2}}{2g} + \frac{w_2{}^2 - w_1{}^2}{2g} = -\int_p^{p''} v\, dp - R \quad . \quad . \quad . \quad (1)$$

To bring into the calculation the losses R due to friction, we multiply the integral with a factor ϵ, which is smaller than 1, and is taken equal for all turbine wheels. We determine ϵ so that the sum of the work of friction of the entire turbine is correctly obtained; that is, we let $\epsilon = 0.75$ to 0.60. The assumed variability of these values influences then only the distribution of the resistances. In going from the case of one load to the case of another, ϵ, strictly speaking, would, of course, also change its value, because of the changed velocities and the steam entrance which is not free from shock.

On the left side we will substitute temporarily for $c_2{}^*$ the somewhat larger exit velocity from the rotating wheel under consideration, that is, c_2, and obtain

$$c_1^2 = c_1'^2 + c_0^2\,;\;\; c_2^2 = c_2'^2 + c_0^2\,;\;\; w_1^2 = w_1'^2 + c_0^2\,;\;\; w_2^2 = w_2'^2 + c_0^2\,;$$

solving for the difference of squares we get for the left side,

$$\frac{1}{2\,g}\Big[(c_1' + c_2')\,(c_1' - c_2') + (w_2' + w_1')\,(w_2' - w_1')\Big]$$

from which we get, on account of the equality of the angles α and α_2 because $w_2 = c_1$ and $w_1 = c_2$, the equation

$$\frac{u}{g}\,(2\,c_1 \cos\alpha - u).$$

As this value is too small on account of having substituted the too large value c_2, we multiply by a factor $\delta > 1$, which is also taken as a constant mean value, and differing very little from 1. The integral on the right side can be simplified, with the slight pressure difference $p - p''$, to its mean value

$$-\int_p^{p''} v\,dp = -\,v\,(p'' - p).$$

If the initial pressure for each turbine wheel is drawn, as in Article 29, as ordinates at a distance $\Delta\,x$, and if we connect the points thus derived by a uniform curve, then $\dfrac{p'' - p}{\Delta\,x}$ can be approximately replaced by the differential quotient $\dfrac{dp}{dx}$.

Then

$$-\int_p^{p''} v\,dp = -\,v\,\frac{p'' - p}{\Delta\,x}\,\Delta x = -\,v\,\frac{dp}{dx}\,\Delta\,x \quad . \quad . \quad . \quad (2)$$

We now insert as an independent the value

$$z = \frac{x}{\Delta x} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

which represents, as can be seen, if we choose such abscissæ lengths x that z is a whole number, the *number of turbines passed through at this time.* Then

$$\frac{dp}{dx}\Delta x = \frac{dp}{dz}\frac{dz}{dx}\Delta x = \frac{dp}{dz}$$

The fundamental equation then is

$$-\epsilon v \frac{dp}{dz} = \delta \frac{u}{g}(2 c_1 \cos \alpha - u) \quad . \quad . \quad . \quad . \quad . \quad (4)$$

and is the differential equation* of our problem.

* We can also use this solution for the interesting problem of the design of a turbine with constant cross-section of flow for all wheels.

From

$$(p + \beta) v = K$$

and

$$G v = f_1 c_1$$

is obtained

$$p + \beta = \frac{K}{v} = \frac{KG}{f_1 c_1} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

We shall introduce for the time the new variable

$$y = \frac{1}{c_1}$$

and obtain with the assumption that f_1 = constant,

$$-\frac{dy}{dz} = \frac{\delta u^2}{\epsilon K g}\left(\frac{2 \cos \alpha}{u} - y\right) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

from which is obtained, even if u is considered = constant,

$$\log \left(\frac{\frac{2 \cos \alpha}{u} - y}{\frac{2 \cos \alpha}{u} - y_a}\right) = \frac{\delta u^2}{K g \epsilon} z \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

and y_a stands for the initial value of y.

The solution gives

$$y = \frac{2 \cos \alpha}{u} - \left(\frac{2 \cos \alpha}{u} - y_a\right) e^{\frac{\delta u^2}{K g \epsilon} z} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

To make an integration possible, we must assume the condition equation of the steam in the simplified form

$$(p + \beta)\, v = K \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

which for our purposes is sufficiently close to the exact value. We get rid of c_1 through the equation of continuity

$$G v = f c_1 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

and obtain

$$\frac{d}{dz}\left(\frac{p+\beta}{G}\right) - \frac{\delta u^2}{\epsilon K g}\left(\frac{p+\beta}{G}\right) + \frac{2\,\delta \cos\alpha}{\epsilon g}\,\frac{u}{f} = 0 \quad . \quad . \quad (7)$$

As in general, up to 10, or even 20 stages, the exponent is considerably smaller than 1, we can solve and neglect the higher powers. If c_{1a} is the first velocity, and c_{1z} an intermediate velocity, we have the simplified formula

$$c_{1z} = \frac{c_{1a}}{1 - \dfrac{\delta u\ (2 \cos\alpha\, c_{1a} - u)}{\epsilon K g} z} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

We could also divide the turbine into groups of $z_1, z_2 \ldots\ldots$ etc., wheels, each with constant cross-section, and obtain for each group a final velocity c_{1e}, calculated from the first arbitrary value c_{1a}. The intermediate values must change according to the hyperbolic law. As the formula represents only an approximation, we must at the end of the process make a check calculation.

The turbine designer who has recourse to more nearly exact values from observation with constructed turbines, can bring these results to correspond closely with practice. By taking from the first design the values of the pressures and the specific volumes for entrance to and exit from single successive groups, the equation of continuity can be written

$$G = \frac{f_{1a}\, c_{1a}}{v_{1a}} = \frac{f_{1a}'\, w_{1a}}{v_{1a}} = \frac{f_{2a}'\, w_{2a}}{v_{2a}} = \frac{f_{1b}\, c_{1b}}{v_{1b}} = \ldots$$

(in which corresponding cross-sections and velocities are denoted by the same signs and from this calculate the more nearly exact values c_{1a}, w_{1a}, w_{2a}, c_{2a}, c_{1b}, $w_{1b} \ldots$ With these values we get

$$h_b = \frac{c_{1b}^2 - c_{2a}^2}{2g} + \frac{w_{2b}^2 - w_{1b}^2}{2g},$$

$$h_c = \frac{c_{1c}^2 - c_{2b}^2}{2g} + \frac{w_{2c}^2 - w_{1c}^2}{2g}.,$$

which are the more nearly exact values of the individual "drop," from which the mean value h_m may be taken and the corrected number of stages Z_0 calculated. We must also observe the somewhat larger drop in going from the last wheel of one group to the first wheel of the next larger group. Finally, with the Parsons design we may take into consideration the quantity of steam flowing through the balancing pistons by suitably decreasing G at the place referred to.

In this are

$$\frac{\delta u^2}{\epsilon K g} = \phi(z)$$

$$\frac{2\,\delta \cos \alpha\, u}{\epsilon g \quad f} = \psi(z)$$

given (represented by a drawing) functions of z, and the general integral form of equation 7 can always be found.

If we place

$$\Phi(z) = e^{\int_0^z \phi(z)\,dz}$$

$$\Psi(z) = \Phi(z) \int_0^z \frac{\psi(z)}{\Phi(z)}\,dz,$$

which expressions are also graphically determined, then

$$\frac{p + \beta}{G} = C\,\Phi(z) - \Psi(z)$$

in which C is an arbitrary constant. The introduction of the obtained integration, which is in general allowable, gives simple values at the limits.

For $z = 0$, p should $= p_1$; that is,

$$\frac{p_1 + \beta}{G} = C\,\Phi(0) - \Psi(0).$$

But as $\Phi(0) = 1$, $\Psi(0) = 0$, we get

$$C = \frac{p_1 + \beta}{G}$$

and

$$\frac{p + \beta}{G} = \frac{p_1 + \beta}{G}\,\Phi(z) - \Psi(z)\,. \quad . \quad . \quad . \quad . \quad (8)$$

For $z = z_0$, the total number of turbines is $p = p_2$; we have, therefore, from equation 8,

$$G = \frac{(p_1 + \beta)\,\Phi(z_0) - (p_2 + \beta)}{\Psi(z_0)} \quad . \quad . \quad . \quad . \quad . \quad (9)$$

As $\Phi(z_0)$ is always > 1, and β mostly a small value, we may say approximately

and

$$\left.\begin{aligned} G &= p_1 \frac{\Phi(z_0)}{\psi(z_0)} \\ p &= p_1 \left[\Phi(z) - \frac{\Phi(z_0)}{\psi(z_0)} \psi(z) \right] \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad (10)$$

from which formulæ is deduced the following law:

The weight of steam flowing per second and the pressure at any one place of the turbine are approximately (for not too great limits) proportional to the initial pressure in the first guide wheel. The steam volume entering per second is Gv_1 = constant, therefore $p_1 v_1$ are between certain limits not varying greatly, from which *follows that the steam velocity in the first turbine remains approximately constant with small changes of load. The last exit velocity, and with it the exit losses, decrease equally with the quantity of steam.*

The influence of the change of u cannot so easily be seen. If we originally are working with a velocity u (Fig. 232), and if we go over to the larger u', then c_1 must decrease, because if c_1 should remain equal, the "drop"

$$h = \frac{c_1^2 - c_2^2}{2g} + \frac{w_2^2 - w_1^2}{2g}$$

would be too large, and the vacuum pressure p_2 would occur much earlier than after the last rotating wheel. According to this, G also must decrease. On account of this, the steam friction decreases because of the smaller flow velocities in the cross-section, and the *efficiency would increase.* Beyond the limits where c_2 has an axial direction, G will again increase.

In like manner with increasing peripheral velocity, c_1 and G must increase, and not, as it would appear, without limits, because, if such were the case, the pressure difference which would be necessary to cause the first entrance velocity c_1 would have been

neglected. But with very small peripheral velocities this entrance drop would consume a considerable part of the pressure, so that, in

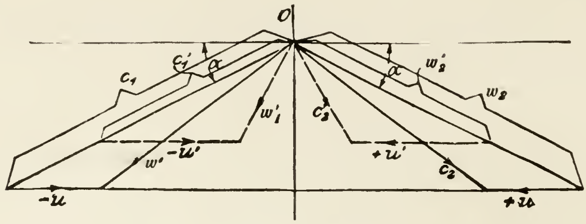

Fig. 232.

combination with the action of increased resistances, the increase of G need not be considerable.

79. RUNNING LIGHT, AND THE LIMIT VELOCITIES OF THE MANY-STAGE TURBINE.

By *running light* we mean the condition of running without any load, but under the influence of the governor; that is, at approximately normal speed of turbine, where the governing valve has full boiler pressure brought to it. The steam consumption of running light or at no load is of vital importance, because practice has shown that the total consumption per hour increased lineally with the effective load. By giving the consumption at full load and at running light we can find also the economy of all intermediate loads.

The consumption of feed water of the *Parsons Turbine* is from 10% to 20% of the normal, as can be seen from the following table taken from the experiments of *Stoney*, which have already been discussed.

TABLE 3.

STEAM CONSUMPTION OF THE PARSONS TURBINE RUNNING LIGHT.

Power	kw.	**52.7**	**108**	**232**	**529**	**1 190**
Corresponding steam consumption,	kg. per hour	**671**	**1 320**	**2 304**	**5 459**	**10 485**
Corresponding steam consumption,	*lb. per hour*	1 476	2 904	5 069	12 009.8	23 067
Consumption running light . . .	kg. per hour	**145**	**136**	**431**	**670**	**1 183**
Consumption running light . . .	*lb. per hour*	319	299	948	1 474	2 614
Consumption running light . . .	%	**21.6**	**10.3**	**18.7**	**12.3**	**11.3**

Not taking into account the heat radiation of the casing, the steam is superheated considerably by the strong throttling of the regulating valve, and it is possible that the product of the specific volumes and the quantity of steam per hour, that is, the total volume per hour at the entrance to the turbine while running light, may increase to double the volume at full load. In this case the flow velocity in the first wheels will also increase. On the other hand, if we compare the conditions at condenser pressure or at pressures coming close thereto, the steam volume, therefore also the steam velocity, has greatly decreased at no load. If the steam friction in the blade channels is proportional to the square of the steam velocity, then the total value of the steam friction running light must be much smaller than at full load, provided we assume entrance without shock. This friction is further decreased because the steam is highly superheated, and remains longer superheated than at full load.

But against these disadvantages we have the facts that on account of low steam velocity, entrance to the rotating and guide wheels occurs with shock for the larger part of all the stages ; and that the steam velocity must decrease from wheel to wheel, instead of increasing as usual. The occurrence hereby taking place is exceedingly complicated, and necessitates a somewhat full discussion. The blades are mostly placed so close together that at least towards the exit the stream must entirely fill the channel. There we can easily calculate the steam from the quantity of steam and the heat condition, which is quite easily determinable. The exit velocity c_1 from the guide wheel then gives with $-u$ the relative velocity w_1 from the rotating wheel, which for the low pressure wheels has approximately the direction as shown in Fig. 236, but with steeper entrance. The eddy currents hereby occurring are a part of the total loss, but then steam shock may be caused by the enlargement that follows the contraction in the rotating blades. Here occurs a compression of the steam which is accompanied by losses and the necessary decrease of the flow velocity of the steam. The losses can so increase that the last wheels furnish no work, or even act as brakes.*

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* There has often been determined the point of intersection O_1 (Fig. 233) in the representation of the steam consumption as a function of the load, by prolonging the steam consumption line, and OO_1 has been taken as the work of running light, but

If a *fully loaded turbine is suddenly unloaded*, and the regulator does not work, the machine will "run away" and will reach a cer-

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this is incorrect. Let a condenser pressure exist at entrance to, and exit from, a turbine. If the turbine is rotated by a motor, there may occur a slight suction, as was already explained previously. But theoretically the turbine would do no work so long as no steam is needed. If we open the steam valve so little that only an exceedingly small quantity of steam can enter, then the absolute velocity at exit from a guide blade, that is, c_1 and the relative exit velocity w_2, will be very small. The absolute exit velocity c_1 is a resultant from w_2 and u, and is nearly as large as u. On the one hand, there occur eddy current losses in the wheel; on the other hand, the absolute velocity of the steam is increased, and therefore a transposition of work to the steam takes place throughout. Only after the quantity of steam has reached a certain value can work be performed in the high pressure wheels, and gives a surplus which finally will overcome the resistances of running light. The point O_1, therefore, does not correspond to the indicated zero power, but is, in fact, already negative.

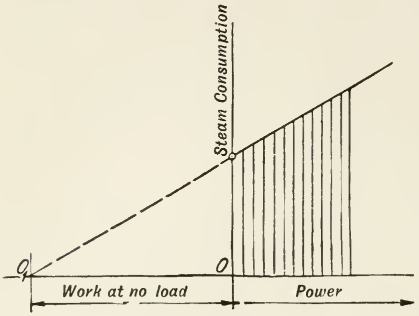

Fig. 233.

The varying of the steam consumption in the region of O power was determined for a turbine *in which all wheels aided in producing work while running light.* In

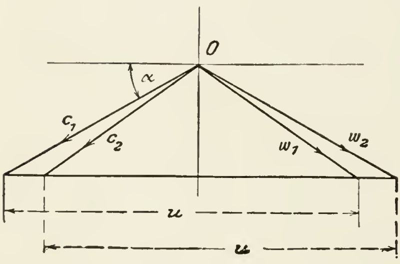

Fig. 234.

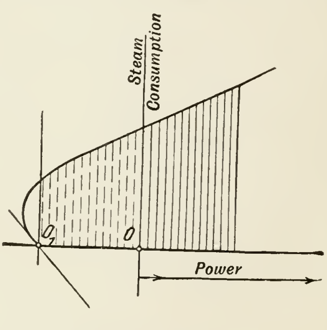

Fig. 235.

order to give to the turbine, according to the assumption, a negligibly small indicated power, there must exist in the velocity diagram, Fig. 234, the almost coinciding velocities c_1, c_2 and w_1, w_2. In our fundamental equation

$$-v\frac{dp}{dz} = \frac{\delta\mu}{\epsilon g}(2c_1\cos\alpha - u) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

we may introduce *v as a constant*, because its change, on account of small pressure differences, would remain small. Equation 1 applies also to *the performance of non-compressible liquids driving a many-stage turbine.*

tain *limit velocity.* The steam load will initially increase while the efficiency will likewise increase until the peripheral velocity has reached the value at which axial exit from the rotating wheels occurs. At the same time the work of friction of the drums and the labyrinth pistons and that of the bearings increases, which limits the otherwise attainable velocity. Let us assume that the steam friction mentioned is normally 5% of the rated power and increases as the third power of the number of revolutions. Then the full power of the turbine will be braked by $\sqrt[3]{\frac{100}{5}} = 2.7$ times the increased number of revolutions. The bearing friction of course also absorbs more work, still only in the simple ratio to the velocity, and need not be considered when the revolutions are increased 3 to 4 times. If we assume, in order to get another limit, that this special work of friction is negligible, then the velocity diagram will again take the approximate form shown in Fig. 234, that is, w_1 and w_2 are nearly equally large, and in the same direction, likewise c_1 and c_2. The steam entrance to the rotating blade channel (and the guide blade channel) occurs with the extraordinary deviation shown in Fig. 236, so that the eddy current resistances nearly utilize the entire drop, and transmits to the wheel only slight work. Without

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With $Gv = fc_1$ we get

$$\frac{dp}{dz} = -G\phi'(z) + \psi'(z),$$

and ϕ', ψ' are always positive functions. By immediate integration we have

$$p = -G\phi(z) + \psi(z) + C,$$

and for the condition that $p = p_1$ for $z = 0$, and $p = p_2$ for $z = z_0$, it follows that

$$p_2 - p_1 = -K_1 G + K_2 \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (2)$$

in which K_1 and K_2 are positive constants.

The power carried to the machine can be placed proportional to the pressure difference $p_1 - p_2$, so that

$$N_i = K_3 G(p_1 - p_2).$$

If we place $(p_1 - p_2)$ from equation 2 in the above, we obtain

$$-\frac{N_i}{K_3 G} = -K_1 G + K_2$$

or

$$N_i = K_1 K_3 G^2 - K_2 K_3 G;$$

that is, at the point $N_i = O$, G does not disappear, but is represented by a parabola passing through the initial point as a sloping tangent, Fig. 235.

experiments this new c_1 cannot be approximated; if we take it equally large with that at normal conditions, then the peripheral velocity values would become 5 and 6 times the normal ones. As the stresses of the rotating parts increase as the square of the

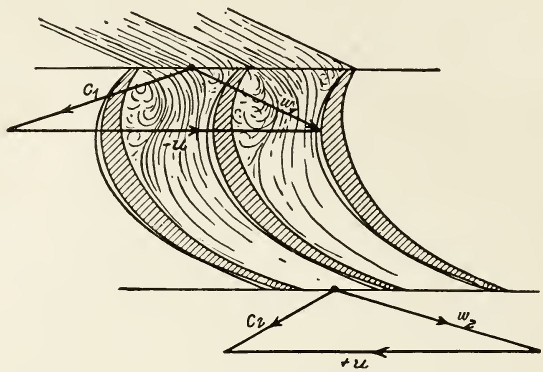

Fig. 236.

number of revolutions, *there exist very slight prospects* (even if we take the first case into consideration) *of building the many-stage turbine so that it will withstand a "running away" without serious danger.*

APPENDIX.

THE FUTURE OF THE HEAT ENGINE.

80. PERPETUAL MOTION OF THE FIRST TYPE.

A MACHINE that creates work, that is, produces work from nothing or delivers more work than is brought to it in any (for example, latent) form, is impossible. The impossibility of this so-called perpetual motion (called the first type to distinguish from the second type discussed below) was divined by science more than one hundred years ago, but received its final confirmation from the *principle of the conservation of energy* enunciated by *Meyer*, *Joule* and *Helmholtz*. This principle to-day furnishes the unshaken foundation of all natural science, hence also of machine construction.

81. PERPETUAL MOTION OF THE SECOND TYPE, AND THE SECOND FUNDAMENTAL LAW OF THERMODYNAMICS.

As often as heat disappears, we must, from the laws of energy, find its equivalent quantity of energy in another form, for instance, as mechanical work. But this transformation is not unlimited and cannot be carried through arbitrarily. It is, above all, dependent on the presence of a temperature drop and a deliverance of heat at the existing lower temperatures. The heat contents of the sea, the atmosphere and the entire earth, represent an immense reservoir of energy, and numerous inventors have undertaken the problem of transforming this heat, always available and free to all, into work without the assistance of a lower temperature; and their endeavors are of course practically impossible. A machine that undertakes to transform heat into work by cooling a given heat reservoir, *without changing anything else in the surroundings*, is called, according to *Ostwald*, *perpetual motion of the second type.*

If the heat reservoir above described is, strictly speaking, not infinitely large, then the mechanical work will be retransformed into heat by the constantly occurring friction and other sources of loss and the way the above law was stated by *Ostwald* is correct in all cases.

*The second fundamental law of thermodynamics says that this perpetual motion of the second type is impossible even with ideal machines, that is, even with frictionless non-conducting engines.**

The proof for this law is not an absolute one, it is cumulative; that is, the results hitherto derived from it have without exception been confirmed by experience. Nevertheless, from a purely logical point of view, it is not at all improbable but that a new discovery may prove an exception; and strictly speaking, we can only call this law an hypothesis. The entirely false conception of how a law of nature must be "proved" has led many inventors to believe, in direct contradiction to the second fundamental law of Thermodynamics, that their invention is an exception, thereby destroying the truth of this law. Against such views we must emphasize that the principle of the conservation of energy has only been "proved inductively;" that is, we can only affirm that which has been found correct in all cases hitherto observed. The second fundamental law was originally enunciated by *Clausius* in a somewhat different form, and only for pure transformation of heat. Later, *Gibbs*, *Helmholtz* and *van't Hoff*, from this and other conclusions derived from the phenomena of chemistry, the galvanic current, and the theory of solutions, have achieved brilliant scientific results which have been confirmed by experience in numberless cases. This second law is therefore found to represent a controlling principle of all natural phenomena, and has, scientifically speaking, the same degree of certainty as the law of conservation of energy.

Of course, we have at all times regions, which on account of their recent discovery cannot be fully investigated. For instance, consider the phenomena of radiation, for which many claim that even the principle of the conservation of energy does not hold good. In these entirely new discoveries, and in these alone the second law of thermodynamics has not yet been verified, because

* The addition that also ideal machines cannot realize perpetual motion is necessary, as we can see later on.

they are so recent and on account of the difficulty of investigation. Still, careful investigators do not doubt that here also the law will apply. Over the entire remaining region of scientific research, the law has been confirmed by innumerable experiments. Therefore, we may earnestly urge the inventor, not to spend his means in carrying out any idea that would contradict the second law of thermodynamics.

Still, on the other hand, it must be emphasized, that the transformation of the heat of the surroundings into mechanical work is not in itself impossible, but, what is of utmost importance, that it still occurs *only as an accompanying phenomenon* to other and in general very much larger transformations of energy. There are galvanic chains that develop more electrical energy than the "heat-manifestations" corresponding to the chemical union of the elements ; therefore, the excess of heat is taken from the surroundings. So long as we have a supply for such galvanic chains of suitable material at our disposal, we have the heat of the surroundings transformed into work "gratuitously." But should they have to be made artificially by chemical means from other elements, we must then transform as much mechanical work into heat, as will more than balance the former gain. These processes are practically useless because the value of the transformed heat per unit weight of the material consumed is generally an exceedingly small quantity.

There also exist purely thermodynamic processes by which the heat of the surroundings may be transformed into work. The simplest and most obvious example would be a compressed gas which at atmospheric temperature expands isothermally. In this the work performed is the exact equivalent of the transformed quantity of heat. If we find a supply of compressed gases in nature, it is possible to produce such work. As soon as the gas must be compressed artificially every economy of this process would cease. We will speak below of other chemical thermodynamic processes of a similar nature, and prove that they also have no importance practically.

Finally, we repeat, that there would be perpetual motion of the second type when there is transformation of heat from any single source unaccompanied by a temperature drop or by any other change whatsoever ; and in this sense only is this type to be considered an impossibility.

82. THE CARNOT CYCLE.

Consider a body of any kind which in order to make this discussion general may be assumed to consist of a mass of materials interacting chemically. Let this body undergo the following process: an adiabatic compression from the temperature t_2 to the temperature t_1; an isothermal expansion at the temperature t_1 with the addition of the quantity of heat Q_1; an adiabatic expansion to the temperature t_2; and finally, an isothermal compression at the temperature t_2, by taking away the heat Q_2 until the *initial condition* has been reached. Such a process is called the *Carnot Cycle. These changes of condition must be capable of occurring in the reverse order;* that is, the uniform temperature of each of the heat reservoirs which furnishes Q_1 and receives Q_2 respectively, should vary only an infinitesimal amount from the two uniform temperatures of the working body; the kinetic energy of this body, that is, the velocity, with which the process is performed, must be negligibly small. Finally, we shall assume that the "engine" in which our body works is frictionless. During one rotation an external work is performed expressed in heat units as $A L$, and according to the principle of the conservation of energy,

$$A L = Q_1 - Q_2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

A second body may go through this same process and between the same temperature limits in the reverse order, and its weight is to be such that the *performed* work is again equal to $A L$; while the cooler heat reservoir in this case is receiving the heat Q_2', the warmer reservoir is delivering the heat Q_1'. Therefore we again have

$$A L = Q_1' - Q_2',$$

so that

$$Q_1 - Q_2 = Q_1' - Q_2' \text{ or } Q_1' - Q_1 = Q_2' - Q_2.$$

As the external work gained during the first process is exactly consumed during the second process, and as both bodies after each have gone through one revolution are in the initial condition, the action of this process consists therein that a certain quantity of heat

has been taken from one reservoir and brought over to the other. If $Q_1' > Q_1$, then the difference $Q_1' - Q_1$ would be taken from the colder reservoir and carried over to the warmer. This surplus, in a third "right-handed" process, should perform work, and by repeating this process we could gain continuous work at the cost of the colder reservoir, without having any other changes occur. But this is perpetual motion of the second type, and therefore impossible. This is demonstrated, if we assume $Q_1' < Q_1$, in which it is only necessary to exchange processes 1 and 2 so that there remains only the possibility that *

$$Q_1 = Q_1', \text{ therefore also } Q_2 = Q_2'.$$

By division, it follows

$$\frac{Q_1}{Q_2} = \frac{Q_1'}{Q_2'}$$

that is, this ratio is independent of the nature of the bodies that are used, if only the same temperature limits, t_1 and t_2, are kept. As the pressure, the volume, the aggregate condition and the chemical composition of the body were not considered, this ratio can only depend on the temperatures; that is,

$$\frac{Q_1}{Q_2} = f(t_1, t_2), \quad \ldots \ldots \quad (2)$$

in which f is an unknown, but for all types of bodies constant, function. Its form we determine, according to *Poincaré*, by assuming *Carnot* cycles between the temperatures t_0, t_1, t_2 and between the same adiabatics, the cycle being arranged as follows: 1, between t_1 and t_2 with the quantities of head Q_1 and Q_2; 2, between t_1 and t_0 with Q_1 and Q_0; 3, between t_2 and t_0 with Q_2 and Q_0; so that the three equations must exist:

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* We might of course immediately say that frictionless machines do not exist, and that the proof has not been rigidly enough carried out; but on the one hand we are building steam engines whose work of friction, including the work of the air pump, s only 5% of normal load; on the other hand, the supposition of an ideal engine for this proof is not too cumbersome, and the proof is therefore more clearly expressed.

$$\frac{Q_1}{Q_2} = f\,(t_1\,t_2), \frac{Q_1}{Q_0} = f\,(t_1\,t_0), \frac{Q_2}{Q_0} = f\,(t_2\,t_0) \quad . \quad . \quad . \quad (3)$$

If we find the ratio Q_1 and Q_2 from the second and third equations and place it in the first equation, we have

$$\frac{f\,(t_1\,t_0)}{f\,(t_2\,t_0)} = f\,(t_1\,t_2) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

This identity can only exist, according to *Poincaré*, if $f\,(t_1\,t_2)$ has the form

$$f\,(t_1\,t_2) = \frac{\phi\,(t_1)}{\phi\,(t_2)} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

in which function ϕ is now unknown. For its determination it is sufficient to find by experiment the ratio $\frac{Q_1}{Q_2}$, for any single body. As such a body, an "ideal" gas can be taken whose equation of condition is,

$$pv = R\,T,$$

in which T is the absolute temperature and R a constant; for such a body the specific heat c_p for constant pressure and c_v for constant volume remain unchanged, and form the ratio

$$k = \frac{c_p}{c_v}.$$

In going over the separate steps of the cycle individually, we find by a very simple calculation,

$$\frac{Q_1}{Q_2} = \frac{T_1}{T_2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

in which $T_1 = 459 + t_1$ ($273 + t_1$ in French units) and $T_2 = 459 + t_2$ ($273 + t_2$) are the upper and lower "absolute" temperatures of the *Carnot* isothermals. The *Carnot* temperature function $\phi\,(t) = \phi\,(T - 459)$ is reduced therefore to the absolute temperature,

$$\phi(t) = T \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

if we let the still arbitrary constant factor = 1.

The quantity of heat transformed into "useful work" is now merely the difference between the heat quantities Q_1 and Q_2; that is,

$$A L = Q_1 - Q_2.$$

The "efficiency" η is the ratio of the useful, gained, energy $A L$ to the total heat Q_1, supplied; as Q_2 assumed the temperature of the surroundings, and therefore became practically useless, we have,

$$\eta = \frac{A L}{Q_1} = \frac{Q_1 - Q_2}{Q_1}.$$

But, according to equation 6,

$$Q_2 = Q_1 \frac{T_2}{T_1},$$

it then follows,

$$\eta = \frac{T_1 - T_2}{T_1} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

from which we have the law:

The thermodynamic efficiency of a Carnot cycle depends purely on the temperature of the isothermals between which the process occurs and is independent of the nature of the working bodies. The utilization of heat is all the better, the higher the temperature of the heat supplied and the lower the temperature at which it is rejected.

83. CYCLE WITH ADDITION AND SUBTRACTION OF HEAT AT ARBITRARILY CHOSEN TEMPERATURES.

Consider any body subjected to any physical or chemical change of condition in which *only reversible processes* occur, and in which the path of the pressure and volume changes are represented by the so-called *pv*-diagram, Fig. 237. Divide the *pv*-plane by a series of adiabatics into infinitely small strips, and let us call

the quantity of heat which is added or subtracted during the actual change of condition between the adiabatics, and which is represented in the figure, by dQ_1, dQ_1', dQ_1'', . . . dQ_2, dQ_2', dQ_2''. . . .

The heats dQ_1, dQ_2 can be taken as a Carnot cycle described by an auxiliary body between the adiabatics a and a' and the infinitely short isothermals J_1 J_2 corresponding to T_1 and T_2, that is, the average temperatures of the actual changes of condition. The heat added and subtracted at J_1 and J_2 respectively, differ from those of dQ_1 and dQ_2 only by an infinitesimal of a higher order, and we have, according to equation 6,

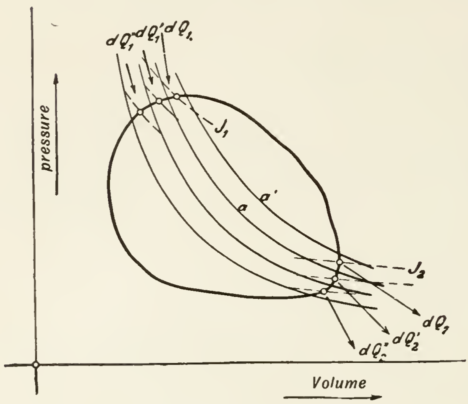

Fig. 237.

$$\frac{dQ_1}{dQ_2} = \frac{T_1}{T_2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (9)$$

The useful work dL, corresponding to the elementary process and represented by the elementary area of the pv-diagram, is in heat units $A\,dL = dQ_1 - dQ_2$; the thermodynamic efficiency is as above,

$$\eta = \frac{T_1 - T_2}{T_1}.$$

The above given law holds good for every elementary process, and we may, therefore, taking everything together, state the law :

The utilization of heat in any arbitrarily chosen cycle with only reversible changes is the better, the higher the temperature at which heat is added, and the lower the temperature it is subtracted.

This law, that appears general and to hold good without exception, is greatly *limited* by the application of a so-called *regenerator*. Let there exist in a cycle a "right-handed" and continuous change of condition at which heat is imparted, and later on a "left-handed" change at which it is taken away, of such a char-

acter that for element after element the temperatures and the exchanged quantities of heat are equally large, while the pressures can remain different. It is now in general possible, by an ideally working *heat exchange apparatus* (according to the principle of counter currents), to add theoretically without loss the quantity of heat given up on the left-handed path to the working body, experiencing a right-handed change of condition, the heats being thus made to circulate and not freshly supplied each time. It is therefore, in controvention to the above generalized law of *Carnot*, a matter of entire indifference whether, for the efficiency of this special process, the quantity of heat in question is added or subtracted at high or low temperatures. A practically useful application of this theoretically very promising idea has not yet been accomplished

84. THE INTEGRAL OF CLAUSIUS.

If we write equation 9 in the form

$$\frac{dQ_1}{T_1} = \frac{dQ_2}{T_2} \quad \text{or} \quad \frac{dQ_1}{T_1} - \frac{dQ_2}{T_2} = 0$$

and combine it with the similarly given values

$$\frac{dQ_1'}{T_1'} - \frac{dQ_2'}{T_2'} = 0$$

$$\frac{dQ_1''}{T_1''} - \frac{dQ_2''}{T_2''} = 0, \text{ etc.,}$$

we get by summation

$$\sum \frac{dQ_1}{T_1} - \sum \frac{dQ_2}{T_2} = 0.$$

But if we consider the quantity of heat dQ_2 taken away, algebraically, that is, insert it as a negative value (in which dQ_2 represents the absolute value), we may write simply,

$$\sum \frac{dQ}{T} = 0$$

as the amount that is distributed throughout all the heat elements if we substitute for the summation the integral sign, we obtain for a cycle with only reversible processes the law of *Clausius*,

$$\left(\int\right)\frac{dQ}{T}=0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (10)$$

in which the parenthesis indicates the integration is extended over the whole closed cycle.

85. ENTROPY.

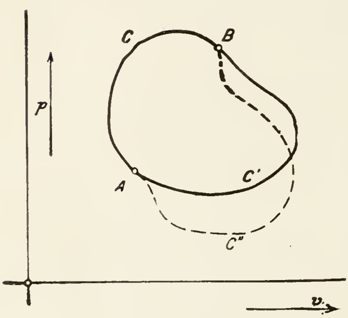

Fig. 238.

Consider 1 kilogram (or pound) of our material to be taken by a reversible process from condition A, Fig. 238, which we shall define as "normal," to condition B, according to curve C; then return to A by the path C', so that a cycle ensues. We now resolve the integral of *Clausius* equation 10, into the part from A through C to B, and the part from B through C' to A, and write

$$\int_A^A\frac{dQ}{T}=\underset{\text{through } C}{\int_A^B\frac{dQ}{T}}+\underset{\text{through } C'}{\int_B^A\frac{dQ}{T}}=0 \quad . \quad . \quad . \quad . \quad (11)$$

If the change occurs from A to B through C', then all elementary quantities of heat involved change their signs, and we have

$$\underset{\text{through } C'}{\int_A^B\frac{dQ}{T}}=-\underset{\text{through } C'}{\int_B^A\frac{dQ}{T}},$$

then equation 11 gives

$$\underset{\text{through } C}{\int_A^B\frac{dQ}{T}}=\underset{\text{through } C'}{\int_A^B\frac{dQ}{T}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (12)$$

The integral of the element $\frac{dQ}{T}$ is therefore independent of the method in which a body is taken from the initial condition and brought to a given final condition, provided the path is everywhere reversible. We call this integral the *increase of entropy of the body between the conditions A* and *B*, and write

$$\int_A^B \frac{dQ}{T} = S - S_0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (13)$$

The value of the entropy at A remains undetermined, and therefore S is only determinable up to an arbitrary constant that must be added. If we call the condition at A the "zero condition," or if we place $S_0 = 0$, then the entropy will be a certain number corresponding to every condition of the body, and can be calculated from the beginning, so long as the condition can be reached by only reversible changes.*

From the definition of entropy it follows

$$dS = \frac{dQ}{T} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (14)$$

and

$$dQ = T\,dS \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (14a)$$

This important equation says *that the (reversible) quantity of heat dQ that is added, is obtained as the product of the absolute temperature and the elementary increase of entropy* during the observed infinitely small change of condition. This law makes it possible to represent the imparted heat graphically as an area; for instance, we draw a coördinate system with S as the abscissas and T as the ordinates. As the entropy S and the temperature are determined by

* It is important to draw attention to the fact that there was no assumption made as to the nature of the working body, that the above definition of entropy also holds good for masses acting chemically on each other, if only their condition is determinable from certain data that correspond to a condition when the chemical forces are in equilibrium. The existence of external equilibrium is not necessary because in masses that are in motion an infinitely small element may be taken as in relative balance with its center of gravity. With mixture of gases we can also neglect the condition of equilibrium of chemical forces, because the entropy of one constituent is not influenced by the presence of the other constituent.

these "parameters of condition" p and v, each point of the pv-plane corresponds to a point of the TS-plane, and there can be constructed a curve of condition (for instance, expansion line) from the first to the second. In this manner, the "entropy diagram," Fig. 239, is constructed. In this rectangle $B' B'' B_1'' B_1' = TdS = dQ$, and the area $B_1 B C C_1$ represents in heat units the entire heat taken up during the change of condition from B to C. If the change occurs in the direction of C to B, the contents of the surface must be taken as negative; that is, the heat was not added to, but *taken away*.

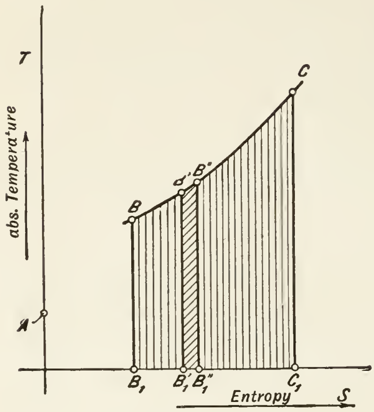

Fig. 239.

From relation 12 it further follows, that $\frac{dQ}{T}$ is a complete differential of both chosen "parameters" for the determination of the condition; as, for instance, of p, v or v, T, etc., so that the complete integration of S is always possible.

86. ENTROPY DIAGRAM FOR STEAM.

For steam, the calculation of the entropy is made by allowing 0° C. (32° F.) and the volume of water, which is considered constant, to represent the "normal condition," the entropy being referred to unit weight of water; we here choose as reversible change of condition the adiabatic compression of water to the desired pressure, and then the addition of heat at constant pressure.

In the fluid condition, if we neglect the unnoticeable increase of temperature and performance of work during the compression of the water, the equation $dQ = c\,dT$ holds good until boiling point is reached at pressure p or temperature T, in which c is the specific heat of water and is independent of the pressure. Therefore, the first part of the entropy is

$$s' - s_0 = \int_0^T \frac{c\,dT}{T} = \tau,$$

while the evaporation at constant pressure, therefore also constant temperature, is

$$dQ = r\,dx \text{ and } s_x - s' = \int_0^x \frac{r\,dx}{T} = \frac{r\,x}{T}$$

so that for saturated steam at "condition T and x," we have

$$s_x - s_0 = \tau + x\frac{r}{T}.$$

At the "limit curve," or curve of constant steam weight,

$$s'' - s_0 = \tau + \frac{r}{T}.$$

In the superheated territory we have at constant pressure p,

$$dQ = c_p\,dT$$

and

$$s - s'' = \int_T^{T'} \frac{c_p\,dT}{T} = c_p \text{ nat. log}\left(\frac{T'}{T}\right),$$

in which T' is the temperature of the superheated steam and c_p is a constant $= 0.48$.

These values are graphically represented for the practically important territory of change of condition in chart 1. The curves $p =$ constant and $x =$ constant have been calculated for a large number of intermediate values. The isothermals $T =$ constant are naturally represented by horizontal lines. For the adiabatics, $dQ = 0$, therefore $s =$ constant; that is, they are vertical lines. The curves $v =$ constant have also been drawn, so that for two determining parts being given, for instance p and v, the third can instantly be found; that is, x or T. Finally, the curves $\lambda =$ constant are drawn, through which, from what has already been said, the calculations are simplified.

87. INCOMPLETE CYCLE WITH REVERSIBLE AND NON-REVERSIBLE CHANGES OF CONDITION.

If a body described reversible changes of condition, its temperature, as explained above, must equal, within an infinitely small difference, the temperature of the heat reservoir from which it receives heat. If we also assume only reversible occurrences in the reservoir, then for each element of change of condition, the change of entropy $dS = \frac{dQ}{T}$ of the working body is equally large, but opposite to that in the reservoir, because T and dQ are also equal for both, but opposite. The change of entropy of both bodies taken together is nothing, and also applies to finite changes of condition. We have therefore the law:

In a purely reversible occurrence, the sum of the entropies of all bodies that in any way participate in the occurrence, remains unchanged.

Conversely, if non-reversible changes of condition occur, the law receives the following supplement, first given by *Gibbs* and *Planck:*

The sum of the entropies of all bodies participating in any one occurrence is, at the end of the change of condition, larger than at the beginning; only in the limit case, in which all parts undergo reversible changes, does the sum of the entropies remain unchanged.

This proof for a closed cycle, that is, a cycle-process with non-reversible thermodynamic transformations, has been referred by *Clausius* to his fundamental principle that heat cannot by itself go from a cold to a warmer body.

For unenclosed (incomplete) cycles of any type, the proof may be had by considering two *Carnot* cycles as explained in Article 82. Let the right-handed one represent a non-reversible change of condition, the left-handed, a reversible, and so that the work performed by the former is just used up by the latter. As non-reversible we define each occurrence whose results, according to *Planck*, cannot be completely neutralized by any method at our disposal; that is, cannot be so neutralized without a change remaining in any other body. That the occurrence cannot be repeated in the opposite direction, is therefore not sufficient; it must above all be impossible to reëstablish the initial condition without other changes.

Under these assumptions the heat $Q_1 - Q_1'$ taken from the warmer reservoir during the cycle process can only be positive, for if it were equal to nothing, we would have finally exactly the same condition of all participating bodies as at the beginning; we would have, therefore, neutralized the non-reversible change of the first process, which contradicts our assumption. But if $Q_1 - Q_1' < O$, then the quantity of heat $Q_1' - Q_1$ would have been taken from the colder reservoir into the warmer without the performance of work; and if this could be performed in a suitable machine, we would have perpetual motion of the second type, which is impossible. There remains, therefore, only

$$Q_1 - Q_1' > 0 \text{ and } Q_2 - Q_2' > 0.$$

On account of the equality of the work, $Q_1 - Q_2 = Q_1' - Q_2'$; that is $Q_1 = Q_1' + \Delta$; $Q_2 = Q_2' + \Delta$, in which Δ represents a positive value.

For the reversible cycle,

$$\frac{Q_1'}{T_1} - \frac{Q_2'}{T_2} = 0,$$

we have, as $T_1 > T_2$,

$$\frac{Q_1}{T_1} - \frac{Q_2}{T_2} = \Delta\left(\frac{1}{T_1} - \frac{1}{T_2}\right) < 0.$$

By the same reasonings as were made in Article 5, we get for any arbitrarily chosen cycle process, the formula

$$\left(\int\right) \frac{dQ}{T} < 0,$$

in which, as we can see from the derivation, T is the temperature of the reservoir while with non-reversible occurrence the equality of the temperature of the body and the reservoir does not form a condition. Let, in the former Figure 238, an unclosed process (incomplete) take place between the conditions A and B with non-reversible occurrences in the working bodies, but only reversible changes in the heat reservoir, which follows the path C. We make this process a closed (complete) one by adding the reversible change of condition in all parts from B to A over the path C'. The integral of *Clausius* gives

$$\underset{\text{through } C}{\int_A^B \frac{dQ}{T}} + \underset{\text{through } C'}{\int_B^A \frac{dQ}{T}} = \underset{\text{through } C}{\int_A^B \frac{dQ}{T}} - \underset{\text{through } C'}{\int_A^B \frac{dQ}{T}} < 0 \quad . \quad . \quad (\alpha)$$

Let S_A and S_B be the entropy of the working bodies at A and B, likewise S_A', S_B' are the initial and final values of the entropy of the reservoir which during the given change of condition from A to B is connected with the bodies (through C). Then

$$S_B - S_A = \underset{\text{through } C'}{\int_A^B \frac{dQ}{T}}; \quad S_B' - S_A' = - \underset{\text{through } C}{\int_A^B \frac{dQ}{T}} \quad . \quad . \quad (\beta)$$

in which the negative sign of the last term must be inserted because dQ is added to the bodies, therefore $-dQ$ denotes algebraically the quantity of heat given to the reservoir. We have, therefore, by inserting the values of equation β in equation α,

$$-(S_B' - S_A') - (S_B - S_A) < 0 \text{ or } (S_B + S_B') - (S_A + S_A') > 0,$$

that is, the sum of the entropies of all the participating bodies in the unenclosed change of condition from A to B is greater at the end of the occurrence than at the beginning, which was to be proved.*

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* In a corresponding proof of *Planck* there is (on page 86 of his Thermodynamics) an obscurity which we shall discuss for the benefit of those acquainted with his work. The question is discussed, what results occur if we assume that the entropy of a gas could be decreased without effecting changes in other bodies. Here we can consider as opposed to the incomplete demonstration of *Planck*, three possibilities. First, the temperature could remain equal, then we would have by the isothermal addition of heat from the surroundings and the expansion of the gas to the original entropy, a perpetual motion of the second type; second, the temperature could be higher, and by adiabatic expansion to the former temperature, we would obtain work from nothing; third, the temperature could be lower, and work must be expended to adiabatically compress to the initial temperature, after which the gas would be permitted to expand without work to the volume of greater entropy, and by periodic repetition would constantly annihilate work. Then *Planck* is not clear in his references to chemical occurrences, and because of this, some might have already reached the strong proof here given. This supplement does not in any way make superfluous the study of the classical work of *Planck*.

88. THE ECONOMY OF THE HEAT ENGINE.

The performance of work by heat motors as known to-day depends above all on the extraction of energy by chemical union (combustion), and therefore we can apply the law of *Planck* in its general sense for the determination of their economy. To our regret, the physical and chemical constants of our working materials are too little known, and we must be satisfied with a few general statements. Especially for the *loss of work by non-reversible occurrences* of processes common to gas motors, the following may be said:

We shall imagine all generation of heat in the given work occurrence to be dependent on chemical processes, and take as the final results (for purposes of retransformation) at atmospheric conditions an abstraction of heat, Q_0 from the surroundings, which takes place at temperature T_0. The useful work is utilized to raise a weight whose entropy does not change. Let the entropy of the working body be S before the transformation, and S' after the same, the increase being therefore $S'-S$. The entropy of the lower heat reservoir experiences an increase $\frac{Q_0}{T_0}$. In all, the entropy has increased the value

$$N = S' - S + \frac{Q_0}{T_0} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

The total energy of the working body is

$$L = U - U' - Q_0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

provided the volumes at the beginning and end of the occurrence are equal. If they are not equal, we have in place of U and U' the so-called potential at constant pressure; that is, the value $U' + pv$, which with vapors is the "heat-contents." The difference of energy $U - U'$ is the "heat value" of the working body per unit weight at constant volume (in general, that at constant pressure). Substituting Q_0 from equation 1 in equation 2, we have:

$$L = U - U' + T_0 (S' - S) - NT_0 \quad . \quad . \quad . \quad . \quad (3)$$

If we have only reversible occurrences, then

$$N = 0,$$

and the useful work is

$$L_0 = U - U' + T_0 (S' - S) \quad . \quad . \quad . \quad . \quad . \quad (4)$$

which actually is

$$L = L_0 - N\,T_0 \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

and which can be expressed in the following law:

With non-reversible occurrences of any (also chemical) type, the useful work of the described processes experiences a decrease equal to the product of the increase of entropy occurring to the participating bodies during the process and the temperature of the reservoir which abstracts the heat (that is, in its widest sense, the surroundings).

The energies in the initial and final conditions, U and U', are fixed by the return of the working bodies; their difference is therefore, as we wish to repeat, to be taken as given, likewise the entropies S and S', therefore also the *maximum useful work*, L_0; from which follows the important law:

With given initial condition of the working body or body-system, and likewise given condition (that is, pressure and temperature) of the surroundings, hence the final condition, we get for the described process, the maximum useful work if each non-reversible change of condition is avoided. The value of this useful work is independent of the type of purely reversible processes by which the transformation takes place.

From equation 4, which gives the useful work with reversible changes of condition of all parts, the following theoretical possibilities may result:

1. The entropy of the working body remains unchanged, that is,

$$S' = S \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

then

$$L = U - U' \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6a)$$

Hence: *the maximum work received is identical with the "heat value."*

2. The entropy is smaller at the final condition than at the initial,

$$S' < S \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

then,

$$L = U - U' - T_0 (S - S') \quad . \quad . \quad . \quad . \quad . \quad (7a)$$

or, *the maximum work received is smaller than the heat value.*

3. The entropy at the final condition is larger than at the initial condition,

$$S' > S \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

then,

$$L = U - U' + T_0 (S' - S) \quad . \quad . \quad . \quad . \quad . \quad (8a)$$

or, *the maximum work received is larger than the heat value.*

With $N = 0$, the heat to be given up to the surroundings is

$$Q_0 = T_0 (S - S'),$$

therefore, $= 0$ in the first case, and positive in the second case. But if $S' > S$ as in the third case, then Q_0 is negative; that is, *heat is taken from the surroundings by the bodies and (immediately) transformed into work.*

There ensues, therefore, the exceedingly important question whether there are bodies and processes that permit the realization of the theoretical possibilities. The ordinary four events shown by an indicator diagram would not be suitable. On the other hand, we might reach a solution by using fluid combustibles having certain hypothetical attributes, for instance, according to a process of the following type:

We would abstract from a certain combustion motor the quantity of heat Q_0 and use it for evaporating the suitably chosen combustible, which boils at the temperature T_0, and which must give a heat of vaporization such that the quantities concerned during one performance of the process would take up just the heat Q_0, or somewhat more. The combustible gases are cooled at atmospheric pressure and exhausted into the atmosphere, and the entire heat value of the combustible, or somewhat more, has been transformed into work.

This theoretical possibility of an efficiency that equals 1 or even larger than 1 has absolutely no practical significance. Not considering the question whether combustibles could be had of such high

heat of vaporization, it is not practical simply because combustibles that would boil at atmospheric temperature would exist with its vapor, that is, must be considered as a "natural gas." If H is the heat value per unit weight of the liquid material, and Q_0 is the heat of vaporization, then the heat value of the gaseous material would be $H' = H + Q_0$ per unit weight. H is transformed into work, and if H is taken as unity, then the efficiency $= 1$. But if the similar value H' is taken as a base, then the work will be, inversely, $H' - Q_0$, and the efficiency would be less than 1. In the latter case we can imagine the combustible to be in gaseous form, and the motor works as an ordinary gas motor. The heat Q_0 must finally be taken away, but we receive exactly as much work as before. This apparently favorable utilization of the given-up heat, Q_0, is actually of no use, and these remarks hold good in general, because the heat that is available for vaporization at the lowest temperature level existing is practically worthless.

Entirely different are the relations when a working body leaves the motor at temperatures which lie above that of the surroundings. This given-up heat has a transformation value on account of its temperature "head," and should, for economical reasons, be utilized.

89. PRACTICAL CRITERION OF HEAT UTILIZATION.

With the already mentioned ignorance of the entropy change of our combustible, it is absolutely necessary to search for simple points of view that would permit of a decision to be reached as to the degree of the utilization of energy.

For the technically important combustion process, it is approximately allowable, as can be proved, to look upon the occurrence *as if the liberated chemical energy of the combustible mixture had heat added to it from the outside.* Instead of allowing it to exhaust into the atmosphere we can further imagine, for instance, with the ordinary four events shown by the indicator diagram, that the products of combustion are retained in the cylinder, and are cooled at constant volume to atmospheric condition; whereupon through further heat abstraction during the return stroke of the piston is formed the line of exhaust, and through imparting heat, the suction line, of an indicator diagram, and the performance of the four events may

be started all over again. From these considerations the combustion motor is changed to a certain degree into an enclosed hot-air motor, and we can apply to the occurrences and energy transformations that are now only looked upon as thermic, the same laws as were derived before.

If we take what has been said, we find that to reach the highest energy utilization of heat engines of any type we can give the following leading fundamental laws:

1. *Decreasing the passive resistances, such as friction, throttling, etc., avoiding heat losses of every type.*

2. *Adding the heat or conducting the combustion at the highest possible temperature, abstracting the heat at the lowest practical temperature, avoiding to greatest possible extent non-reversible changes of condition.*

3. *Utilization of the given-off heat, and the application of regenerators, where the kind of combustible and the work processes permit, so long as we can construct practical and efficient regenerators.*

Of the known suggestions for improving the thermic working processes, according to the above points of view, the following deserve a short discussion.

90. LATEST SUGGESTIONS.

Avoiding the addition of heat at low temperatures to the feedwater, by complicating the classical *Carnot* process of the steam engine so that the exhaust steam is only allowed to condense to a certain degree of moisture, and with adiabatic compression to the boiler pressure, by means of a compressor, the mixture is transformed into water at the steam temperature of the boiler. This suggestion, originating in the classical thermodynamics and whose application even *Thurston* * lately said was desirable, has in itself much that is enticing. If we, for instance, investigate a steam engine working with "dry" saturated steam between 12 kg. per sq. cm. (170.7 pounds per square inch) and 0.2 kg. per sq. cm. (2.8 pounds per square inch) with reference to the attainable gain, the application of an air pump compressor promises a saving of heat of about 10% ; with 0.1 kg. per sq. cm. (1.4 pounds per square

* Transactions Amer. Soc. Mech. Eng., 1901.

inch) back pressure the gain increases to 15%. Still, we must designate all hopes for this process as utopian, because the necessary compressor must be of a size nearly equal to the low-pressure cylinder of our steam engine, and its work of running light, in combination with the other resistances, would consume the entire gain. There is also the difficulty, that with compression in an ordinary piston engine, the steam and water separate, and a very incomplete temperature exchange results.

A heat regenerator has lately been suggested by the German patent No. 129 182 of *Lewicki v. Knorring, Nadrowski* and *Imle,** in order to utilize the exhaust of steam turbines. The process consists in taking the still strongly superheated exhaust of a turbine and leading it into a heating chamber that according to the patent record is placed in the water or steam space of a boiler, to there evaporate water or superheat the steam. *Lewicki, Jr.*, states † that in his experiments with the application of superheated live steam at 460° to 500° C. (860° to 932° F.) the steam exhausted at 309° to 343° C. (588° to 649° F.). It is obvious that the regaining of the excess of heat here exhausted represents a saving. *Lewicki* gives the following values:

		One-half Peripheral Admission.	Full Peripheral Admission.
Steam temperature	C.°	**460**	**500**
Steam temperature	*F.°*	860	932
Steam pressure at turbine entrance	kg. per sq. cm.	**7.0**	**7.0**
Steam pressure at turbine entrance	*lb. per sq. in.*	99.6	99.6
Steam back pressure	kg. per sq. cm.	**1.0**	**1.0**
Steam back pressure	*lb. per sq. in.*	14.2	14.2
Steam consumption per h. $p._e$ hour	kg.	**14.1**	**11.5**
Steam consumption in English h. $p._e$ hour	*lb.*	31.4	25.6
Heat consumption per h. $p._e$ hour	Cal.	**11 270**	**9 390**
Heat consumption in English h. $p._e$ hour	*B. t. u.*	45 305	37 748
Temperature of exhaust	*C.°*	**309**	**343**
Temperature of exhaust	*F.°*	588	649
By regeneration, the gained quantity of heat per h. $p._e$ hour	Cal.	**1 415**	**1 340**
By regeneration, the gained quantity of heat per English h p_e hour	*B. t. u.*	5688	5 387
Or in % of total heat	%	**12.5**	**14.3**

* Zeitschr. d. Ver. deutsch. Ing., 1902, p. 783.

† Zeitschr. d. Ver. deutsch. Ing., 1901, p. 1716.

The gain, therefore, where analogous relations exist, would more than pay for a regenerative heating apparatus. Still, it is to be emphasized that the turbine of *Lewicki* ran with too low peripheral velocity, and that the strong superheat of the exhaust was not caused by shocks in the blade channels, but, chiefly, by the retransformation of the exit energy of the steam into heat. If the peripheral velocity is increased, then the superheat of the exhaust would be smaller, and the regenerator can return less heat. We gain relatively more useful work for a given expenditure of heat; the total effect is better if we have the choice between poor (hydraulic) efficiency of the steam turbine and regeneration of a large quantity of heat on the one hand, or good hydraulic efficiency but regeneration of a smaller quantity of heat on the other hand; the latter arrangement would be most efficient.

A third suggestion, that I would like to call a *cycle process with permanent superheat*, is, that we let the highly superheated steam expand isothermally with constant further heating, in order to gain the advantage of adding heat at the highest temperature. If we imagine this process so conducted with superheated steam at 400° C. (752° F.) and 12 atmospheres (176.4 pounds per square inch) pressure, that the final adiabatic expansion at 0.1 kg. per sq. cm. (1.42 pounds per square inch) leads back to the saturated condition, we get as compared to simple superheating to 400° C. (752° F.) an immediate adiabatic expansion to 0.1 kg. (1.42 pounds per square inch), a gain of about 12%. The quantity of heat added along the isothermal is about 30% of the quantity necessary for evaporation and superheating. To our regret, the practical application of these processes to the many-stage turbine is prevented because of the cooling and friction losses in the steam pipes, even with close connection between motor and boiler. Also, the idea of continuously superheating by burning a mixture of gas and air which is gradually imparted to the steam, and so by a *close union of steam and gas turbine* to decrease the heat losses, is proved by closer investigation as impracticable.

The same thoughts are shown in the German patent No. 122 950 (year 1899), representing the idea of the physicist *Pictet, who injected hydrocarbons into a highly heated and compressed mixture of steam and air*, ignited them, and used the products in a piston motor for the performance of work. If *Pictet* works with exhaust,

then his machine is a petroleum motor with water injection ; but if he applies condensation, then the air pump must have such large dimensions that the advantages of the higher initial superheat, which is the chief object of the process, are again neutralized. Here becomes apparent the unpleasant circumstance, that the chief motor must have greater power in order to supply the work of the air pump and compressor, and therefore become correspondingly larger. We have therefore a consumption for the load of running light of the auxiliary machines, and the increased work of running light of the main engine, that again absorbs all advantages, as can be proved by calculation.

Finally, we have in the choice of fluids of high boiling point with the ordinary steam engine process a means of adding heat at high temperatures ; and we shall mention the patents of *A. Siegle* and the steam engine of *Schreber*. The former lets a hydrocarbonate which is difficult to evaporate, for instance solar oil, that evaporates at from 350° to 450° C. (662° to 842° F.), perform work in a steam motor, after which the residual oil-water evaporates in a surface condenser built like a steam boiler, and is utilized as a driving force in the usual way. *Schreber* suggests, as a first trial, *aniline* because of its good thermic attributes ; that is, the advantageous relations of the heats of evaporization and of the liquid. If we wish to utilize the advantages of adding heat at high temperature, this ratio must not be too small. *Schreber* * further emphasizes the necessity formerly overlooked of utilizing the high heat contents of the waste gases of the fuel by economizers, which introduces a further element of complication.

This addition of one or more steps to the ordinary steam engine process seems exceedingly seductive, and the improvement in efficiency greater than for any other of the schemes already mentioned. This steam engine (Mehrstoff-Dampfmaschine) deserves, no doubt, the highest consideration, and it would be worth while to spend more time and money for the purpose of testing it. But we must expect to meet with great difficulties, among which we might mention the determination of the entire composition of this suggested material. Aniline expecially has such highly poisonous

* *Dingler's* Polytechnisches Journal, Nov., 1902. Since then fully discussed in the excellent work, The Theory of the Multiple-fluid Steam Engine (Mehrstoff-Dampfmaschinen), Leipsic, 1903.

properties and such an unspeakably strong odor, that for thi sreason, even chemists doubt the industrial use of this material.

Because the intention of increasing the height of temperature was in part impracticable, and leads to exceedingly long and wearisome experiments, inventive genius has turned to increasing the depth of temperature, and tried by means of a *heat-abstraction machine* or *waste-heat engine* to utilize the last difference between condenser pressure and cooling water temperatures. The process consists of condensing the saturated steam in a surface condenser, in evaporating sulphurous acid and allowing it to perform work in a piston engine. The vapor of the sulphurous acid is then also condensed in a surface condenser by cooling water. The verification of this suggestion lies in the fact that the steam engine generally works with a vacuum which exceeds 0.1 kilogram per square centimeter (1.42 pounds per square inch), and even often exceeds 0.2 kilogram (2.84 pounds per square inch). But these pressures correspond to a temperature of about 45° to 60° C. (113° to 140° F.), while the mean temperature of the cooling water is often 10° to 20° C. (18° to 36° F.) lower. Therefore, theoretically, there is to be gained a temperature drop of 30° to 50° C. (54° to 90° F.), which even by a *Carnot* engine, with, for instance, 180° C. (356° F.) upper temperature limit, would give a gain of $\frac{35}{135}$ and $\frac{60}{120}$, that is, 26% and 50% respectively. We are informed by the announcements of *Josse* as to the progress of the waste-heat engine, and know that here, too, considerable practical constructive difficulties are to be overcome.*

As a matter of fact, it is by no means simple to perfect the condenser arrangements of a steam engine so that it can work continuously with dilutions that have a boiling point of 10° to 15° C. (50° to 59° F.). But if this could be accomplished, then the entire tem-

* There is a waste-heat engine in the power house, Markgrafenstrasse, of the Berlin Electrical Works that has been installed for some time, and is in regular service. According to a report I received from the above works, the engine from Dec. 1, 1901, to May 31, 1902, has been in operation 1507 hours, and has delivered a mean useful power of 91 kw. A large number of machines, with capacities up to 400 h.p., are included in this arrangement; one of 200 h.p. capacity has been since Oct., 1902, in constant practical use. The greatest danger lies in the liability of the condenser leaking, whereby the sulphurous acid is oxidized by the water to sulphuric acid, and the wrought iron parts are so eaten away (for instance, in one night) that it is impossible to use the condenser again. The construction of the stuffing-boxes seems to fill the requirements.

perature drop of the steam engine may be utilized; however, it is impossible, on account of the low temperature, to prevent the increase of condensation in the last cylinder. The steam engine must also sacrifice the last possible element of expansion, for the waste heat engine utilizes it almost entirely. On the other hand, the waste-heat engine finds other applications also; for instance, a gas waste-heat motor being built.

The physicist *Pictet* suggested a peculiar way of lowering the temperature drop of a non-condensing steam engine. He intends to heat compressed air to the temperature of the steam and to mix this with the steam leading to the engine. If the mass-ratio of air to steam is about 2 to 1, then, according to *Pictet*, the latter supplies about $\frac{1}{3}$ of the existing total pressure. If the total pressure were 1 pound per square inch, then the steam has about $\frac{1}{3}$ pound per square inch pressure, and therefore the exhaust into the atmosphere would expand just as much as though a condenser were used. From this *Pictet* concludes that the steam consumption of this non-condensing engine approaches nearly that of a condensing engine. But even if we can see a gain herein, we cannot doubt that the installation of a condenser must give better results.*

* This suggestion of *Pictet*, while it perhaps holds good theoretically, and not considering the practical difficulties, overlooks *two fundamental losses, that can never be avoided in an engine working with a mixture of different vapors or gases.* The mixture of steam and air that can only occur in a steam cylinder (or in a reservoir) because of the otherwise unavoidable rusting of the boiler, may or may not be complete; that is, in some parts complete, in others not. Where it is complete, the steam expands at exhaust to 1 atmosphere, and not to partial pressure, leaving the machine as wet steam at 100° C. temperature, and heats the surrounding air particles to an equal degree. But where the mixing is complete, another loss takes place on account of the increase of entropy of the mixing parts, as each experiences reduction of heat contents. In order to determine this latter loss mathematically, we must investigate the *process of diffusion occurring* at constant pressure, as represented in Fig. 240. At A the steam enters, at B the air; at C the mixture leaves. If we apply to the mixture contained between the sections A, B and C the principle of the conservation of energy, we have the following simple law:

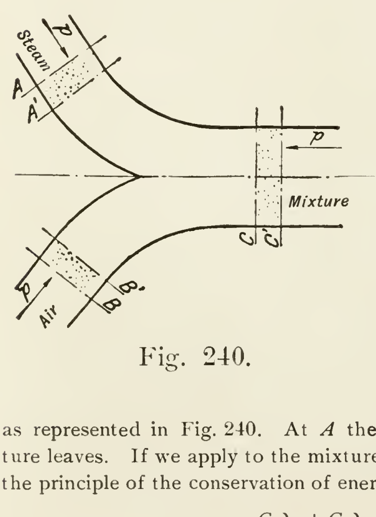

Fig. 240.

$$G_1 \lambda_1 + G_2 \lambda_2 = G_1 \lambda_1' + G_2 \lambda_2',$$

Gas Motors run by producer gas have a decided advantage, as they generally reach consumption figures of 3 200 calories per h. p.$_e$ hour (12 875 B.t.u. per English h.p.$_e$ hour), and have even reached 2 800 calories (11 265 B.t.u.); that is, nearly 23% of the *total possible thermic utilization.* Still, this motor type to-day is limited only to the existing combustibles, namely, coke and anthracite. However, *Deutz* has obtained good results with the brown-coal gas producer, but the production of power-gas from ordinary black coal, our chief combustible, seems not yet fully developed.

Considered thermodynamically, the furthest advanced are finally the motors for fluid combustibles, and in particular those of *Banki* and *Diesel.*

The latter, according to measurements by *Prof. Lundholm,* has reached with a 3-cylinder machine of 120 h.p., a consumption of petroleum of 0.173 kg. per h.p. hour (0.385 pound per English h.p. hour), therefore a heat utilization of 36.8% referred to the effective load. The mechanical efficiency is estimated at 85%, and we see from these figures that the motor, since its first introduction by *Prof. Schröter,* has made considerable progress. The purely thermic process can hardly be improved upon, because of the existing completeness of combustion; still, the mechanical efficiency has been greatly increased by decreasing the size of the auxiliary air pump, and we might expect, from the experience of large steam engines, that this efficiency will be even greater with larger units.

A very enticing suggestion originated with *Friedenthal;* * the

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in which λ^1 is the heat contents of the steam, λ_2 the heat contents of the air $= c_p T$ before the mixture; λ_1', λ_2' the same after the mixture, G_1, G_2 the weight of steam and air respectively. Further, the volumes of the steam and air passing through are equal, then

$$G_1 v_1' = G_2 v_2'.$$

We now calculate the entropy, $S = G_1 s_1 + G_2 s_2$ before the mixing, and the entropy, $S' = G_1 s_1' + G_2 s_2'$ after the mixing. The product of the increase of entropy $S' - S$ and the absolute temperature T_0, that is, $(S' - S)\ T_0$, gives the loss of work occurring due to expansion to the temperature T_0 by mixing. We have then, for $G_1 = 1$, and $G_2 = 2$ at 10 atmospheres absolute initial pressure, saturated steam, and air of equal temperature after diffusion, the part pressure of the steam as 4 3 atmospheres, that of the air 5.7 atmospheres, the common temperature as 446° C. (835° F.) absolute, the increase of entropy, 0 16 units, therefore with expansion to 0° C. (32° F.) a loss of $(S' - S)\ T_0$ = about 44 calories (174.6 B. t. u).

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* Proceedings of the German Physical Society, 1902, Vol. 18.

liquid to be used as fuel is evaporated in a return tubular boiler, made as nearly perfect as possible, at constant volume in order to increase the temperature to the highest attainable maximum (beyond the critical temperature). At this point an expansion is caused in the cylinder of a steam engine, and carried down to atmospheric pressure, whereby *Friedenthal* hopes at the same time to reach atmospheric temperature, or even go below it. If we assume the former, the vapor part of the expanding mixture is conducted to the boiler and there burned, and the liquid part is pumped into the boiler, with the necessary additional supply. The heat generated by combustion must just be sufficient to evaporate the required quantity of liquid. If a fuel is found that will satisfy these conditions, then the entire heat value is transformed into work. Still, this problem stands to-day in a position similar to the processes mentioned in Article 88. We can state the quantity of heat that has been transformed into work if we end the process by condensing the vapor portion through the abstraction of the heat Q_0. The fluid as well as the vapor portions have described an ordinary cycle whose work output is $H' - Q_0$, so long as H' is the heat value of the vapor state of aggregation. But this difference is identical with the heat value of the fluid combustible, and we see that here also the efficiency would $= 1$, if we refer the work gained to the heat value of the fluid combustible which would boil at atmospheric condition, but that the work output remains equal whether we begin with the vapor state of aggregation or with the liquid state. Actually, we have before us a steam engine process with exceedingly high pressure; still, it is immaterial so far as the gain of work is concerned whether we use the combustible or any other fluid as a working body. But this deprives the process of all interest, and it might in addition be mentioned that it cannot be practically carried out, and the assumption that the adiabatic expansion at atmospheric pressure reaches a temperature below that of the surroundings is an impossible one.

Also, the idea originating with *Friedenthal*, of mixing known combustibles, for instance alcohol with water, and by their evaporation, to take up the entire waste heat, contains an assumption that cannot be realized.* We can best see this if we imagine this

* The author also in his first edition made use of the incorrect assumption, and thanks *Prof. Mollier* for drawing attention to the mistake.

process applied to a gas motor, where it must likewise lead to the same goal. Let us imagine, then, a suitable quantity of water evaporated by the remaining gases; then the relation is such that by means of the condensation, at atmospheric pressure in a perfect regenerator the steam quantity leaving per cycle could evaporate the entering quantity of water per cycle. The quantity of heat belonging actually to the combustible gases, and to the surplus of vapor temperature, would only increase the temperature of the new charge from cycle to cycle; a normal condition is therefore impossible. The same is the case if we entirely neglect the water and carry over to the fresh charge the remaining heat by means of a perfect regenerator.

It is different with the process already applied by *Simon*, to utilize the heat of the residual gases for evaporating water at such a pressure that the resulting vapor during expansion will perform work in common with the combustion gases. Here, theoretically, we can be sure of a gain, even if practically it seems doubtful. *Güldner* * thinks that the fundamental thought of such a "vapor-gas-engine" cannot be so easily disregarded; while the temperature drop of the residual gases is relatively small the smaller it is, the better the gas motor works thermally, and would require exceedingly large heating areas. It would be better also to utilize, for vapor production, the heat given up by the cylinder walls to the cooling water; that is, to use the water jacket as a vapor boiler. With the necessary high vapor pressure, the temperature of the walls would be so high that the lubrication of the rubbing surfaces would appear impracticable.

Even if, therefore, these latter suggestions offer no, or only doubtful, prospects, we have reached to-day, thanks to the collaboration of science and practice, some very satisfactory results with motors working with liquid fuel. On the other hand, the utilization of coal, our chief source of energy, is still very imperfect; and even if it were possible to change each type of coal to a gas, it can hardly rival the *Diesel* motor in getting the heat value out of the fuel, on account of losses which would be caused during the process of gas production. There arises, therefore, the legitimate question, whether or not we are following the wrong path, and should give up the actual

* Verbrennungsmotoren, 1903, p 31.

building of motors, and confine ourselves to the problem of the direct production of electricity from coal. In order to discuss this question with authority, I have referred it to the well-known electrochemist, *R. Lorenz* of Zürich, to whom I am indebted for the following statements, which I present in summarized form.

In order that a material in a galvanic element may be useful from an electromotive standpoint, it must go over into solution to a condition of the so-called ions. It has been possible to dissolve coal in fluids, but it is questionable if the solution were in the form of ions ; that is, electrically loaded atoms or atom groups. Because of this, there have not been found any, or only doubtful, electromotive forces. The same is the case of the carbon oxide element, and we may also say here that the electromotive action is doubtful. Besides this, there are other indirect methods that may be used, as, for instance, the suggestion of *Nernst* to transfer the energy of the coal by means of a blast furnace to iron or zinc, and then to consume these metals in galvanic elements. There must therefore be elements constructed in which the mentioned metals must form a reducible salt with carbon. This is the case with the "precipitating elements" discovered by *Lorenz*, but the scientific investigation of which has not been fully completed. Finally, we might in an indirect way utilize the decrease of electromotive force with the temperature, in the reversible galvanic chain (accumulators) in such a manner that we might charge a highly heated element of low voltage during the addition of heat, and discharge it at high voltage after being cooled, and while heat is being abstracted. The difference between the added and subtracted quantity of heat would, according to *Carnot's* law, be transformed into electrical energy. Even elements working in molten electrolytes would only be useful in the region of 500° to 860° C. (932° to 1580° F.), which corresponds to a theoretical efficiency of about 35% ; but here the quantity of heat necessary to heat and cool the element, as compared to the useful heat, is so great that the unavoidable losses must strongly influence the efficiency. Therefore a union with other elements is necessary, in order to utilize the drop of temperature to that of the surroundings, and also to utilize the heat contents of the fire gas of the first process, leaving at about 900° C. (1652° F.). Even for the first experiment, there is needed highly complicated and extensive apparatus.

If I understand correctly the very noteworthy reports of *Mr. Lorenz*, there is still a series of preliminary problems awaiting solution before the main problem of direct transformation can be effected; but indirect transformation requires extended and complicated installations, without, so far as can be seen, promising any useful gain.

There is, therefore, no immediate danger threatened to motor building; but we are wholly thrown on our own resources in defending our position. For many eyes are directed to a motor which will combine the high thermal results of the gas engine with the constructive advantages of the steam turbine, and for these reasons will now be briefly discussed. It is the gas turbine.

91. THE GAS TURBINE.

The work process that naturally suggests itself for a gas turbine is as follows: gas and air are separately compressed by means of a compressor to a more or less high pressure, burned in a chamber at constant pressure and then led directly to the turbine. The system of the turbine is theoretically a matter of indifference; the expansion is first carried on to atmospheric pressure. This process corresponds to the well-known cycle of *Brayton*, of which gas motor theory proved that exactly the same thermal efficiency at assumed constant specific heat existed there as in the ordinary explosion process, provided the final pressure of compression of the latter is as high as the combustion pressure of *Brayton*. *The ideal gas turbine would therefore give the same economy as the ideal four-cycle motor*, and there remains only the question of how large in each case the losses of work and of cooling become in practical construction. The work of compression for gas and air is equally large on the average, and the needed consumption of power for this purpose is not much different, if we consider that the rods of the turbine compressor are light, but require a gear wheel arrangement. The remaining losses of work of the piston motor might be somewhat less.

We must, on account of the high temperatures, use turbines with a single pressure stage, hence nozzle turbines. If we calculate on equally high cooling losses, the same energy remains at our disposal. In the gas motor the energy appears as the actual indicated

work of which we lose in large units about 20% because of resistances during suction and exhaust, as well as the engine resistance under friction load, receiving thereby 80% as effective work at the shaft. In the steam turbine we must subtract the nozzle and the blade frictions, the losses of exhaust, and the wheel friction to get the effective power. The sum of these losses is in the well-known single stage steam turbine, more than 40%, and the gas turbine has the advantage that its cooling losses may become smaller than those of the gas motor, if we are able to isolate the combustion chamber internally so that a water cooling may be omitted. But now comes the cardinal difficulty, that a work of this type results in very high temperatures at the end of expansion, through which the life of the wheel blades is threatened. If we mix with the air for combustion atomized water which must be evaporated, then the temperature can be kept lower; but the efficiency drops in the same ratio.* The utilization of the waste heat for evaporating the injected water in order to save the latent heat was here made use of; but in all, it seems questionable whether a gas engine of the described type could enter into favorable competition with the piston motor.

It is much different with the scheme already suggested to combine the turbine with the ordinary four-cycle motor in such a manner that the explosion gas is led to the turbine during the expansion period, and at the same time performs work in the cylinder. We could, no doubt, carry the expansion to atmospheric pressure and seemingly reach without trouble what the compound gas motor, because of the decided cooling of the working gases, formerly tried in vain to accomplish. The theoretical gain contrasts strongly with the poor utilization of expansion work in the turbine, for the increased losses in the nozzle must work with greatly varying pressure ratios. Furthermore, the intermittent working is for many reasons unfavorable, while the difficulties with the temperature also remain, as in the equal pressure turbine.

An advance in the thermal utilization of heat will not be brought about by the gas turbine; still, it will receive much consideration on account of the prospects offered for the utilization of the fuel on which, until now, steam engineering depends. The tar

* See the complete calculation of *Lorenz* in Zeitschr. d. Ver. deutsch. Ing., 1900. p. 252, which will give approximately the same results for varying specific heats.

and asphaltum type of substances that are formed during the production of gas from bituminous coal, and make the working of gas motor impossible, can, with a steam turbine, be burned in an enclosed generator under pressure, and thereby rendered harmless. As the steam turbine, without bringing an actual betterment of steam economy, has entered into the industry because of its constructive simplicity, so will it be with a gas turbine, which is constructively simpler than the gas motor, provided it will only exceed the steam motors in efficiency. The constructive difficulties that have to be overcome in a large gas motor because of the immense piston pressures and heat expansion of the complicated cylinder heads (cracks galore!) are well known. A safe gas turbine would in this respect be an improvement. As is known, experiments are in progress to supplant the piston compressor by a rotating one, as, for instance, in Parsons' patent, who takes his turbine for this purpose with reversible flow and direction of rotation. The first designs did not promise much, because Parsons with only 1.4 atmosphere (20.6 pounds per square inch) gauge pressure reached only 60% efficiency. Our experiments also show that the compression of flowing vapors or gases occurs with greater resistances than the expansion. The constructive simplicity of such an arrangement could not be exceeded; but only when the mechanical efficiency of the compression reaches an exceedingly good value, and when the utilization of the flow energy in a turbine has been considerably increased, or when substances have been found of sufficient stability beyond red heat, will the gas turbine be taken up in the industrial world.

The interest which the motor occasions will permit us to give the following short mathematical discussion.

92. CALCULATION OF A UNIFORM PRESSURE GAS TURBINE.

The calculation of a gas turbine by a graphical analytical method is exceedingly simple, Fig. 241, if we allow ourselves to consider the specific heats of the gas to be constant for the construction of the adiabatics, while their changeability is taken into account during constant pressure. We draw in one pound (kg.) of the gas and air mixture suitable for combustion, and compress the

same either according to the adiabatic AB or as in an ideal compressor, according to the isothermal, AB_1. The contents of the areas $ABB'A'$ and $AB_1B'A'$ respectively are the indicated compression work, and are measured in foot pounds (meter kilogram) and designated by L_k and L_k' respectively. For these, *only as an example of the way in which this calculation is to be regarded,* we make the further assumption, that the mixture in its passage to the combustion chamber at adiabatic compression, cools to the initial temperature, T_1. We will imagine the heat of combustion as a quantity Q_1, added externally, and refer to the fact that according to the *Hütte,** in the presence of the theoretical quantity of air, the quantity of heat developed per cubic meter of the mixture varies very little for different kinds of gas; as, for instance, Dowsen gas has about 600, and illuminating gas about 800 calories; and correspondingly less with a surplus of air. Of this heat value of the mixture, H heat units per cubic feet (cu. m.), there is lost ζH through cooling; and there remains

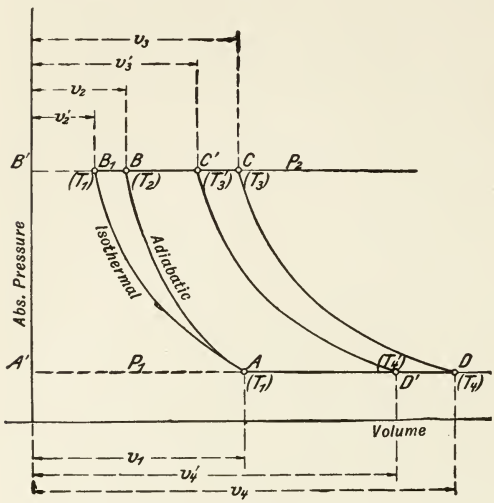

Fig. 241.

$$H' = (1 - \zeta) H$$

so that

$$Q_1 = H' v_1 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

where v_1 is the volume in cubic feet (cu. m.) at the pressure of 1 pound per square inch (1 kg. per sq. cm.) ($=A'A$), because H' is also referred thereto. The final temperature, T_3, we get from the relation

$$Q_1 = c_p (T_3 - T_1) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

* Engineer's Pocketbook " Hütte," published by the Academic Society, " Hütte," of Berlin, Germany.

where $\bar{c}_p$ stands for the mean value of the specific heat for the temperature interval T_1 to T_3. The corresponding volume $B' C = v_3$ we find from the equation

$$v_3 = v_2' \frac{T_3}{T_1}.$$

The adiabatic expansion CD gives the work area $A'B'CD$, from which the theoretical exit velocity c at the nozzle exit can be determined from

$$\frac{c^2}{2g} = \int_{p_1}^{p_2} v\,dp = \text{area } A'B'CD \quad . \quad . \quad . \quad . \quad (3)$$

Now by drawing the velocity diagram and approximating the wheel friction, as was explained with the single stage steam turbine, determine the efficiency η_w and the power that is delivered at the shaft $= L_w$. Let area $A' B' C D$ be represented by L^0; then

$$L_w = L_0\,\eta_w \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

From this work there must be subtracted the power necessary to drive the compressor; that is, $\frac{L_k}{\eta_k}$ where η_k is the mechanical efficiency of the compressor. The effective power, L_e, is therefore

$$L_e = \eta_w L_0 - \frac{1}{\eta_k} L_k \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

The consumption of heat is Q_1 in heat units, or $\frac{Q_1}{A}$ in foot-lbs. (m. kg.); therefore, the total efficiency of the "dry" working turbine is

$$\eta_0 = \frac{A\,L_e}{Q_1} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

We now inject into the combustion chamber, per pound (kg.) of the gas mixture, y pounds (kg.) of water, which evaporates, and is superheated to the temperature T_3', for whose calculation the following equation will serve:

$$\bar{c}_p\,(T_3 - T_3') = y\,[q_3 - q_0 + r_3 + c'\,(t_3' - \tau_3)\,] \quad . \quad . \quad (7)$$

In this, q_3 is the heat of the liquid corresponding to the partial pressure p_3 of the vapor, r_3 is the external heat of vaporization, τ_3 the temperature of saturation, $c' = 0.48$ of the specific heat, q_0 the initial heat of the liquid. We find from the equation of condition approximately

$$p_3 = p_2\,\frac{47\,y}{29.3 + 47\,y} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

if we place for the gases of combustion the constants for air. The volume of the total mixture decreases to

$$v_3' = \frac{T_3'}{T_3}\,v'_3 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (9)$$

The exponent of the adiabatic $C'D'$ for constant specific heat can easily be calculated, and is

$$k' = \frac{c_{1p} + y c_{2p}}{c_{1v} + y c_{2v}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (10)$$

in which subscript 1 refers to the mixture, and subscript 2 to the vapor. With these values temperature T_4' may be approximately calculated, and is the temperature existing in the turbine chamber, and is important on account of the life of the blades. The work is

$$L_0' = A'B'C'D' \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (11)$$

corresponding to the work of the total weight, $1 + y$ pounds (kg.), and gives the new (theoretical) exit velocity c' by the formula

$$(1 + y)\,\frac{c'^2}{2\,g} = L_0' \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (12)$$

The delivered work at the shaft is the power of the turbine wheel, and is

$$L_w' = \eta_w'\,L_0' \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (13)$$

and the new effective power is

$$L_e' = \eta_w' L_0' - \frac{1}{\eta_k} L_k \quad . \quad . \quad . \quad . \quad . \quad . \quad (14)$$

Finally, the total efficiency is

$$\eta_0' = \frac{A L_e'}{Q_1} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (15)$$

CALCULATION BY MEANS OF THE HEAT CONTENTS.

The heat contents of a gas for constant pressure is, according to the general formula 1*a*, Article 14, per unit weight of gas,

$$J = u + A\,pv = \bar{c}_v\, T + A R T = (\bar{c}_v + A R)\; T = \bar{c}_p\, T. \quad (16)$$

in which $\bar{c}_p$ is the mean value of the specific heat between the absolute zero temperature and T. With this value, the various phases of the gas turbine processes are exceedingly easy to follow.

We shall imagine the temperature of all places to have been calculated, and designate the heat contents at the points A, B, B_1, C, D, by using these letters as subscripts to J, as J_A, J_B, J_{B_1}, J_C, J_D, etc. Then follows from the consideration of the cycle process $A'\,B'\,BA$, the indicated compression work in heat units under adiabatic compression, to which we will confine ourselves,

$$A L_k = J_B - J_A \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (17)$$

We shall again assume that the mixture compressed to T_1 (point B_1) is cooled, because $A\,B_1$ is an isothermal, we have

$$J_{B_1} = J_A \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (18)$$

a. WITHOUT WATER INJECTION.

The heat added by the combustion is

$$Q_1 = J_C - J_{B_1} \cdot \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (19)$$

from which J_c is determined. The available work of the turbine is

$$A\,L_0 = J_C - J_D \quad \ldots\ldots\ldots \quad (20)$$

If we let Q_2 be the quantity of heat that has been extracted from the products of combustion along the path $D\,A$, that is,

$$Q_2 = J_D - J_A,$$

then we can also let the available work be

$$A\,L_0 = (J_C - J_{B_1}) - (J_D - J_{B_1}),$$

or place J_{B_1} for J_A in the second parenthesis ; and, as is immediately apparent,

$$A\,L_0 = Q_1 - Q_2 \quad \ldots\ldots\ldots \quad (20a)$$

The theoretical exit velocity c is found from the equation

$$A\,\frac{c^2}{2\,g} = J_C - J_D = \bar{c}_p\,(T_3 - T_4) \quad \ldots \quad (21)$$

The effective power in heat units is

$$A\,L_e = (Q_1 - Q_2)\,\eta_w - \frac{A\,L_k}{\eta_k} \quad \ldots\ldots \quad (22)$$

The total efficiency is (with adiabatic compression)

$$\eta_0 = \frac{(Q_1 - Q_2)\,\eta_w - A\,\dfrac{L_k}{\eta_k}}{Q_1} = \frac{Q_1 - Q_2}{Q_1}\,\eta_w - \frac{A\,L_k}{Q_1}\,\frac{1}{\eta_k} \quad (23)$$

β. WITH WATER INJECTION.

T_3' is to be taken according to formula 7 and T_4' for adiabatic expansion with exponents as in equation 10. The partial pressure p_4 of the vapor at point D' is calculated by the formula

$$p_4 = p_1\,\frac{47y}{29.3 + 47y} \quad \ldots\ldots\ldots \quad (24)$$

and serves for the determination of the quantity of heat Q_2, which is abstracted from the mixture in order to cool it from condition D' to condition A. It is

$$Q_2 = y\,[0.48\,(t_4' - \tau_4) + r_4 + q_4 - q_1] + \overline{c_p}\,(t_4' - t_1)\,. \quad (25)$$

In this τ_4 is the temperature of saturation corresponding to p_4, and r_4 and q_4 are the external heat and the heat of the liquid ; q_1 is the heat of the liquid at temperature t_1, and we shall assume that $t_1 < \tau_4 < t_4'$, which generally occurs. In this we may neglect the slight remaining quantity of water in vapor form at temperature t_1.

We now have

$$A\,L_0 = Q_1 - Q_2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (26)$$

$$A\,(1 + y)\frac{c'^2}{2g} = Q_1 - Q_2 \quad . \quad . \quad . \quad . \quad . \quad . \quad (27)$$

$$A\,L_e' = \eta_w\,(Q_1 - Q_2) - \frac{1}{\eta_k} A\,L_k \quad . \quad . \quad . \quad . \quad . \quad (28)$$

$$\eta_0' = \eta_w \frac{Q_1 - Q_2}{Q_1} - \frac{1}{\eta_k}\frac{A\,L_k}{Q_1} \quad . \quad . \quad . \quad . \quad . \quad . \quad (29)$$

By means of these equations, we may follow the changes of efficiency when the chosen pressure of compression and the quantity of injected water change.

We can easily derive the formulæ for the case when the water is partially evaporated by the waste gases at the pressure p_2, and then the vapor introduced into the combustion chamber. This method, though but slightly flexible, is more favorable than, perhaps, the method of decreasing the temperature by increasing the excess of air, because the air must be already compressed, while the water is in a fluid condition, hence must be forced into the heating body with negligibly small consumption of work.

The efficiency increases with the pressure of compression to a certain limit, and then again decreases ; but possesses with the usual assumptions for η_w and η_k, unsatisfactorily small values.

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